

The Association for Mathematics Education of South Africa



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30 June - 4 July 2025

University of Limpopo
(Turfloop Campus)
Polokwane
Limpopo Province

Editors: Paul Mutodi, Dina Mamashela, Ranta Langa, Thalangwini Thikathali, Siphwe Sihlangu



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Foreword

Mathematics serves as a gateway to majority careers, especially in Natural Sciences, Engineering, Technology and Accountancy. Access and acquittance in these fields rely on competence in both basic and advanced mathematical skills and knowledge. Equitable mathematics education ensures that all students, regardless of race, gender, socioeconomic status, language, or ability, have access to meaningful and empowering math learning experiences. The goal is to make math relevant, engaging, and liberatory and not just procedural or performative. In order to promote equitable mathematics education, it is essential to prioritise relevance and engagement in the teaching and learning process. This means providing opportunities for students to see the practical applications of mathematics in their everyday lives and to engage with the material in meaningful ways that resonate with their interests and experiences.

One way to promote relevance is to incorporate real-world examples and contexts into the mathematics curriculum. This could involve using data and statistics from current events, discussing how mathematics is used in various professions, or exploring historical contributions of underrepresented groups to the field of mathematics. By connecting the material to students' lives and interests, educators can help them see the value and importance of mathematical concepts.

Engagement can also be fostered through active and collaborative learning experiences. This could involve group projects, hands-on activities, and interactive technology tools that allow students to explore and apply mathematical concepts in a more dynamic and engaging way. By creating a classroom environment that encourages curiosity, exploration, and creativity, educators can help students develop a deeper understanding and appreciation for mathematics.

Overall, by promoting relevance and engagement in mathematics education, educators can create a more inclusive and equitable learning environment where all students have the opportunity to succeed and thrive. With a focus on connecting mathematics to real-world contexts and fostering active engagement in the learning process, we can help students develop the critical thinking skills and problem-solving abilities needed to navigate our increasingly complex and data-driven world. The current theme, ‘Envisioning Equitable Mathematics Education: Promoting Relevance and Engagement’ was conceptualised against this background.

One important way to foster relevance and engagement in mathematics education is through real-world applications. Studies have indicated that students are more inclined to connect with and comprehend the relevance of mathematical topics when they can see how they apply in real-world settings. For instance, instructors may demonstrate to their students how mathematics is applied in real-world situations by using examples from areas like finance, engineering, and medicine.

Teachers can encourage greater engagement with the subject by giving students more freedom and opportunities to investigate mathematical concepts independently. For instance, instructors may use project-based learning exercises that enable students to use mathematical principles to solve real-world issues and problems.

Promoting relevance and engagement in South African mathematics classrooms is crucial for enhancing learner performance, decreasing dropout rates, and making math more meaningful by



connecting curriculum content to students' real-life experiences and future aspirations, thereby fostering intrinsic motivation and active participation. A number of presenters are engaging with some of these issues that arise from those and strategies that can be used to address them. Our panellists and plenary speakers were also invited to address issues affecting quality mathematics teaching and learning from various perspectives that include higher mathematics learning, teacher education, international perspective, curricular issues and learners support material. The workshops that form part of these proceedings offer hands- on experiences through which facilitators share successful strategies and activities that they have successfully developed to improve the quality of mathematics teaching and learning in different phases and context.

This congress, like any other, will hopefully inspire you to take up the challenges of quality mathematics teaching and learning beyond the five days in which the activities unfold. You will continuously use the two volumes as a critical resource material in your classrooms, seminars, workshops, planning sessions, and so on.

Lastly, we remain thankful for the presenters and their reviewers for their immense contribution to the 30th Annual National Congress of the Association for Mathematics Education of South Africa. We are forever inspired! Enjoy!



Review Process

The papers accepted for publication for the 2025 AMESA Congress were subjected to blind peer review by at least two experienced mathematics education reviewers. The academic committee considered the reviews and made a final decision on the acceptance or rejection of each submission, as well as changing the status of submission. Authors of accepted submission were given the option of not to have their accepted long papers published in the AMESA 2025 Proceedings, to keep open the possibility to submit it to a journal. They were requested to submit an extended abstract rather than their full submission, and this extended abstract will be published in the Proceedings for publication.

Number of submissions: 100

Number of plenary paper submissions: 6

Number of long paper submissions: 34

Number of short paper submissions: 5

Number of workshop submissions: 24

Number of 'How I teach' paper submissions: 14

Number of poster submissions: 1

Number of Maths Market: 20

Number of submissions accepted: 96

Number of submissions rejected: 4

Number of submissions withdrawn by authors: 2

We thank the reviewers for giving their time and expertise to reviewing the submissions.

Reviewers:

Benadette Aineamani	Chipoi Mangwiro	Benjamin Tatira
Alex Jogymol	Mildret Ncube	Msebenzi Rabaza
Mogege Mosimege	Wisani Hlungwani	Tšhegofatšo Makgakga
Anthony Essien	Hlengiwe Mhlungu	Nel Maboko
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Vasuthavan Govender	Ranoha Moosa	Morongoa Tau
Jojo Zingiswa	Paul Mutodi	Aaron Tau
Piera Biccard	Kgaladi Maphutha	
James Sibanda	Duncan Mhakure	
Eva Makwakwa	Dimakatso Muthelo	



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ENVISIONING EQUITABLE MATHEMATICS EDUCATION: PROMOTING RELEVANCE AND ENGAGEMENT

Satsope Maoto

University of Limpopo, South Africa

This plenary address attempts to make sense of the theme: “Envisioning equitable mathematics education: Promoting relevance and engagement” within issues of access, marginalisation, learner identity, power, social justice, decolonised society. Equity and engagement in mathematics education are complex issues. Although these two concepts are equally important, the argument, in this paper, is skewed to tackling the question: how does equity play out in a mathematics learning environment? It is hoped that issues relating to relevance and engagement would emerge within the discussion. Four mathematical tasks are used to illustrate whether it is possible or impossible to address the dimensions of equity equally or fully in any given situation.

Key words: mathematics education; equitable; relevance; engagement

INTRODUCTION

Teachers and learners of mathematics are much more complex; they cannot leave their social constitution behind them. Thus, their historic, social, political, and economic contexts cannot be split off, and discarded allowing the cognitive to function alone (Klein, 2005). The knowledge that we produce as researchers and policies that policy makers conclude on are situated and thus, would remain partial, locatable, and critical. There is the recognition that there is no one truth or right way to do things. Knowing this, researchers are open to new possibilities and the idea of small, locatable yet potentially significant change.

WHAT IS MATHEMATICS EDUCATION?

A simplistic view of mathematics education is that it is classroom based because it speaks to teaching, learning and assessment – this is where mathematics education originated. It was during this early origin stage that much of mathematics education research concentrated on describing and analysing the individual, intellectual agency of students/learners in diverse mathematical contexts. Valero (2004a) argued that such research has constructed a view of the learner as a “schizomathematicslearner”. She concluded that such a discursive object portrays students/learners as mathematical cognitive agents, decontextualized from the social, historical, political, and cultural arenas where they exist. The focus and interest in understanding one aspect of students’ thinking has almost eliminated the other components of students as fully real, living, and acting human beings. Valero (2004a) argued that the notion of cognitive, intellectual agency has to be encompassed with a notion of political agency understood as the students’ action in complex social situations where mathematical initiative



is one of the multiple possible ways of influencing their life conditions. An interesting challenge for research, then, was to find ways to enlarge the notion of agency in order to connect the micro-context of the mathematics classroom with larger context of action in which students participate (and where exclusion/inclusion is also in operation). In other words, the challenge was to link the individual learner (and his/her intellectual agency in mathematics) with his/her larger social setting, within which disadvantage on the grounds of race and ethnicity has been historically constituted (Valero, 2004b).

We have passed the origin stage of mathematics education, hence, it is our responsibility to envision equitable mathematics education, among others, by promoting relevance and engagement. If mathematics education was limited to teaching, learning and assessment, it would not impact curriculum reform like the introduction of the **‘Mathematics Teaching and Learning Framework for South Africa: Teaching Mathematics for Understanding’**, (Department of Basic Education (DBE), 2018). The framework was developed to present new thinking about mathematics teaching, learning and assessments and intended to help teachers implement the South African Curriculum and Assessment Policy Statement (CAPS) in ways that support understanding (DBE, 2018). The Framework model and the supporting exemplars are provided to offer guidance to teachers to transform their teaching. Successful implementation – also at the core of mathematics education research – depends on national (and sometimes international) comparative assessments, technology, and economics. Such are driven by critical issues on mathematics education which in South Africa are inclusive of relevance of mathematics to context. Inclusive in this context is real life application of mathematical models. This is helpful in breaking the critical neutrality of statistical outcomes inferences that are dependent of tested approaches without bringing in new variables that can yield outcomes that are relevant to engagement of mathematics education with social justice and democracy. This component of mathematics education if overlooked strips it off its identity as addressing socio and political issues – where issues of power come into play. Mathematics education is not just about power, it is also about practices among which are how matric results are analysed. Mathematics education has evolved to address the social ills of societies.

EQUITABLE MATHEMATICS EDUCATION

There has been a growing focus on equity-related research in mathematics education practices, theories, curricula and policies (Allestaht-Snyder & Hart, 2001; Gutiérrez, 2011; Vithal, Brodie & Subbaye, 2024; Lee, & Herner-Patnode, 2025). The argument was that teachers work with students in individual classrooms but cannot work in isolation to accomplish broadly conceived goals for equity in mathematics education. Administrators, teacher educators, researchers, and policymakers must all understand the issues, take initiative, and commit to supporting students and teachers in a collaborative effort to address the complex issues involved in achieving mathematics for all. Although some progress has been observed in research on equity in mathematics education, there are still large inequalities within and across countries and that these have real consequences for many of the poorest and most vulnerable learners (Vithal, Brodie & Subbaye, 2024).



Since equity in mathematics education is a complex issue, the definition of equity begins with two premises (Alleksaht-Snider & Hart, 2001). Firstly, all students, regardless of their race, ethnicity, class, gender, or language proficiency, will learn and use mathematics. Secondly, all people who are involved with and interested in the education of children must become aware of the social, economic, and political contexts of schooling that can either hinder or facilitate mathematics learning for underrepresented students. Equity means fairness, not sameness (Gutiérrez, 2011). So, when we look for evidence that we are achieving equity, we should not expect to find that everyone ends up in the same place. Equity in mathematics education requires: (a) equitable distribution of resources to schools, students, and teachers, (b) equitable quality of instruction, and (c) equitable outcomes for students (Alleksaht-Snider & Hart, 2001). Equity is achieved when differences among subgroups of students in these three areas are decreasing or disappearing (ibid.). Teachers' knowledge of mathematics, their preparation to teach mathematics, and skills for teaching diverse students are all aspects of equitable instruction. Mathematics achievement, course enrolment, interest, motivation, and valuing of mathematics are all important types of student outcomes linked to equity in student access to mathematics education.

Discussions on equity centre around four dimensions: access, achievement, identity, and power (Gutiérrez, 2011; Moschkovich, 2013), which are interrelated. Put by Vithal, Brodie and Subbaye (2024), access refers to the resources available to support mathematics learning; achievement refers to student outcomes, which are crucial in determining students' futures; identity refers to the holistic growth of learners as social and cultural beings that mathematics education can and should promote; and power refers to how social relations between different hierarchies play out in mathematics classrooms. Using these four dimensions, equitable mathematics education should aim for access (the resources available to students), achievement (assessment outcomes), identity (connecting to students' social and cultural backgrounds), and power (how students can use mathematics to improve the world) (Lee, & Herner-Patnode, 2025). However, Gutiérrez (2011) cautions that although these four dimensions are necessary for true equity, they are not going to be equally or fully present in any given situation. For example, teachers cannot be expected to address power issues every day in the classroom in ways that are meaningful to every student. Similarly, when identity or power issues are being brought to the surface, at times the connection to mastering dominant mathematics may take a lower priority. The goal is to attend to and measure all four dimensions over time (ibid.).

RELEVANCE AND ENGAGEMENT IN MATHEMATICS EDUCATION

There are studies that have shown that when teachers carefully select relevant mathematics content that connects to students' lived experiences, students become encouraged to engage with the concepts at hand (Abdulrahim & Orosco, 2020; Tinto, 2023). Viewed by Tinto (2023), students who are more engaged are, on average, more likely to succeed. He considers the phrase “on average” as the key term and argues that not all students who engage succeed. Nor do all students who do not engage fail. This is the case because engagement is more complex and varied than typically recognised (ibid.). Earlier research viewed engagement as having three dimensions: behavioural, emotional/affective and cognitive (Santos, & Barmby, 2010). Wang,



et al. (2020) added the fourth characteristic: social and argued that these four interrelated-yet-distinct dimensions make engagement a unique construct. Behavioural engagement is about the active participation in learning activities, persistence, and effort, emotional engagement is the students' attitudes (e.g. perceived value, interest in) towards the activities, cognitive engagement can be seen as the 'psychological investment' (Santos, & Barmby, 2010), attention, willingness to go beyond what is required, and employing the necessary learning strategies for the comprehension of complex ideas; and social engagement includes the quality of social interactions with others, as well as the act of forming, maintaining, and enjoying relationships while learning. Thus, engagement is a dynamic and reciprocal developmental process between context, self, and relevant outcomes that serves as a pathway to learning (Wang, et al., 2020).

Not all engagements are seen by students as positive. When that is the case, they can undermine persistence. Conversely, when engagements are seen as positive, they can promote persistence. Argued by Lee and Herner-Patnode (2025), increased student engagement leads to improved academic outcomes and enhanced self-efficacy, enabling students to use mathematical language effectively and appreciate the significance of mathematical concepts in daily life. Students who are given opportunities to explore relevant issues can develop confidence in their mathematical abilities.

CREATING AN EQUITABLE LEARNING ENVIRONMENT

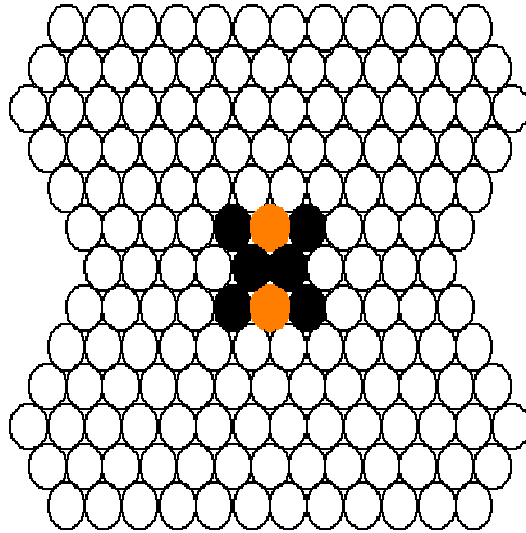
Drawing from how a concept of equity is defined, an equitable learning environment could be described as where all students are accepted, valued, and empowered to achieve academic success and ultimately understand the importance of and can demonstrate empathy to other people (Bayles & Morrell, 2018). Creating a learning environment that is inclusive, culturally responsive, and caring requires a pedagogy that encourages multiple student perspectives, values, and beliefs to be shared openly, safely and respectfully. In a mathematics learning environment, both teachers and students bring with them their experiences, cultural understanding, beliefs, and values. It remains the responsibility of the teacher to shape, normalise, provide access to the learning environment to the extent where all students take responsibility for their own learning and self-assessment. The teacher's responsibility, also, remains to formulate equitable mathematical tasks that learners with a wide range of abilities can access. Such tasks should nurture logical and mathematical reasoning, creativity and imagination, critical thinking, and are open to multiple interpretation and extendable (Maoto et al. 2023). They have the potential to be differentiated to accommodate a variety of students, and modifiable to accommodate varied concepts. They are powerful in broadening students' problem-solving skills through understanding of underlying mathematical concepts and principles, as well as making connections between them. If well designed and appropriately implemented, such tasks could contribute largely to students' engagement (ibid.). Students are to be provided with opportunities to experience learning relevant to their lives, engage in rich discussions, freely inquire, and safely and respectfully challenge authority when necessary (Bayles & Morrell, 2018).



Illustrative mathematical tasks

Activity 1

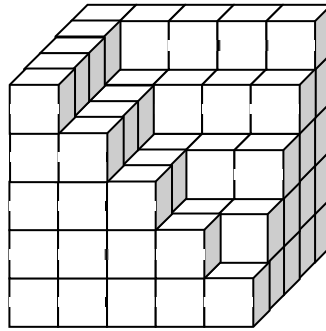
Study the pattern below carefully and respond to the accompanying questions.



- Create a pattern similar to the one on the left on the grid building provided on the right.
- Explain in layman's language how the pattern grows.
- How many black beads will you need to build the 4th pattern?
- How many white beads will you need to build the 4th pattern?
- How many beads will you need to build the 4th pattern?
- Show other ways in which you could have found the number of beads in 5 above.
- Which of those ways do you prefer and why?
- How many black beads will you need to build the 10th pattern?
- How many white beads will you need to build the 10th pattern?
- How many beads will you need to build the 10th pattern?
- Reflect on how you solved h); i) and j) above. State those critical steps or issues that had to resolve in the process of determining the solution.

Activity 2

How many cubes are there in this construction? Show your calculation.



Activity 3

A contractor who does not meet the deadline on the construction of a building is fined R800 per day for each of the first 10 days of extra time, and for each additional day thereafter the fine is increased by R160 each day. If the contractor is fined R20 160, by how many extra days was the construction time delayed?

- a) First analyse the activity and decide how it could be solved.
- b) Work out the activity and show the procedure you followed to arrive at the answer.

Activity 4

In how many ways can five people sit around a circular table?

Reflecting on the four illustrative mathematical tasks

- 1) Analyse each task and comment on whether it does/does not address the dimensions of equity (access, achievement, identity, and power). Provide reasons for your responses.
- 2) In a case where the task does not address equity, how could it be improved such that it becomes an equitable task?
- 3) What should the teacher do to create an equitable learning environment for the success of each of these four illustrative tasks?

The argument by Gutiérrez (2013) must be considered that the meaning of a mathematical concept cannot be extracted from the meaning of the mathematical task that is presented to and interpreted by students. However, being open to the multiple meanings that students place on mathematical practices and offering an educational setting where those meanings can be valued and built upon is a step in the right direction. It is thus important to pay attention to the views of both teachers and students, as they offer a critique of what has been normalised in school. In this way, we open the possibility not just for teaching mathematics in more equitable ways, but also for a radical reform in mathematics (ibid.).

CONCLUDING THOUGHTS



For a long time, we have blamed teachers, students, the curriculum, and even the system for inequities in mathematics education. Rarely do we consider that we (the mathematics education research community) could be part of the reason why very little in the way of significant changes ever arise in the kinds of achievement, participation, and relationships with mathematics that so many oppressed/disadvantaged students experience. In equitable mathematics learning environments, learning encompasses more than socialising students into particular mathematical practices; learning also includes changes to student identities and power, what Gutiérrez (2013) refers to as “*sociopolitical turn*”. Thus, when analysing opportunities to learn, we should consider not only students’ access to mathematical content and discourse practices, but also their access to positional identities as knowers and doers of mathematics (Esmonde 2009). Of course, participation in mathematical discourse practices may influence one’s identity in the classroom and one’s identity may reciprocally influence how one participates in the group.

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FROM CLASSROOM PRACTICE TO SYSTEMIC SUPPORT: ENVISIONING EQUITABLE MATHEMATICS EDUCATION IN LIMPOPO'S FOUNDATION PHASE

Rebecca Maluleke

Limpopo Province Department of Basic Education

Abstract

This paper reflects on the theme of the 2025 AMESA Congress—“*Envisioning Equitable Mathematics Education: Promoting Relevance and Engagement*”—from the dual perspectives of a Foundation Phase mathematics teacher and a Subject Advisor in Limpopo Province. Drawing on practical examples and systemic observations, it explores how equitable, relevant, and engaging mathematics instruction can be achieved in rural contexts. The paper identifies key challenges—including insufficient teacher training, underuse of resources, and overcrowded classrooms—and offers recommendations grounded in both classroom experience and district-level support.

Background

Trends in International Mathematics and Science Study (TIMSS) in 2019 indicates that 37% of South African learners acquired basic mathematical knowledge and a lower 28% in grade 5. TIMSS also outlines that 63% of learners had not acquired basic mathematical knowledge. This means South Africa still has a way to go to improve mathematics basic knowledge. In addition, the 2011 DBE survey measured curriculum coverage (volume of work covered) by counting the number of exercises executed over a specified period, per subject (Mathematics Grade 6 and Grade 9). Nationally, 31% of Grade 6 learners met the minimum standard of four Mathematics exercises per week. The average number of written Mathematics exercises nationally was 2.5 (DBE School Monitoring Survey 2011)

For Grade 9 learners, nationally, 91% of learners did not meet the minimum standard of four Mathematics exercises per week and the average number of written Mathematics exercises was only 1.8 exercises per week (DBE School Monitoring Survey 2011). With the introduction of the Curriculum and Assessment Policy Statement (CAPS) in 2012, better guidance is provided in terms of curriculum coverage throughout the year. However, compliance to curriculum coverage at times still poses a challenge to the DBE. It is in the linear nature of MST subjects that if the required number of topics in a learning programme is not covered, learning of the topic will, in subsequent years, be even more challenging for the learner.

The accumulated deficit will therefore widen as the learner progresses through the grades, because the basics were not covered sufficiently well or not covered at all. It is therefore crucial that learners are exposed to and master the full curriculum each year to build competence to manage these subjects in subsequent grades. In the district we have 373 schools (226 Primary, 152 Secondary and 3 Special schools). Out of 373 schools, 340 schools are in quintiles 1-3. The district is generally rural based. We experience more socio-economic challenges like less parental involvement to support learning as most learners come from child headed families. There are also other the socio-economic challenges: Poverty, unemployment, drug abuse,



gender-based violence, teenage pregnancy and the children that are cared for by their grandparents.

District Profiling

The district is profiled in terms of:

1. District Mathematics staffing:

	GET	FET
Officials		
Curriculum coordinator	1	
DCESSs	1	1
SESs	0	3
Comments: There is no SES in the GET at the moment		

2. Learner performance

GRADES	2024 Mathematics Final Results				
	Wrote	Pass	P%	Fail	F%
3	15589	13942	89	1647	11
6	16143	12953	80	3190	20
9	16147	4677	28	11470	72
12	3703	3172	86	531	14
Comments: Grades 9 and 12 have underperformed, differentiated intervention should be done to improve learner performance.					

1. Introduction: Why This Theme Matters

The theme of equitable mathematics education is particularly relevant in the Limpopo Province, where persistent learning gaps, under-resourced schools, and underqualified teachers pose significant challenges. As a mathematics Subject Advisor who previously served as a Foundation Phase teacher, I have observed first-hand the disconnect between policy and classroom realities. This paper draws from both roles to discuss how relevance, engagement, and equity can be actualized in mathematics teaching.

2. Clarifying Roles: Teacher and Subject Advisor

My professional journey began as a Foundation Phase teacher, where I developed a strong belief in contextualized and child-centered approaches to mathematics. I currently serve as a Senior Education Specialist (Mathematics) in the Mopani East District, supporting over 70 schools. My responsibilities include monitoring curriculum coverage, facilitating teacher training, mediating national policies such as the Annual Teaching Plan (ATP) and “Teaching Mathematics for Understanding” (TMU), and developing CAPS-aligned learning tasks.



3. Core Concepts: Equity, Relevance, and Engagement

The Congress theme captures three essential principles in foundational mathematics education:

- **Equity:** Ensuring all learners—regardless of background, language, or ability—have access to quality mathematics instruction.
- **Relevance:** Making mathematics meaningful by connecting it to learners’ lived experiences and cultures.
- **Engagement:** Encouraging curiosity and active participation through interactive and learner-centered approaches.

These elements are foundational in overcoming the barriers that many Limpopo learners face daily.

4. Classroom Practice: Lessons from Teaching

As a Foundation Phase teacher, I developed strategies to engage learners in mathematics through storytelling, visual representation, and contextual games. One memorable example involved introducing subtraction using a story:



“On my way to school along Nsovo Street, I saw three houses burning. Each row on that street has ten houses. How many are left?”

Learners responded with excitement, naming real people from the street, counting with their fingers, and even peering out the windows. This approach fostered immediate engagement and helped children grasp the concept of subtraction through lived experience.

Other strategies included:

- **Mathematical storytelling** to frame problems in familiar contexts.
- **Game based learning and number lines with animals** to visualize quantity and operations.





Ncuva

Magava

ACTIVITIES TO BE INTRODUCED

Ncuva	Grouping and sharing
Morabaraba	Critical thinking and problem solving
Rope skipping	Counting on and patterns
Hopscotch	Counting, skip counting, number pattern
Magava	Number patterns, counting multiplication

The role of indigenous games in learners' lives goes beyond physical activity and encompasses their cognitive, social, and emotional development.

According to Ankebé Kruger, a professor in counselling psychology at the North-West University (NWU) Potchefstroom campus, these games instill values such as happiness, freedom, friendship, democracy, responsibility, obedience, and mutual help, which are essential for healthy adult life.

In addition to fostering positive values, participating in traditional games can also have a significant impact on learners' behaviour and emotional state.

Kruger emphasises that indigenous games can help reduce negative feelings, prevent behavioural problems, and mitigate acts of violence among learners. "Moreover, indigenous games offer cognitive benefits as well. These games require strategic thinking, quick decision-making, and problem-solving, enhancing children's cognitive abilities," she said.

- **Personalization:** Incorporating learners' hobbies (e.g., football, TV shows) to foster ownership and interest.

These approaches created inclusive environments where all learners—especially those from underprivileged backgrounds—felt seen and capable.

5. Advisory Practice: Lessons from Systemic Support

As a Subject Advisor, I work to scale such effective practices by:

- **Mediating the ATP and TMU frameworks** to help teachers understand progression and pacing.



2023/24 ANNUAL TEACHING PLANS: MATHEMATICS: GRADE 1: TERM 3

Mathematics time allocation per day: 1 hr 24 mins + 5 = 7 hrs per week OR 1hr 30 mins x 4 days plus one 1-hr lesson per week = 7 hrs	
Whole class activity - Counting, mental math (consolidation of concepts already taught) - New concept teaching - Classroom management (allocation of independent activities) Independent group-guided teaching and independent work (inclusive of the differentiated teaching of new concepts - oral, practical and written activities only) The teacher is also meant to plan well for effective teaching and assessment for learning to inform any remediation and further teaching.	6 mins + 10 mins 20 mins 34 x 3 groups = 48 mins Third group does substantial independent written work
Suggested group teaching plan:	
MONDAY Group 1 and 3	TUESDAY Group 2 and 3
WEDNESDAY Group 1 and 3	THURSDAY Group 2 and 3
FRIDAY Whole class teaching	

TERM 3	WEEK 1 AND 2 DIAGNOSTIC ASSESSMENT 2	WEEK 3 AND 4	WEEK 5 AND 6	WEEK 7 AND 8	WEEK 9 AND 10
CONTENT AREAS AND TOPICS	NUMBERS, OPERATIONS AND RELATIONSHIPS				
	Counting - integrate with number patterns and mental maths				
NUMBER CONCEPT DEVELOPMENT	- Count forwards and backwards in 1s, from any number between 1-50 - Count forwards in multiples of 10s, 5s, 2s from any multiple of 10, 5, 2 between 0 and 50 - see the relationship with addition - emphasise more than - ascending order	- Count forwards and backwards in 1s, from any number between 1-60 - Count forwards in multiples of 10s, 5s, 2s from any multiple of 10, 5, 2 between 0 and 60 - see the relationship with addition - emphasise more than - ascending order	- Count forwards and backwards in 1s, from any number between 1-70 - Count forwards in multiples of 10s, 5s, 2s from any multiple of 10, 5, 2 between 0 and 70 - see the relationship with addition - emphasise more than - ascending order	- Count forwards and backwards in 1s, from any number between 1-80 - Count forwards in multiples of 10s, 5s, 2s from any multiple of 10, 5, 2 between 0 and 80 - see the relationship with addition - emphasise more than - ascending order	- Count forwards and backwards in 1s, from any number between 1-90 - Count forwards in multiples of 10s, 5s, 2s from any multiple of 10, 5, 2 between 0 and 90 - see the relationship with addition - emphasise more than - ascending order
	Mental maths number range 10. Ask quick maths questions to promote quick thinking (techniques: Put large number first in order to count on, number line, doubling and halving, build up and break down)				
Building number sense	- Order a given set of numbers 1-10 - emphasise to read and compare to	- Order a given set of numbers 1-12 - emphasise to read and compare to	- Order a given set of numbers 1-15 - emphasise to read and compare to	- Order a given set of numbers 1-15 - emphasise to read and compare to	- Order a given set of numbers 1-15 - emphasise to read and compare to

- **Facilitating workshops** on assessment design, curriculum interpretation, and instructional strategy.

Lesson Study

- Ogegbo and Gaigher (2019:5) describe lesson study as a form of PD strategy that is used to support teachers' learning by ensuring that they improve content knowledge, skills to apply classroom practice, and contribute to teacher change in beliefs and attitudes. Ozdemir (2019:49) explains lesson study as process whereby teachers work together to formulate a research lesson that is presented in class by one teacher while other teachers give feedback to the one who presented the lesson. From what I have observed in Japan, I have seen this and is supported by Ozdemi
- Additionally, in lesson study, a team member explores the best method that can be used to teach or present a lesson in such a way that it can be well understood by learners. Other teachers reflect on the lesson with a view to improving it (Phiri, 2020:68). In this study, lesson study refers to when teachers at the same school and subject share their teaching skills where one teacher presents a lesson and others give comments to assist the teacher to improve their teaching practice.
- **Versioning diagnostic assessments** into Xitsonga to ensure linguistic access for learners.

I have also seen the value of providing differentiated tasks and structured lesson plans that reflect real classroom conditions.

Lesson Plans equate to quality time for mathematics teaching has a decisive impact development of mathematical concepts and skills. The activities learners engage in should, however, not be "keep busy" activities, but should be well planned for and clearly focused on the mathematics concepts as outlined in the curriculum / ATPs.

6. Reflections: Lessons Learned

Several key insights have emerged from my dual roles:

- **Connecting math to real life** builds learners' confidence and critical thinking.
- **Games and visuals** cater to diverse learning styles and enhance enjoyment.
- **Language-sensitive teaching** supports better engagement and comprehension.



- **Encouraging mathematical talk** promotes peer learning and deeper understanding.
- **Flexible and adaptive lesson planning** allows instruction to meet learners where they are.

These strategies foster equitable participation and learner-centered classrooms.

7. Challenges: Barriers to Equity and Engagement

Despite these gains, several structural challenges inhibit equitable mathematics instruction:

- **Learner Backgrounds:** Teachers often fail to consider learners' linguistic and cultural contexts. Gay (2022) notes, culturally responsive teaching requires knowledge of students' backgrounds and deliberate inclusion in instruction.
- **Teacher Qualification:** Many Foundation Phase teachers, especially in Limpopo, lack specialization in mathematics. Like myself, many are trained in general education, not specifically in math.
- **Capacity Building:** Advisor training may last several days, yet teacher workshops often receive only 2–3 hours. This is insufficient for deep pedagogical development.
- **Resources:** As the Department, we provide materials (e.g., Math Boxes, Six-Bricks, tablets), I observed that they are frequently underused or poorly maintained.
- **Infrastructure:** Overcrowded classrooms limit hands-on learning. A 1:40 teacher-learner ratio remains aspirational in many areas.
- **Multicultural Competency:** Iwuanyamwu (2024) highlights that teachers often lack the humanistic and pedagogical skills needed to serve diverse classrooms.

8. Recommendations

To support equitable, relevant, and engaging mathematics education, I propose the following:

1. **Understand Learner Contexts:** Teachers should research learners' cultural and social backgrounds and use this knowledge in planning lessons.
2. **Prioritize Specialist Training:** Recruit and appoint Foundation Phase teachers with mathematics specialization; Subject Advisors should also hold math qualifications.
3. **Expand Training Duration:** Extend in-service teacher training to several days and consider weekends or holidays for deep-dive sessions.
4. **Audit Resource Usage:** Replace internal reporting with external audits to ensure accountability and proper resource use. Integrate procurement with SASAMS to manage supply-demand balance.
5. **Improve Infrastructure:** The Department should prioritize classroom decongestion and maintain a maximum class size of 40 learners to enable effective teaching.

9. Conclusion

Addressing marginalization in mathematics education requires commitment at both classroom and systemic levels. By connecting lessons to learners' lived experiences, supporting teachers with sufficient time and training, and auditing resource use, we can realize the vision of equitable, relevant, and engaging mathematics education. Empowering teachers and learners alike ensure a more just educational future for all.

Africanising the Curriculum



The South African government began to make changes to the apartheid-based curriculum as soon as the first democratic elections were over. The curriculum went through four major policy cycles. In the first curriculum change, a process of cleansing and purification occurred, removing offensive language, racial stereotypes, and incorrect content (Chisholm 2005; Jansen 1999). By the fourth curriculum change in 2009, a greater emphasis on Indigenous knowledge could be found in the preamble to the Curriculum Assessment Policy Statements. However, the prescribed content was still largely presented as neutral knowledge. For Van Wyk and Higgs (2011, 177), a curriculum based on Indigenous African knowledge systems empowers educators and Biccand and Meeran 23 learners, giving them confidence in their own abilities and “a sense of pride in their own ways of being in the world.” While we concur that there has been a plethora of curriculum changes since 1994 to incorporate South Africa’s change to democracy, the question remains: Have we included an Africanised curriculum that encompasses the knowledge, values, and practices of Africans into our curriculum? Pinar (2006, 2014) believes that the curriculum should be embedded in the culture of the country. This notion is echoed by Msila (2021) who tackled the intellectual debate of Africanising the curriculum and concluded that the use of indigenous knowledge improves.



REFLECTIONS ON PROMOTING EQUITABLE AND SOCIALLY JUST MATHEMATICS TEACHING APPROACHES

Pete Wright

University of Dundee, Dundee, Scotland, United Kingdom

In this paper I present key findings from four research projects which aimed to promote equitable and socially just mathematics teaching approaches. I outline the design of the projects, all of which were based on a critical model of participatory action research. I draw on the findings from the projects, together with the underlying theories, to identify and exemplify what I believe to be four essential components of socially just mathematics teaching. I offer these insights to readers so that they may judge for themselves the extent to which the findings are transferable to their own contexts and can inform their own practice.

INTRODUCTION

I am truly honoured and excited to be invited to present my thoughts and ideas at the 30th Annual National Congress of The Association for Mathematics Education of South Africa (AMESA). At the same time, I admit to being somewhat nervous about presenting my views on equitable and socially just mathematics education to an AMESA audience. Equity and social justice have been a passion of mine throughout my career in education. My appreciation that political change and education are inseparable was developed through becoming active (as a student and then a young teacher) during the 1980's in UK-based organisations, such as the Anti-Apartheid Movement and Nicaragua Solidarity Campaign, that aimed to change policies and public opinion at home, as well as to build solidarity with liberation movements around the world. My strong beliefs that education can and should be used as a means for making the world a better place, and that all students should have access to a good quality education regardless of their background, influenced the development of my teaching practice during subsequent years. Becoming a teacher educator in 2011, and completing my doctoral studies in 2015, provided an opportunity to engage more systematically with theory and research around equitable and socially just mathematics teaching. This helped reinforce my commitment to many of the teaching approaches I had previously adopted (at the time because they seemed to resonate with my beliefs and values) as I began to better understand the rationale and justification behind them.

My nervousness stems from an appreciation that what might be applicable to one context (or appropriate in one part of the world) might not be as applicable (or appropriate) in another. Shenton (2004) argues that to promote the 'transferability' of research findings, the researcher should provide sufficient contextual details about the research setting, the researcher's own background and the backgrounds of the research participants. This will then enable the reader to make an informed decision about the extent to which the findings may be relevant to their own situation and inform their own practice. Hence, my aim in this paper is to explain, in as transparent a way as possible, the research that I have carried out in my own context (working



mainly with schools and teachers in London and other parts of the UK) and leave it to you to judge the relevance of the findings to developing practice in your own situation.

Shenton (2004) also argues that the researcher's reflexivity and critical subjectivity should also be clearly communicated in the reporting of research. Over the years, I have grown to appreciate the significant role that power relationships play in maintaining and reproducing inequity and how power is an inherently corrupting force. Indeed, I have admired and respected several political leaders who have stood up to repression, only to be disappointed when, soon after assuming power, they become complicit in maintaining the same repressive systems that they once opposed. I therefore believe it is imperative for those advocating for equity and social justice to acknowledge any privileged positions they hold, and to reflect critically on how power relations may influence their positioning and actions towards others. I have consistently tried to do this as a teacher (in how I relate to students) and as a researcher (in how I relate to participants). For the same reason, it is important to acknowledge my position as a teacher educator and researcher based in a country in which inequity exists between different groups within society, whilst the population (as a whole) benefits disproportionately from an inequitable world governance and trade system.

Having made my positioning clear, in the remainder of this paper, I will present key findings from four research projects I have led which involved working collaboratively with teachers (and in one case teacher educators) to explore how theories relating to equity and social justice in mathematics education can be applied in developing classroom practice. I will draw on underlying theories to present what I believe to be four essential components of socially just mathematics teaching. I will relate each component to relevant findings from the four research projects and exemplify what each might look like in practice. At this point, I would encourage you to pause and try to imagine what a *socially just* mathematics classroom might look like for you. You might also consider what a *socially unjust* mathematics classroom might look like. I invite you to write down some of your thoughts in which you distinguish one imaginary classroom from the other.

THE FOUR RESEARCH PROJECTS

All four research projects that I refer to adopted a *critical model of participatory action research* (PAR) based on a series of *plan-teach-evaluate* cycles. PAR involves carrying out research *with* teachers, as collaborative partners, rather than carrying out research *on* teachers, as subjects (Wright, 2021). It recognises that both academic researchers (with their research expertise) and teacher researchers (with their in-depth knowledge of the classroom context and their learners) have essential and complementary roles to play in research that aims to bring about positive social change. Because PAR pays closer attention to teachers' perspectives and classroom constraints, it is more likely to generate findings of greater interest and relevance to other practitioners.

The *critical model* of PAR (Skovsmose & Borba, 2004) assumes that the *current situation* (CS), in this case existing mathematics teaching practice, should not be taken as given. The *imagined situation* (IS) is an alternative vision of what could be, in this case an equitable and



socially just mathematics classroom, identified by drawing on research literature and previous experiences. The *arranged situation* (AS) is where an attempt is made to put some aspect(s) of the *imagined situation* into practice, taking account of the realities and constraints of the *current situation*. In this case, the *imaginary situation* involved the teacher researchers designing, teaching and evaluating research lessons in which new ideas, activities and strategies were tried out. This, in turn, led to changes in their thinking and classroom practice and hence a modified *current situation*, feeding into the next PAR cycle (see Figure 1 below).

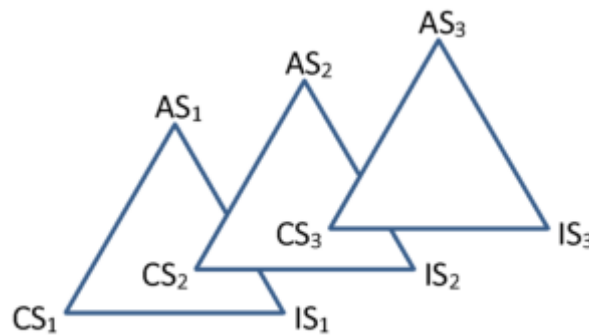


Figure 1: Critical model of PAR with 3 cycles (Skovsmose & Borba, 2004, p. 221)

All four research projects involved establishing a research team of teacher researchers and academic researchers. Research team meetings focused on engaging with research literature relevant to socially just mathematics teaching, designing research lessons and data collection tools (such as student surveys, interviews and video recordings) which were used to evaluate the lessons (Wright, 2024a). The involvement of teacher researchers in designing as well as implementing data collection tools was a key characteristic of the research projects (Wright, 2024a). This contributed towards greater levels of reliability of student data, since relationships of trust were already established between teachers and students. Further data were collected through recording reflective discussions during research team meetings and through interviews with teacher researchers, which were designed to capture their experiences of participating in the projects. Thematic analyses were then carried out on the data (Braun & Clarke, 2022).

The Teaching Maths for social Justice (TMSJ) research project was carried out as part of my doctoral studies conducted from 2013 to 2014 (Wright, 2016a; Wright, 2017). I collaborated with five mathematics teachers in four ethnically diverse secondary (age 11-16) schools in London within areas of relatively high levels of socio-economic deprivation. They were all in their second year as qualified teachers having previously completed the same pre-service teacher education course that I taught on at University College London Institute of Education. The research focused on exploring strategies for translating theories relating to social justice into classroom practice through designing, trialling and evaluating a series of activities. The findings demonstrated how, through critically reflecting on existing practice within a community of inquiry, teachers can transform their thinking and classroom practice. They began to recognise the need to tackle systemic inequities by challenging existing structures and discourses, including around the grouping of students by prior attainment, and by promoting progressive pedagogies that engage all students and develop students' agency.



The Visible Maths Pedagogy (VMP) project was a collaboration with two teacher researchers from 2017 to 2019 in an ethnically diverse secondary school in London (Wright, Fejzo, & Carvalho, 2022). Both were experienced teachers who I had previously worked with as a student teacher and mentor. The project aimed to develop strategies that would enable all students to recognise their teacher's intentions when adopting *progressive* teaching approaches. These approaches were already well established within the mathematics department, however there were concerns that some students, particularly those from marginalised groups, found it difficult to engage with them. The findings demonstrated how teachers can help all learners to develop a greater appreciation of their pedagogic rationale through prompting open discussions with students around the reasons for using their teaching approaches. This, in turn, helps all students achieve greater success when experiencing progressive pedagogies.

The Primary Maths and Social Justice (PMSJ) research project was a collaboration with two co-researchers, six teacher researchers and six other teachers from 2021 to 2023 in two culturally diverse primary (age 5-11) schools in London (Wright, Kelly, & Hilton, 2024). It is common practice in primary schools in England for students to spend significant amounts of time exploring social justice issues (such as climate change or racism) through cross-curricular projects. In contrast, mathematics is commonly taught as a discrete subject, and students often struggle to apply the mathematics they learn in these lessons to solving real-life problems. The project explored how teachers can bridge this gap by making stronger connections between social justice issues tackled in cross-curricular projects and mathematics learning. The findings demonstrated how teachers can help students to develop a greater capacity to use the mathematical knowledge they have previously learned in constructing collective arguments for change in relation to social justice issues (referred to as *socio-mathematical agency*). In doing so, students (and teachers) developed more positive dispositions towards mathematics as they began to appreciate how it can be applied in more meaningful and relevant ways to their own interests and experiences.

The Promoting Socially Just Mathematics Teaching (PSJMT) research project is an ongoing collaboration with 12 colleagues from different universities across England and Scotland that began in 2024. The project aims to explore how mathematics teacher educators can promote socially just pedagogies and practices amongst the student teachers and experienced practitioners they work with. The outcomes include designing a series of professional development workshops for teachers, which are freely accessible (currently in draft form) at www.mathsocialjustice.org/prof-dev. Initial findings suggest that most teachers are enthusiastic and willing to engage with issues of equity and social justice in teaching mathematics, although pressures and constraints they face in the classroom often result in them losing sight of these aims. Reflecting critically on equity and social justice, in particular how these relate to issues which are a key priority for schools (including inclusion, engagement and problem solving), can enable teachers to better understand and overcome these constraints.

Having presented key findings from the four research projects, I will now outline how these findings, and the theories which informed the design of the projects, relate to four essential components of socially just mathematics teaching which I have identified.



PROGRESSIVE PEDAGOGIES

By *progressive* pedagogies I mean adopting teaching approaches that encourage students to tackle open-ended tasks, to discuss ideas and share different methods, and to collaborate in solving mathematical problems. Gutstein (2006) highlights how such approaches enable learners to acquire *mathematical power* by developing their agency in applying mathematical ideas and processes to solve real-life problems. Whilst mastering and practising mathematical procedures is important, experiencing how to pose questions, make conjectures and identify which procedures to use in unfamiliar situations are essential if students are to make effective use of the mathematics they learn outside of school. Worryingly, there has been a recent resurgence in England of teacher-led pedagogies, led by advocates of *Direct Instruction*, who argue that knowledge is inherently powerful regardless of the learner's agency in using it (Wright, Fejzo, & Carvalho, 2022). They draw on Bernstein's (2000) argument that students from disadvantaged backgrounds, due to their upbringing, are less able to recognise how to respond appropriately to (less-structured) *progressive* pedagogies and hence achieve less success when they are employed. Such teacher-led approaches result in increasing levels of anxiety and alienation amongst students as they perceive the mathematics they learn as lacking purpose and meaning (Williams & Choudry, 2016).

The VMP project succeeded to some extent in tackling the problem of students (particularly those from disadvantaged backgrounds) misinterpreting the intentions of the teacher by making the rationale for employing *progressive* pedagogies more explicit to learners. This was achieved through various strategies including holding discussions with students in which this rationale was the explicit focus, for example by asking: "*Why do you think I asked you to ...?*". The Fraction Flags activity (see Figure 2) serves to exemplify this approach (Wright, 2024a). I would encourage you to pause reading this paper whilst you discuss the three prompts below with a colleague.



Figure 2: Flags of Czechia and Seychelles

- Discuss in small groups what fraction of each flag is coloured blue, white and red.
- Find more than one way of explaining your answers.
- Make notes on somebody else's reasoning ready to present these to the whole class.

Students were asked to work on the Fractions Flags activity in small groups. They were encouraged to share ideas with each other and to come up with their own questions to ask. Once they had found one method of finding the answers, they were encouraged to look for alternative methods. The focus was on explaining and justifying their solutions rather than finding the



correct answer. When required, the teacher would intervene with prompt questions such as: *“What information do you need to know to be able to work it out?”* After most groups had found several methods for finding the solutions, some students were asked to present another student’s reasoning to the rest of the class.

After the activity was completed, the teacher then tried out a strategy for making the teaching rationale more explicit to learners, in this case facilitating a discussion with the whole class by using the prompt question: *“Why do you think I asked you to present somebody else’s reasoning?”* The project’s findings suggested that some students initially misinterpreted the teacher’s intentions, for example, the following two responses suggested students believed the teacher was merely enforcing listening: *“To see if you’re listening to your friend.” ... “Make sure you’re listening and learning.”*

Other responses from students suggested reasons that more closely reflected the teacher’s primary reasons, which were to encourage students to share ideas with each other and to engage with and learn from other people’s methods: *“So that we understand other’s point [of] view.” ... “Because you can get two different perspective[s] and it may help you finalise your idea.”*

Over the course of the project, teacher researchers reported an increasing proportion of students who were able to correctly identify the teacher’s reasons for adopting specific teaching approaches. This was particularly evident for those from marginalized groups. There was also evidence to suggest that students were better able to respond appropriately and achieve success when experiencing *progressive* teaching approaches. During an interview with one of the teacher researchers, Keira (a female student of Afro-Caribbean heritage from a family with low socio-economic status) describes how engaging with another strategy for making the teacher’s pedagogic rationale more explicit (in this case a card sort) helped make the purpose of learning clearer to her:

“Because it helps us, like, understand more why we do it, because ... some people would be like ‘Ah, what is this going to help us with in the future?’, and stuff like this. And it would just help you, like, understand more, and like why we’re doing this, why we’re learning this, why it’s going to help us.”

MEANINGFUL INQUIRY

Gutstein (2006) applies Freire’s (1974) model of *education for critical consciousness* in arguing that students should be encouraged to use mathematics to explore issues of inequity and injustice which they encounter in their own lives and in society more widely. As well as highlighting its problematic nature, for example in contributing towards economic crises, Skovsmose (2024) emphasises how mathematics can also be used as a critical tool in struggles for social justice. Mathematical models can be used to monitor and report the extent of climate change or to measure and report inequalities within society (see for example the Lorenz Curve and Gini Coefficient in Figure 3).

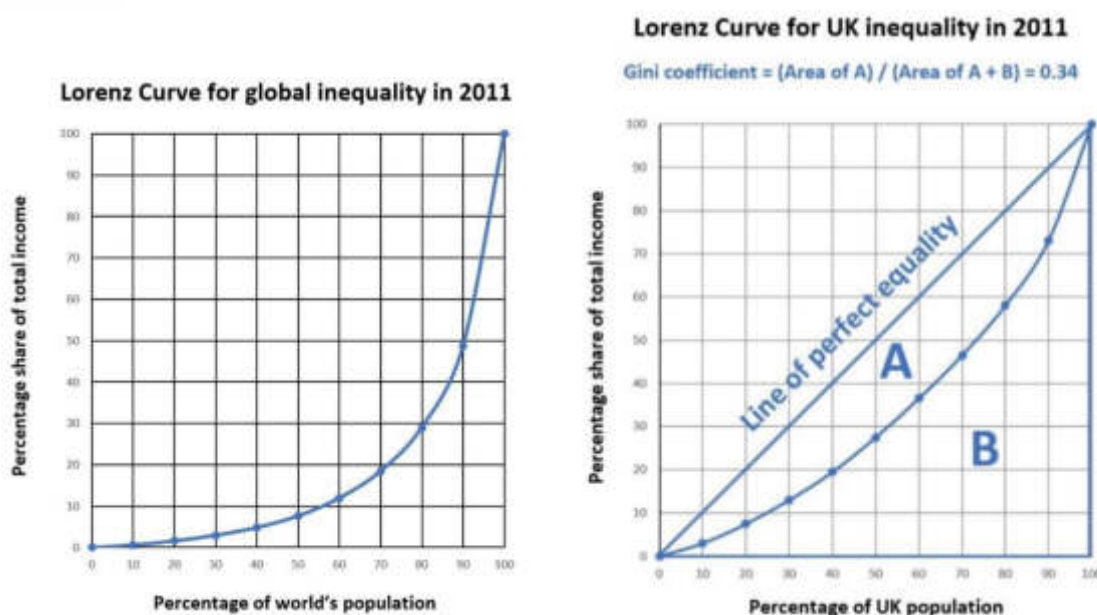


Figure 3: Lorenz curve and Gini coefficient for 2011

The findings of the TMSJ and PMSJ projects demonstrated how secondary and primary students (respectively) respond positively when encouraged to use mathematics to explore real-life social justice issues. In both cases, students' levels of engagement increased dramatically, particularly amongst those who had become disengaged or alienated from the subject, as they began to appreciate the relevance of mathematics to social issues which they experienced directly or felt passionately about. Whilst students commonly encountered word problems in mathematics lessons, the real-life contexts were often superficial in nature and of little interest to them. The teacher researchers noticed how students (even the youngest) care passionately about issues of fairness and social justice and were keen to explore connections between these and mathematics.

"I tried a few things with my bottom set and their motivation has just been so high in those particular lessons that I've had to very rarely like tell them to get on with things or to do things." (Anna, teacher researcher, TMSJ project)

"In general, I do not enjoy maths as I think I'm not very good at it. Today I enjoyed the maths lesson as I enjoyed finding out about fair trade and I liked seeing all the statistics of the money different people make from a bar of chocolate." (Year 9/age 13-14 student, TMSJ project).

"The children were very engaged. I think the fact that they were attributing it to a real-life problem or challenge. So, they really took ownership over setting their own investigations and then understood very much how maths can be related to real life." (Layla, Year 6/age 10-11 teacher researcher, PMSJ project)

However, teachers also faced significant constraints in introducing these ideas into their mathematics lessons. These included the prescriptive nature of the mathematics curriculum, with large amounts of content to be covered, and limited time for planning, particularly as there were very few existing resources to draw from. This required genuine links to be established



between social justice issues and mathematical content. For instance, calculating the Gini coefficient (see Figure 3) involves finding an irregular area between the (straight) *line of perfect equality* and the *Lorenz curve*. This is a very rich mathematical task which allows for various ideas to be applied, including approximating the area either by counting whole and/or part squares or by using rectangles, triangles and trapezia. I invite you to pause and have a go at this for yourself.

These constraints were to some extent resolved by collaboratively planning and sharing of resources between members of the research teams. The teacher researchers in both projects reported how the mutual support provided by other members of the research team encouraged them to try out new (and sometimes risky) ideas with their classes:

“It’s given me the confidence to step off the scheme of work treadmill, of getting through different topics or chapters, and actually saying: ‘Well, these topics, say cumulative frequency, or percentages, I can fit these within a project on something to do with these kids’ world, or to do with our world as a whole.” (Brian, teacher researcher, TMSJ project)

“The collaboration, the meetings with other teachers. I liked it when we shared all ideas, and we heard what other year groups were doing ... If you’re trying to do something new, you just need to get a group of like-minded people, who will have different skill sets, to kind of work with each other, but also challenge and suggest new ideas.” (Rose, Year 5/age 9-10 teacher researcher, PMSJ project)

Both the TMSJ and PMSJ projects generated teaching ideas and resources which have been shared more widely through two publications (see Wright, 2016b; Cotton, Jagdev, Kaur, & Wright, 2024). Note some of these (and similar) teaching ideas are freely available via the TMSJN website www.mathsocialjustice.org (see *Activities* and *Resources* pages, and other project websites which can be found via the *Links* page).

COLLECTIVE AGENCY

There have been several calls recently for more humanistic school curricula that cultivate the critical understanding and collective knowledge needed to enable future generations to address the social, economic and environmental challenges facing global society (OECD, 2018; UNESCO, 2015). This requires learners to develop not only greater understanding of social justice issues, but also agency in generating arguments for bringing about change for the better. Mathematics can play a key role in supporting such arguments. In cases where mathematics is used to solve problems in real life, this is usually done by people working in teams rather than as individuals. However, many mathematics lessons are characterised by students working individually on closed tasks without the opportunity to develop collective agency. We highlighted in a previous paper the need for students to develop *socio-mathematical agency*, defined as the capacity to use mathematics effectively in arguing collectively for social change (Wright, Kelly, & Hilton, 2024). The findings from the TMSJ and PMSJ projects demonstrated how this might be achieved in practice.



During the TMSJ project, teacher researchers developed the *Making a Change* activity which involved students working in small groups, choosing a social justice issue of interest to them, researching it, and using mathematics to construct an argument in support of a change they would like to see made. They were then asked to present their argument to the rest of the class. Groups chose a range of issues to explore including *human trafficking*, *food distribution*, *discrimination* and *the death penalty*. The PowerPoint presentations created by the groups to summarise their arguments can be accessed at: www.maths-socialjustice.weebly.com/project-teaching-resources.html. This activity generated very high levels of excitement and engagement amongst students, including those who had previously shown little interest in mathematics:

“They were all so passionate about the things they were presenting about, was the key thing, and the fact that they got to actually tell everyone what they found out.” (Anna, teacher researcher, TMSJ project)

In the PMSJ project, teacher researchers designed several activities including one that prompted students to consider how to use counters in deciding collectively how to arrive at a *fair choice*, for example, when deciding which fun activity to choose as a class reward for the hard work done during the week. Students were divided into groups and each group was given 10 counters. They were given time to discuss how to allocate their counters to the available options. When all groups had decided, they were invited to place their counters on a board such as that shown in Figure 4.

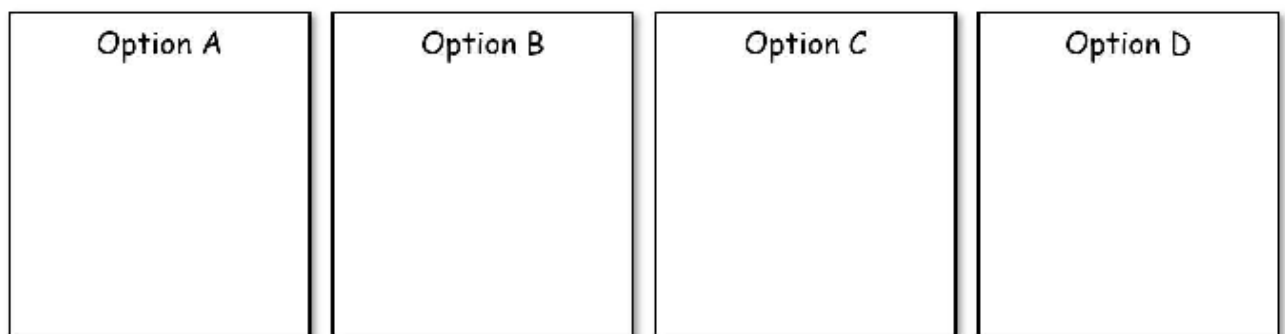


Figure 4: *Making a fair choice*

Students were then asked to reflect on the *fairness* of the choice made by the class, using the following prompt questions (see Wright, 2023):

- *How did you feel about the result? Do you think your view was taken into account? What about other people's views? Why?*
- *How big should the groups be? Why? How many counters should be allocated to each group? What if the groups were different sizes?*
- *How many counters should each group allocate to their first/second/third/fourth choice? Why? How should we use the results to make a decision? Why?*
- *What other questions should we consider?*

The findings demonstrate how students (regardless of age) can develop *socio-mathematical agency* by working collaboratively to generate an argument for change:



“I think the major impact is just that they're actually learning how to give an opinion ... And understanding that they can apply maths to situations that are not just in the classroom, and things that affect them.” (Emma, Year 1/age 5-6 teacher researcher, PMSJ project)

“And then they worked with their talk partner to talk about their views. And there was some quite heated discussion amongst them. And at that point, they were really keen then to come up with some different ideas, which they thought might be fairer.” (Kate, Year 1/age 5-6 teacher researcher, PMSJ project)

CHALLENGING MYTHS AND DISCOURSES

Bourdieu's theory of *reproduction* outlines how schools contribute towards maintaining inequitable power relations between different groups in society through assigning greater value to the cultural resources acquired by children from more privileged groups (Bourdieu & Passeron, 1990). This systemic inequity is concealed by presenting the school as a level playing field in which success is attributed to innate ability rather than the systemic advantage afforded to some children. Mathematics plays a key role in this reproductive process, with a persistent and strong correlation between socio-economic status and mathematical attainment, whilst mathematics qualifications determine access to higher-status university courses and better-paid employment (Wright, 2016a). Skovsmose (2011) highlights the need for both teachers and students to develop a critical understanding of mathematics and to reflect on questions relating to the nature of the subject, its purpose and its position in society. Teachers and students should engage in dialogue and challenge common myths and discourses, including claims that mathematics is culture and value-free, is all about right/wrong answers, and that mathematical ability is innate (Coles & Sinclair, 2022).

All four projects demonstrated how critical reflection, which was an essential component of all four research designs, can lead to a greater willingness of teachers to question their own prior assumptions and challenge discourses they encounter in schools. This was a key focus of the PSJMT project. The professional development resources that were developed incorporate opportunities for teachers to reflect on their initial thinking and, after engaging with the theories presented and with the classroom activities modelled during the workshop, to reflect on how their thinking has changed over the course of the workshop (see example slides from the *Socially just approaches to problem solving in mathematics* workshop in Figure 5).

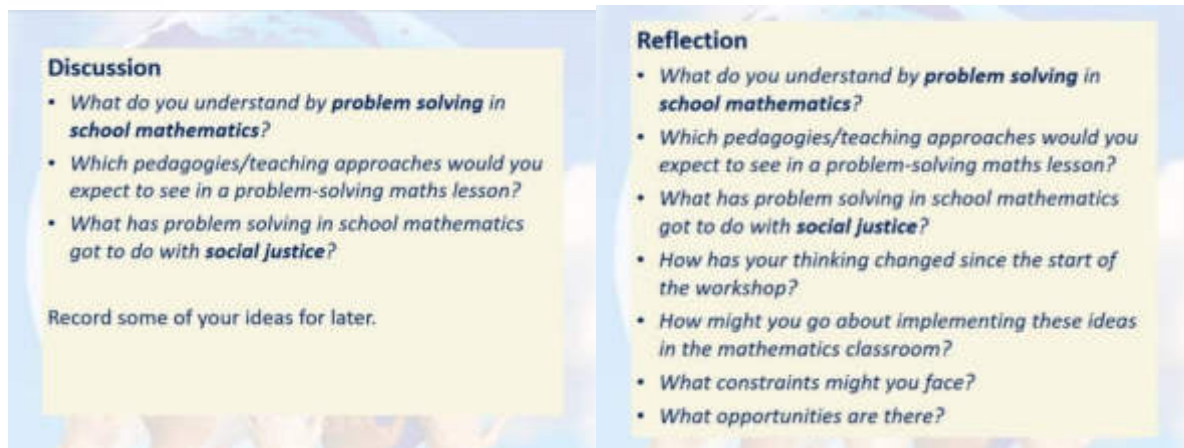


Figure 5: PSJMT workshop – initial Discussion and final Reflection slides

At this point, I would encourage you to pause and to reflect on your earlier thoughts about what distinguishes a *socially just* from a *socially unjust* mathematics classroom (which you were invited to record at the start of this paper). Consider how your thinking has changed having engaged with this paper (mirroring the process that teachers go through when attending the PSJMT professional development workshops).

IMPLICATIONS FOR PRACTICE

I have presented the findings from four research projects and related these to what I believe to be four essential components of socially just mathematics teaching, which are summarised below. I invite you to reflect on the findings from these projects and to judge the extent to which the findings are transferable to your own context. I hope that this paper has helped you to reflect in some meaningful way on your practice.

Essential components of socially just mathematics teaching (Wright, 2024b, p. 26):

- *Employ collaborative, discursive, problem-solving, and problem-posing pedagogies which promote mathematical sense-making and the engagement of all learners with mathematics.*
- *Promote mathematical inquiries that resonate with learners' real-life experiences and that help them develop greater understanding of their social, cultural, political, and economic situations.*
- *Facilitate mathematical investigations that develop learners' individual and collective agency, enabling them to take part in future social action for the public good.*
- *Challenge common myths surrounding school mathematics, expose processes that lead to the marginalisation of some learners, and open up to scrutiny what it means to be successful.*

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Entrenching and disrupting performativity regimes: Impact on learning (mathematics) in post-apartheid South Africa

Anil Kanjee

Tshwane University of Technology

The challenge of addressing learning needs of all learners in South Africa, and in particular learners from poor and marginalised backgrounds has been an ongoing one that has yet to be adequately addressed in the post-apartheid education. A fundamental problem in addressing this challenge has been the use of performativity-based accountability regimes that privilege curriculum coverage and performance over effective learning and teaching. The impact of these regimes manifests across all subjects and grades and manifest in complex and often contradictory ways. While policymakers expect improvements in efficiency, academic excellence and equity, in practice performative accountability can lead to unintended negative consequences. The implications of this approach have resulted in the failure of large numbers of learners in attaining basic levels of performance across the schooling system, as evidence in the recent TIMSS and Systemic Evaluation results. To address this problem, this paper argues for developing alternative practices that challenge dominant performativity-based accountability regimes. These practices must focus on addressing the fundamental challenge of equity and quality across the schooling system, not just in mathematics. Thus, issues discussed in this paper extend beyond mathematics to systemic and policy issues. Where relevant, that impact key pedagogical practices of teachers and opportunities created for learners in mathematics are highlighted.

The paper begins with a brief history of assessment policies and practices, from the colonial period to the apartheid and the post-apartheid eras, highlighting its impact on establishing performativity-based accountability systems that dominate current practices across the schooling system. This is followed by an overview of the different accountability systems impacting teaching and learning. Next, key policies and initiatives implemented for improving the schooling system are discussed. These include relevant assessment and curriculum policies, the different Action Plans as well as the decolonise education movement, interventions by teacher unions, the new spaces provided by the COVID-19 pandemic and the adoption of Assessment for Learning as a pedagogical strategy. The paper also examines prevailing tensions and contradictions arising from policy implementation by different role-players as they challenge dominant performativity-based accountability regimes. The paper concludes by calling for a shift from control and compliance-based accountability to a more developmental, democratic, and context-sensitive approach for creating supportive environments that address the learning needs of all learners, especially those from poor and marginalised communities.

Key words: *Performativity; Accountability; Assessment for Learning; Pedagogy; Learning*



Declaration on the use of AI

AI was used in this paper to:

- Identify additional references to consult.
- Review content of previous papers written by the author.
- Provide summaries of relevant documents.
- Reduce word count and revise text already written.
- Check references used in the text.
- No new ideas were generated for this paper using AI.

INTRODUCTION

Since the first democratic election in 1994, the post-apartheid government faced the immense task of building a democratic society that ensured equal participation and human rights for all citizens. Within the education sector, the key challenge was to implement a new system designed to promote democratic governance, equitable resource distribution, and redress historical inequalities. In practice, this involved unifying the fragmented, racially divided system, removing discriminatory policies, implementing a new funding framework to rationalise resource allocation while also addressing historical inequities and introducing a new curriculum aligned with democratic constitution. The new legislative framework sought to eliminate the 19 racially based education departments into a single multi-tiered governance structure. The National Department of Education assumed responsibility for policymaking, resource allocation, and establishing regulatory frameworks. The nine Provincial Education Departments were tasked with implementation, managing budgetary resources, and employing personnel. At the school level, democratically elected School Governing Bodies were tasked with providing support for financial management and admissions and improving teaching and learning. In addition, several landmark policies were enacted to introduce a new curriculum aimed on addressing the ideals enshrined in the new post-apartheid curriculum.

From the outset, the focus of the Department of Education has been addressing the twin challenges of equity and quality in the schooling sector. This focus has served as the underlying basis of the transformation agenda, and which laid the legislative foundations intended to reshape the educational landscape within the country. Notwithstanding the myriad of policies, acts, guidelines implemented, and supported by various programmes and projects over the last three decades, the issue of equity and quality still remains an elusive goal that has yet to be achieved. One of the primary contributory factors pertains to the ongoing challenge of improving learning and teaching for all learners, especially learners from poor and marginalised backgrounds. This challenge persists despite the significant progress made in improving access to schooling for all children, supporting teacher professional development initiatives, addressing disparities in resource allocations and facilities across schools.

A review of the key policies enacted to improve teaching and learning since 1994 reveals a



fundamental tension in the development and implementation of the curriculum and assessment policies, a tension driven by a focus on measurable outputs such as test scores and curriculum completion on the one hand, and on the other hand, the broader goals of improving learning for all children. The prevalence of such tensions within the South African education system is not surprising. The application of accountability regimes has become a dominant force shaping education systems across the world. While such approaches were introduced with the intention of improving educational quality and ensuring system transparency, they have often led to unintended consequences that undermine the very learning they aim to promote. In South Africa, accountability has been increasingly tied to mechanisms that measure school performance through standardised testing, curriculum coverage, and other quantitative indicators. The reliance on performance metrics has created pressures on educators to "teach to the test," narrow the curriculum, and prioritise compliance over pedagogical innovation. These pressures are particularly acute in under-resourced schools, where the stakes of poor performance are highest, yet the structural support to meet those demands is weakest. As a result, the accountability systems designed to raise standards has, paradoxically, entrenched educational inequalities and stifled genuine instructional improvement. This is especially evident in mathematics education, where historically disadvantaged learners continue to score far below their privileged peers, reinforcing cycles of marginalisation in STEM fields.

This paper seeks to interrogate these tensions by examining South Africa's prevailing accountability frameworks. In addressing these tensions, this paper proposes a shift in how accountability is conceptualised and enacted within the South African context. Instead of emphasising control and compliance, it calls for a more developmental, democratic, and context-sensitive approach to accountability—one that aligns with the country's constitutional commitment to equity and social justice. The paper is organised into five sections. Following this introduction, Section 2 lays the conceptual foundation by defining key terms such as accountability and performativity. Section 3 traces the historical trajectory of assessment and accountability in South Africa, from colonial and apartheid-era policies to post-apartheid reforms. Section 4 highlights the key tensions and contradictions in policy and practice as manifested in current accountability regimes. Section 5 concludes with a brief synthesis and list ideas for the way forward in transforming accountability in South African education.

Accountability, performativity, and learning

Key definitions and concepts

In this paper we use the definition presented in the UNESCO Global Monitoring Report (UNESCO, 2017), which views accountability as a process aimed at helping individuals or institutions meet their responsibilities and reach their goals, comprising: (i) Clearly defined responsibilities; (ii) Obligation to provide an account of how responsibilities have been met; and (iii) Legal, political, social or moral justification for the obligation to account. Within the education sector, the common forms of accountability systems include *Bureaucratic*, *Professional responsibility*, *Results-based* and *Social Accountability* (Flórez-Petour & Rozas-Assael; Hamilton, Stecher & Klein, 2002). *Bureaucratic accountability*, also known as



regulatory or legal accountability, holds schools accountable to higher levels of administration for their adherence to procedures and state regulations. For example, the South African Schools Act (SASA) of 1996 established mandatory compliance requirements for all schools, with the Department of Basic Education (DBE) conducting regular audits of teacher qualifications, curriculum implementation, and infrastructure standards. Schools receive formal evaluations through the Whole School Evaluation (WSE) process and must develop improvement plans based on findings (Christie, Butler & Potterton, 2007). *Professional responsibility* requires adherence to professional norms and standards. **For example**, the South African Council for Educators (SACE) maintains a professional code of ethics and conduct, requiring all teachers to be registered and participate in the Continuing Professional Teacher Development (CPTD) system (Mestry & Singh, 2007).

Results-based, also known as test-based or performativity-based accountability, in which schools are accountable to parents, learners, and the general public based on their learner performance outcomes. **For example**, the Annual National Assessments (ANAs) (now discontinued) focussed on assessing learner attainment in Mathematics and Languages across several grades, while National Senior Certificate (matric) examinations serve as a high-stakes accountability measure, where schools' results are used to identify underperforming schools, that are placed under departmental management or face leadership changes if they consistently show poor matric results (Spaul & Jansen, 2019). ***Social accountability*** refers to systems in which communities and parents monitor and participate in school governance. **For example**, School Governing Bodies (SGBs), mandated by South African School Act, include parent representatives who oversee school budgets, fee policies, and infrastructure development. Moreover, public interest organisations like Equal Education have mobilised communities to demand accountability for school infrastructure standards according to the "Norms and Standards for School Infrastructure" regulations, using social pressure and sometimes legal action to ensure compliance (Equal Education, 2023).

In practice, these accountability systems often operate simultaneously within school systems, creating complex webs of responsibilities and reporting relationships that are particularly challenging for under-resourced schools serving marginalised communities. The balance between these approaches significantly impacts teaching practices, resource allocation, and ultimately learning outcomes. Over-reliance on any single approach can produce unintended negative consequences, such as teaching to the test (performance accountability) or rigid procedural compliance without innovation (bureaucratic accountability) (Flórez-Petour & Rozas-Assael, 2019).

Performativity and test-based accountability

Hamilton, Stecher and Klein (2002) define test-based accountability (TBA) systems as “a set of policies and procedures that provide rewards and/ or sanctions as a consequence of scores on large-scale achievement tests (p. 2). The authors note that such systems are generally characterised by four interrelated components: (i) Goals that are presented as a statement of desired individual or system performance, e.g. curriculum or performance standards; (ii)



Measures that list the quantitative indices of performance associated with the goals, e.g. results of standardised tests; (iii) Targets that specify the desired levels of attainment or improvement required, e.g. minimum score on the test; and (iv) Incentives that indicate the consequences for attaining or failing to attain the, e.g. rewards or sanctions. While the key reasons for implementing TBA usually include the improvement of learning outcomes, as well as ensuring equity across the system and improving efficiency of the education system (Browes, 2020, Camphuijsen Møller and Skedsm, 2020; Lane, 2020), the underlying rationale is based on measuring learner performance using standardised tests and using those measures to create incentives and sanctions for education officials, teachers and/or schools. (Koretz, 2017). However, the implementation of TBA system and how it impacts on teacher practice may differ substantially within and between education systems.

It is important to acknowledge that accountability is, on its own, neither good nor bad. Rather, the effects of any accountability system depend on the policy and political agendas to which it is attached, how it is used, the goals, values, and purposes it serves, and the assumptions it makes about who should be held accountable for what, to whom, under what conditions, with what consequences, and brokered through what power relationships (Cochran-Smith 2021;). Moreover, the implementation of accountability systems involves complex and contradictory processes that require all role-players, especially school actors, to translate and negotiate policy agendas that are attached to different accountability policy tools into specific school practices (Pagès, 2020). Browes (2020) argues that TBA policies will be interpreted and enacted differently in schools at different ends of the performance spectrum, and that teachers' experience of pressure is more closely connected to their schools' response to accountability measures. Similarly, Pagès (2020) notes that within accountability systems, teachers and principals interpret and make sense of different policy messages according to collective and individual subjective accounts. Within the context of education Perryman (2006) notes that 'performativity is linked with the increased accountability and surveillance under which teachers find themselves and their schools being judged in terms of outcome and performance' (p. 150). However, performativity regimes manifest in different ways and at multiple levels across different aspects of the education system.

Ball (2003) defines performativity as

a technology, a culture and a mode of regulation that employs judgements, comparisons and displays as means of incentive, control, attrition and change - based on rewards and sanctions (both material and symbolic) (p. 216).

Accordingly, Ball (2003) argues that performativity regimes create new institutional cultures based on competition that are further entrenched using targets and incentives to promote and regulate specific behaviours and practices. The central feature of these performativity regimes is the "translation of complex social processes and events into simple figures or categories of judgement" (Ball, 2003, 217). Within this context, information is regularly collected, and often made public, for use in monitoring performance of individuals. Drawing on Lyotard's law of



contradiction, Ball (2003) argues that working within performativity driven system creates contradictions between “an increase in the volume of first order activities (direct engagement with students, research, curriculum development) required by the demands of performativity, and the ‘costs’ in terms of time and energy of second order activities that is the work of performance monitoring and management” (p. 221). The consequences of which result in greater focus and emphasis on collection of information for supporting systems of monitoring and control at the expense of fulfilling core responsibilities and tasks, like learning and teaching.

The predominant manifestation of performativity regimes is the use of test-based accountability (TBA) or high stakes testing (HST) systems to drive educational policy and practice comprise (Camphuijsen Møller & Skedsmo, 2020; Hamilton, Stecher, and Klein, 2002; Koretz, 2017). Curricula and performance standards are prescribed at the national level, teacher monitoring and evaluations conducted at the school level, monitoring curriculum coverage and implementation of scripted lessons at the district level while learner performance results are often publicly reported within and across schools, districts and provinces as is the case with the matriculation examinations (Reddy, 2008).

Performativity and pedagogy

Alexander (2008) defines pedagogy as “the observable act of teaching together with its attendant discourse of educational theories, values, evidence and justifications. It is what one needs to know, and the skills one needs to command, in order to make and justify the many different kinds of decisions of which teaching is constituted” (p. 59). Similarly, Westbrook et al. (2013) argue that effective pedagogy refers to “those teaching and learning activities which make some observable change in students, leading to greater engagement and understanding and/or a measurable impact on student learning” (p. 8). Within the context of the classroom environment, teachers’ pedagogical practices manifest in the complex sets of interactions that take place on a daily basis. These practices include how teachers introduce lessons, the questions teachers ask and the opportunities provided for learners to respond, the tasks and activities that teachers set and the support provided for learners to engage with these tasks and activities, the classroom climate that teachers seek to create, the types of learning that teachers seek to promote as well as the types of assessments applied and the manner in which teachers use assessment information for identifying and addressing the specific learning needs of their learners (Alexander, 2008; Westbrook et. al, 2013). In practice, this means that teachers must: (i) be well versed with the national curriculum content; (ii) possess the requisite subject content knowledge; (iii) apply relevant teaching approaches to engage all learners and (iv) effectively use appropriate assessment methods and techniques to determine and improve learners’ level of understanding.

Hay, Tinning and Engstrom (2015) note that the relationship between assessment and pedagogy has been most overtly articulated through the assessment for learning (or formative assessment) approach. The authors argue that the fundamental premise is that assessment provides teachers with information on the progress of their learners’ learning so that appropriate adjustments in



pedagogy and curriculum can be made to optimise future learning” (p. 8). Specifically, Hay et. al. (2015) note that the use of formative assessment allows teachers to determine whether the intended outcomes they had planned to achieve have been realised, and whether to adjust their classroom practice to achieve the intended outcomes. For Hay et al. (2015) formative assessment thus “becomes an important mechanism for promoting pedagogical work through the capacity it offers for establishing the degree of alignment between pedagogical intent and pedagogical consequences and the factors that may be contributing to any misalignments” (p. 8).

Within the education sector, the impact of performativity systems in schools is complex and often contradictory. While policymakers expect improvements in efficiency, academic excellence, and equity, in practice performative accountability can lead to unintended negative consequences (Gurova, & Camphuijsen, 2020). These include curriculum narrowing, teaching to the test, teacher-centred pedagogy, and gaming strategies such as focusing resources on learners near passing thresholds and holding back learners who are unlikely to successfully complete examinations. For teachers’ performativity regimes leads to change in practices at both the organisational level as well as at the level of teachers (Ball, 2003; Browes, Camphuijsen Møller & Skedsmo, 2020). On a regular basis, teachers have to contend with numerous judgements and targets that emanate both from within the school as well as from outside the schools. More concerning is that these regimes have also created contradictions and tensions that impact on teachers’ pedagogical practice. For Holloway and Bass (2018), these performativity regimes have led to the (re)conceptualisation of teacher performance as that which can be quantified and measured, and thus reduced teacher pedagogy to classroom practices that can be made measured, evaluated, and compared. The consequence is that teachers

“are no longer encouraged to have a rationale for practice, account of themselves in terms of a relationship to the meaningfulness of what they do, but are required to produce measurable and ‘improving’ outputs and performances” (Ball, 2003, 222).

The consequences for teacher practice are two-fold. First, the tension between performance and meaningful pedagogical contributions. As Ball (2003) notes, teachers are aware that their pedagogical practices will not be captured by or valued within the specified measures and targets, while at the same time, these specified measures and targets will distort their practice. Second, teachers’ primary role and responsibility, which is to support learners to identify and address their specific learning needs, is unlikely to be prioritised nor supported across the education system. Given the predominant emphasis on external targets and measures within performativity regimes, and often contrasting emphasis and implications of these measures and targets at different levels or among different role-players within the system, it is more likely that teacher practices will focus on areas and/or practices that will enhance attainment of targets and demonstrate improved performance.

Assessment Practices in South Africa: Colonial Period to Post-Apartheid



This section provides a brief overview of the historical context within current accountability policies and practices that need to be understood. Beginning with the colonial period, followed by the apartheid period (pre-1994) and the post-apartheid period, the history of assessment represents one of the most systematic examples of how assessment systems and practices have been used to entrench racial hierarchies and justify social control. This evolution demonstrates how educational assessment, far from being a neutral technical exercise, served as a powerful tool for maintaining economic exploitation and social stratification.

British Colonial Period (pre-1948)

Assessment practices in South Africa during the British colonial period were heavily influenced by, first, the establishment of examination systems within the British colonies, and second, the influence of the psychological assessments. Outside the schooling sector, the need for formal credentials led to the establishment of the first examining bodies, beginning with the Public Service Board in 1850, which was soon replaced by the Board of Examiners in 1858. This board issued certificates in literature and science designed to correspond with UK university degrees. Within the education sector, the system was organised along the four colonial territories at the time, Cape, Natal, Transvaal and Orange Free State, (Boucher, 1969; Ruperti, 1976; Malherbe, 1925, 1977; cited by Kanjee & Makgamatha, 1998). Public examinations became central to the assessment system from the 1870s. Three main examinations were introduced under the University of the Cape of Good Hope: the School Elementary examination at primary level (roughly Standard VI), the School Higher examination (later known as Junior Certificate), and the Middle Class examination (later Matriculation). Standard Six examinations at the end of primary school served as qualifications for apprenticeships and were initially conducted through inspection.

Early assessment of Standard six pupils took the form of an external examination through inspection conducted by the inspector from the Education Department (Malherbe, 1925; 1977; Bot, 1951, cited in Kanjee & Makgamatha, 1998). Malherbe states that an inspector visited a school once a year to administer tests to pupils and inspect their schoolwork. These tests were graded for each standard and often printed on little cards. No teacher was allowed access to these tests. Performance on these tests determined who was to pass and who was to fail. School subjects that were easily measurable such as arithmetic and spelling were given preference while less measurable subjects such as geography and history were excluded. This type of examining not only lacked reliability, it also gave the schools, teachers, and pupils a false sense of values. The primary school external examination formed part of the payment-by-results system (Natal DoE, 1949; Bot, 1951; Malherbe, 1925; 1977; 1951, cited in Kanjee & Makgamatha, 1998). According to this system, the teachers were paid on the basis of the strength of inspection results. Both the examinations and the system of payment-by-results made the teachers and the pupils fearful of the inspectors. The Standard Six external examination ceased to exist in the Transvaal in 1942, in Natal in 1948 and in the Orange Free State in 1956 (Malherbe 1977, 1951, cited in Kanjee & Makgamatha, 1998). By that time compulsory education had been raised to Standard eight, the latter of which became an exit point for employment. Following the formation of the Union of South Africa, the Joint



Matriculation Board (JMB) was established in 1918. The JMB served as the first national certification council to standardise common examinations systems and practices across the provinces, and to administer the end of schooling matriculation examination (Lolwana, 2006). While UMALUSI currently serves as the certification council, the matriculation examination, now known as National Senior Certificate, still serves as the end of schooling examination in South Africa.

The introduction of psychological testing around 1916, stemming from the interest in intelligence measurement and the eugenics movement (Foxcroft and Davies, 2008) had a profound impact on education testing practices. According to Malherbe (1977, cited by Kanjee & Makgamatha, 1998) the use of psychological instruments in assessing scholars started around 1916 under American influence. Throughout the 1920s and 1930s, various institutions including the Research Council of the Department of Mining and Industry, the National Bureau for Educational Research, and universities like Stellenbosch and Grey University College in Bloemfontein drove test development process (Foxcroft & Davies, 2008; Kanjee & Makgamatha, 1998). A critical point noted by Foxcroft & Davies (2008) is that these tests were developed and standardised exclusively for white populations, establishing a troubling precedent that would persist for decades. Foxcroft & Davies (2008) also report that the first recorded abuse of psychological tests in South Africa was by the administration of the Army Beta Test, which was standardised only for white children to black children. In explaining the lower mean performance of Black children, Fick (1929, cited in Foxcroft & Davies, 2008) argued that performance differences were likely due to innate ability differences between racial groups. However, the authors report that this misuse of psychological testing drew sharp criticism from the 1936 Interdepartmental Committee on Native Education, which argued that intelligence tests constructed for white populations could not be validly applied to a different population group. This early critique highlighted the fundamental flaw in applying culturally specific tests across diverse populations, yet such warnings went largely unheeded, setting the stage for decades of discriminatory testing practices that would only intensify under formal apartheid.

Apartheid Era - Post-1948

When the National Party assumed power in 1948, assessment practices became explicitly segregated along racial lines, reflecting the broader apartheid ideology. Throughout this period, the deeply inequitable nature of these assessment systems both reflected and reinforced the apartheid ideology, providing significantly more resources and opportunities to White students compared to other racial groups. Separate education departments were established for Whites (under control of four provinces), Africans living in South Africa as distinct from Africans living in the homelands, (Department of Bantu Education), Coloureds (Department of Coloured Affairs), and Indians (Department of Indian Affairs). This fragmentation extended further with education departments for each of the ten homelands, creating distinctly different assessment regimes based on an elaborate system of tests and examinations by which control and entry into the economy was regulated (Swartz, 1992). It is thus no surprise that the emphasis in education was geared towards rote learning, and thus exam orientated, for both Black and



White South Africans. The development of critical thought and active learner participation in the learning-teaching process was actively discouraged; rather, learners were viewed as mere passive receivers of information (Kallaway, 1984).

Kanjee and Makgamatha (1998) note that White schools gradually abolished Standard Six and Eight public examinations, eventually leaving only Standard Ten as an external examination¹. African schools required "continuation passes" (first or second class) at Standard Six to proceed to secondary education, and from 1975, implemented Standard Five as the final primary school examination. Coloured schools maintained partly external Standard Eight examinations alongside fully external Standard Ten examinations. Indian schools similarly had externally controlled examinations at Standards Eight and Ten, with certificates awarded upon successful completion. The matriculation examinations, administered at the end of Standard Ten by the JMB, continued to serve as the end of schooling certificate, and was used both for selection into the higher education sectors as well as the world of work. The JMB, re-instituted in 1955, became the primary standardizing authority for matriculation across all racial departments, though provincial education departments set their own matriculation examinations subject to JMB recognition. In 1986, the South African Certification Council (SAFCERT) replaced the JMB, and in 2001, SAFCERT was replaced by UMALUSI (Lolwana, 2006).

During the apartheid era, psychological testing in South Africa became deeply entrenched in the country's system of racial segregation, evolving from earlier discriminatory practices into a sophisticated apparatus of social control Nzimande (1995). The ideological foundation of apartheid-era testing, pioneered by Simon Biesheuvel, was rooted in the psychology of racial and ethnic differences and operated on assumptions of inherent differences amongst the various race groups in South Africa (Nzimande, 1996). The discriminatory use of psychological tests during the apartheid era had a profound and lasting consequences, fundamentally undermining the legitimacy of testing among the majority of South Africans. The impact of the race-based practices was manifested within schools through the respective psychological services units within the different education department. This period saw the development of separate measures for different racial groups, example IQ testing instruments, with the National Institute for Personnel Research (NIPR) and later the Institute for Psychological and Edumetric Research (IPER) - both eventually incorporated into the Human Sciences Research Council (HSRC) - becoming the dominant forces in test development (Kanjee & Makghamatha, 1998).

Post-1994 to 2020 (pre-COVID 19)

In addressing the vestiges of the apartheid system and the introduction of a non-racial, non-sexist, democratic education system in South Africa, a plethora of policies and acts were implemented to provide high-quality education to all children. Key among these include the: National Education Policy Act (1996) that created mechanisms for developing and implementing cohesive national education policies; the National Qualifications Framework (NQF), instituted through the South African Qualifications Authority (SAQA) Act (1995),

¹ At the time, 12 years of schooling comprised: Grades 1 and 2, and thereafter Standards 1 to 10, with Standard 5 making the end of Primary Schooling and Standard Ten marking the end of Secondary Schooling.



which established a national framework for educational qualifications, the South African Schools Act (1996) which aimed to transform institutional arrangements across the system to promote the fundamental rights to education enshrined in the new Constitution; as well as the National Curriculum Statements (1997) and the General Education and Training Assessment Policy (1998), both underpinned by an outcomes-based education approach (Spren & Valley, 2010), designed to move education beyond apartheid-era pedagogical approaches and toward a system that could deliver both equitable access and educational quality.

The NQF aimed to create a unified national framework for learning achievements, improve access and progression across education and training pathways, enhance quality, and address the historical discrimination in educational and employment opportunities. Formal schooling comprised of the General Education and Training Band (Grades R–9) and Further Education and Training (Grades 10–12, with Grade 12 as the final exit point), stipulating qualifications, outcomes, and standards for learner assessment. Complementing the NQF, Outcomes-Based Education (OBE), adopted through Curriculum 2005, represented a radical shift towards a learner-centred, results-oriented educational delivery. This approach emphasised clear identification of learning outcomes, progression based on demonstrated achievement, accommodation of diverse learner needs through varied strategies and assessment tools, and providing learners with sufficient time and support to reach their full potential. Both the NQF and OBE fundamentally relied on assessment as a central mechanism, grounding curriculum design, content, and delivery in the assessment of essential knowledge, skills, attitudes, and values, and focusing assessment definitions on learning outcomes and phases.

Despite this ambitious intent, a review of the policies during this period revealed a growing trend towards stronger accountability and performativity regimes that continue to dominate the schooling system in South Africa. Jansen (2001) notes that the OBE approach implemented in post-apartheid South Africa was a top-down reform model that restructured the education system around measurable learner performance outcomes rather than the content delivered by teachers. Rooted in a managerial logic, Jansen (2001) argues that the OBE approach promoted a standardised outcomes across all education level that prescribed what learners should be able to do as a result of instruction. In practices this was manifested in prescriptive performance indicators such as the 66 specific outcomes in Curriculum 2005 (DoE 1997), over 140 competencies in teacher education specified in the norms and standards for teachers (DoE, 2000), and performance-based qualifications in the National Qualifications Framework (SAQA, 1995).

Key Assessment and Curriculum policies pre-COVID era

- National Curriculum Statements (DoE, 1997c, 1997d, 1997e, 1997f);
- the Assessment policy in the general and further education and training bands (DoE, 1998); the Revised National Curriculum Statements (DoE, 2002);



- National Policy on Assessment and Qualifications for Schools in the General Education and Training Band (DoE, 2007);
- Foundations for Learning Campaign (DoE, 2008);
- Action Plan to 2014² (DBE, 2010a);
- Delivery Agreement of the Minister of Basic Education (DBE, 2010b);
- National Protocol for Assessment: Grade R-12 (DBE, 2011a);
- Curriculum and Policy Statements, popularly known as CAPS, (DBE, 2011b);
- South African Democratic Teachers Union – AfL Professional Development Programme (Kanjee, 2012)
- Policy on the Organisation, Roles and Responsibility of Education District (2013);
- Action Plan to 2019 (DBE, 2015);
- Annual Teaching Plans (DBE, 2016);
- Systemic evaluation model for the basic education sector (2017);
- Identification Management and Support of Underperforming Schools (DBE, 2017);

Figure 1: Key Assessment and Curriculum Policies and programme Implement 1994 – 2015

Curriculum and assessment policies

The underlying rationale of the assessment (DoE, 1998, 2005, 2007; DBE, 2011a) and curriculum policies (DoE, 2002, DBE, 2011b, 2011c) was to transform apartheid era systems and practices that sought to deprive black South Africans access to resources and opportunities and stifled their intellectual development to meet the needs for a cheap source of labour (Mathonsi, 1988; Nzimande, 1995; Swartz, 1992). The seminal assessment policy (DoE, 1998) advocated for an assessment for learning approach to make the shift from the high-stakes examinations-based system to an assessment system in which both formal and informal assessments are used for improving learning and teaching. However, early education debates and policies in South Africa paid minimal attention to the crucial issue of assessment, both at the systems and classroom levels (Kanjee, 1998; Pahad, 1996, Potenza & Monyoko, 1998). The practical consequences of this oversight meant that the implementation of the new curriculum and assessments policies laid the foundation for the unintended consequences that continued to impact on teaching and learning within the schooling systems for the last three decades. Kanjee and Sayed (2013) report that while notions of assessment for learning and/or formative assessment have been promoted across different versions of these policies, the current system is strongly dominated by policies and practices that are “*assessment focussed, and measurement driven*”.

Within schools, the School-Based Assessment (SBA) constituted the basis on which all assessments were conducted. In practice, SBA in comprises all formal assessment tasks (i.e.

² Performance agreements, linked to the Action Plan targets, were signed between the President of the country and the Minister for Basic Education.



class tests, assignments, projects, end-of term tests as well as end-of-year examinations) conducted by schools and teachers during the academic year as part of the continuous assessment component specified in the National Curriculum and Assessment Policy Statements (CAPS) (DBE, 2011a; DBE, 2011b, c). These assessments are compulsory for all learners and are used for purposes of monitoring progress, promotion, and at the Grade 12 level, certification. The implementation varies by educational phase, with Foundation Phase (Grades R-3) employing mostly informal, integrated assessments like observation checklists and oral responses, while Intermediate and Senior Phases (Grades 4-9) utilise more formal tasks including tests, projects, and investigations that constitute 75% of the final promotion mark. In the Further Education and Training Phase (Grades 10-12), SBA becomes highly standardised and externally quality assured, contributing 25% to final marks in most subjects. Across different subject areas and grades, the number and types of formal assessment are clearly prescribed as are the guidelines for reporting and recording of results (Kanjee & Sayed, 2013; Moloi & Kanjee, 2018). Table 1 provides an example of the formal assessment prescribed for the Intermediate (grades 4, 5 and 6) and Senior Phase (grades 7, 8 and 9).

Table 1: Formal assessment prescribed mathematics (Grades 4 to 6)

	Forms of assessment	Minimum requirements per term				Number of tasks per year	Weighting
		Term 1	Term 2	Term 3	Term 4		
SBA	Tests	1	1	1		3	75%
	Examination		1			1	
	Assignment	1			1	2	
	Investigation				1	1	
	Project			1		1	
	Total	2	2	2	2	8	
End of the year examination						1	25%

The SBA system serves multiple accountability functions across different levels of the education system. At the teacher level, educators must develop and administer tasks aligned with national standards, subject to moderation by district officials and subject advisors, with non-compliance potentially resulting in interventions. Schools use SBA results for improvement planning and are monitored by School Management Teams, School Governing Bodies, and district offices, with underperforming schools identified for additional support or intervention programs. At the systemic level, SBA data feeds into the Education Management Information System (EMIS), informing provincial and national reports on learner achievement and curriculum coverage, while moderation processes form part of national strategies to improve assessment integrity, particularly for high-stakes Grade 12 examinations. Moreover, the assessment policy (DBE, 2011a) as well as the CAPS (2011b,



c) also prescribes a framework for reporting and recording assessment results comprising three key elements, namely rating codes, descriptions of competence and percentages (Table 2). A critical limitation of the prescribed reporting framework is the lack of any descriptions regarding what learners should know and be able to do, and thus limited information is provided to assist learners, parents and teachers in identifying and addressing the strengths and weaknesses of their learners (Moloi & Kanjee, 2018).

Table 2: Reporting requirement specified in the CAPs for Grades (Grade 7 to 9)

Code	Description	Percentages
7	Outstanding achievement	80 – 100
6	Meritorious achievement	70 – 79
5	Substantial achievement	60 – 69
4	Adequate achievement	50 – 59
3	Moderate achievement	40 – 49
2	Elementary achievement	30 – 39
1	Not achieved	0 – 29

More recently, the National DBE, in collaboration with Provincial Education Departments, also introduced the Annual Teaching Plans (popularly referred to as ATPs) to assist teachers with the effective implementation of the curriculum (Figure 2). First implemented in 2016, the ATPs comprise sets of a schema with dates and notional hours for the content topics teachers need to teach for each school week, and when and how it should be assessed (GDE, 2016). Bertram, Mthiyane, and Naidoo (2021) note that this approach focuses on curriculum coverage, which is often attained at the expense of real learning.



Mathematics Annual Teaching Plan 2016 Grade 2, Term 1									
Cycle	WEEK	Date	COUNTING	MENTAL MATHS	WEEKLY CONCEPT	WORD SUMS	DATE COMPLETED	Percentage Completion 2.5% per week = 25% per term	SBA
1	1	13 – 15 Jan	Count reliably objects to 50	Number concept: Range 15 Order a given set of numbers	Number symbols and number names • Recognise • Identify • Read • Write	Solve problems: • drawings concrete apparatus			
	2	18 -22 Jan	Count reliably objects to 100	Range 99 Compare numbers more/less	Numbers to 25 • Describe • Compare • Order • Position	Solve Problems: • Building up • Breaking down			
	3	25 -29 Jan	Count forwards / backwards • 1s up to 100	Know which number is • 1more/less	Place value: Numbers to 25: • Recognise • Know • Decompose • Identify	Solve Problems: • Doubling • halving			Oral
	4	1- 5 Feb	Count forwards /backwards • 10s up to 100	Know which number • 2more • 2less	Performing Calculations • Concrete/drawings	Solve Problems: • Number lines			Practical:



Figure 2: Grade 2 Mathematics ATP for term 1 (Obtained from GDE, 2026).

In their review of curriculum reform process in South Africa, by Muller and Hoadley (2019) note that the current Curriculum and Assessment Policy Statements (CAPS) represents a performance-based curriculum that “provides high specification of subject content to be covered, as well as clear stipulations regarding the sequencing and pace of coverage (p. 119). Bertram, Mthiyane, & Naidoo (2021) occur with Muller and Hoadley (2019) arguing that the initial version of the curriculum was under stipulated in terms of content, the current CAPS is much more structured. These authors highlight a key concern that this approach to curriculum implementation will result in focus on coverage only or on learners’ understanding of the content.

Grade 12 Matriculation Examinations

The Grade 12 Matriculation examination, currently known as the National Senior Certificate (NSC) examination, serves as the final school-leaving examination in South Africa, marking the culmination of 12 years of schooling for learners. This final secondary school examination can be traced back to the early colonial periods when it was used as a measure of educational attainment and as a means of selecting university entrants. The Joint Matriculation Board (JMB), established in 1918, was initially responsible for administering these examinations across the former provinces, before being succeeded by the South African Certification Council (SAFCERT) in 1986. SAFCERT was replaced by UMALUSI that continues to oversee national norms and standards for certification and determines admission requirements for higher education (Lolwana, 2006). The matriculation examination, now NSC, has a profound impact on the lives of learners in South Africa, primarily serving as the most significant determinant of access to higher education, and as well as the job market. Achieving a Bachelor's-level pass is crucial for pursuing university studies, and these qualifications decisively influence the opportunities available to young South Africans.

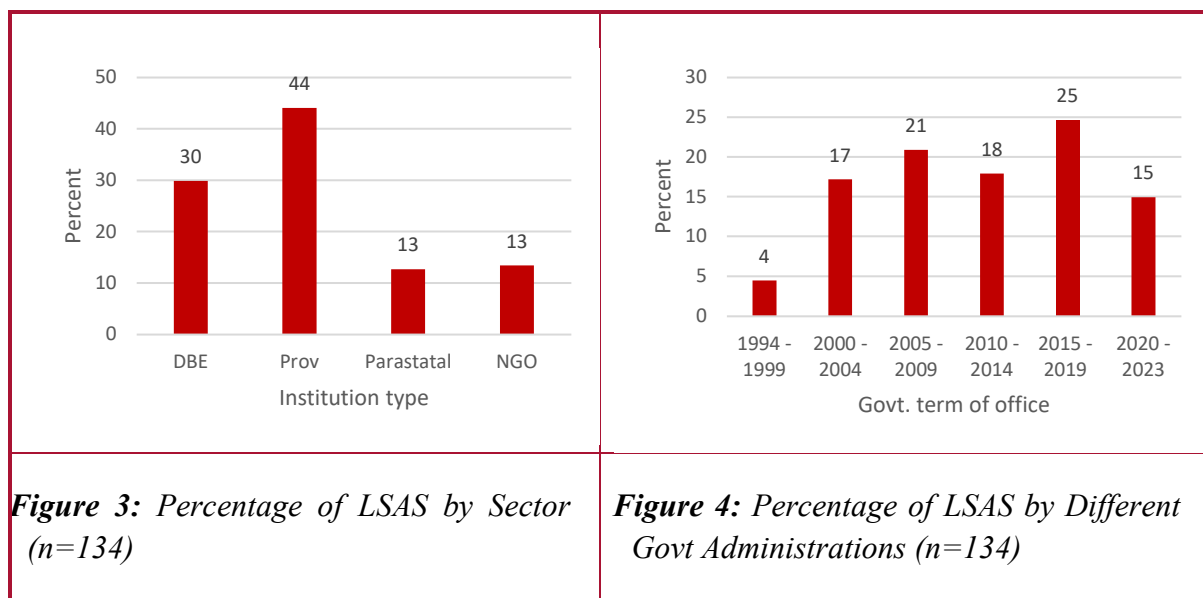
The NSC examination is also utilised as a tool for improving the quality of education through assessment, monitoring, and supervision across the schooling system. The DBE employs an "inclusive basket" of indicators for evaluating Grade 12 results, moving beyond a sole reliance on the traditional pass rate which can be distorted by various factors (DBE, 2022a). In this regard, the Action Plan to 2024 notes that this comprehensive approach aims to create a "results-oriented mutual accountability" culture within the education sector through setting specific targets for learners, such as increasing the number of those eligible for Bachelor's programmes and those achieving passes in mathematics and physical science (DBE, 2022a). The results also inform provincial and district authorities, enabling them to identify underperforming schools, and target interventions to schools most in need, as well as providing parents with better information on their children's educational progress.

Large-Scale Assessment Studies in South Africa

Since the beginning of the post-apartheid era, large-scale assessment studies have been a feature of the schooling system in South Africa, typically comprising (i) national assessment



studies (NAS), (ii) Systemic Evaluation Studies (SES), and (iii) International assessment studies (IAS) also referred to as cross-national assessment studies. To date there have been a total of 134 large-scale assessment studies, implemented at the district, provincial or national level, conducted in South Africa (Kanjee, 2024a)³. These include studies implemented by the national and provincial departments of education, parastatals as well as NGOs, and focuses primarily on assessment of learner performance in languages and mathematics. The primary purpose for conducting these studies varies from monitoring and evaluation of the system to monitoring and evaluation of projects and/or programmes.⁴ The majority of these studies (Figure 3) were implemented by the National (34%) and Provincial Departments of Education (44%). After the end of the first term of office for the newly elected democratic government, the percentage of LSAS increased from 17% to 25% between 2000 and 2019 across the different administrations of government in the post-apartheid era (Figure 4). A key gap in the literature is that no information is available on the costs of these studies. However, estimates place the costs at between one and two billion rand over the last 30 years. The 2019 TIMSS study, for example, cost R45 million while the latest PIRLS cost R42 million as revealed in the DBE tender notices.



A disaggregated analysis of the LSAS implemented across the different terms of office of the government (see Figure 5) reveals that during the first two terms of office, the majority of these LSAS were implemented, and funded, by NGOs and/or parastatals, with the Human Sciences Research Council, a government parastatal organisation responsible for the majority of these studies. In the later terms of office, specifically, the last three terms, all LSAS were implemented under the auspices of, and funded by, the Provincial and National Departments of Education. The majority of studies during this period comprised the annual Western Cape Education Department Provincial Assessment Studies as well as the Annual National

³ The information presented in this section cuts across the timelines specified in the headings given the context in which the LSAS were implemented.

⁴ Excludes pilot studies or studies undertaken to validate instrument



Assessments, a census study implemented in all grades from 2012 and 2014 by the DBE.

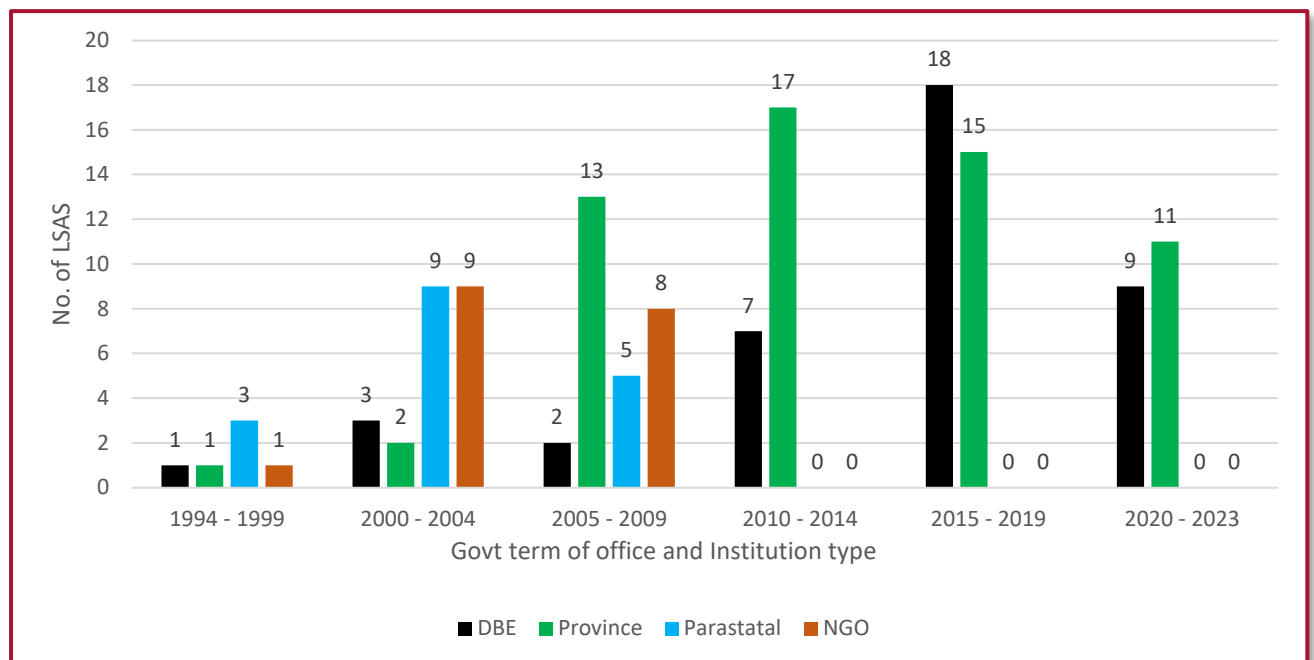


Figure 5: Number of LSAS by Government Term of Office and Institution Type (n=134)

Notwithstanding the significant increase in the number of learner assessment studies implemented, there has been limited use of this information to support data-driven decision-making aimed at identifying and addressing key challenges impacting quality and equity within the education system. While the underlying rationale for the limited use of assessment information sector varies across the different stakeholders, key reasons include the limited and/or timeous access to assessment data (Moloi & Kanjee, 2012); poor reporting and dissemination of assessment results (Kanjee & Moloi, 2016); and limited capacity in the analysis, interpretation and incorporation of assessment data into decision-making at both the classroom and systems levels (Kanjee, et. al., 2024a; 2024b).

The Foundations for Learning Campaign (FLC)

Intended as a four-year campaign, the Foundations for Learning Campaign (FLC) was launched in 2008 to increase the average performance of all Grade 1 to 6 learners in Languages and Mathematics to not less than 50% (DoE, 2008). To attain this goal, the Department of Education: (i) provided all teachers with the appropriate resources for effective teaching and learning, e.g., structured lesson plans and work schedules; (ii) stipulated rigid requirements for teaching time and daily activities; (iii) required all teachers to assess, track and record learner progress and achievement on a monthly basis, and to report this information to the district office on quarterly basis. District officers were also required to submit these reports to the provincial office; and (iv) introduced standardised testing for all Grade 3 and 6 learners to measure progress made to achieving the set targets (DoE, 2008). Kanjee and Sayed (2013)



contend that the FLC introduced the notion of ‘census testing’ to the education landscape in South Africa for holding school heads and teachers accountable for the performance of their learners. However, the implementation of the FLC was beset with a myriad of problems and it was ultimately abandoned (Meier, 2011). Interestingly, the FFLC also specified that the “for the duration of the campaign, South African will not participate in regional or international assessment studies assessing learner competency levels in Literacy and Numeracy in the GET Band (DBE, 2008, 5).

Action Plan to 2014 and Action Plan to 2019.

First published in July 2010 for public comment (DBE, 2010), and finalised in January 2012, the Action Plan 2014: Towards the Realisation of Schooling 2025⁵ (DBE, 2012a) proposed a sector-wide plan for South Africa's schooling system. The plan is intended mainly for two groups: managers responsible for implementation and people responsible for monitoring the success of implementation (DBE, 2012). The Action Plan to 2014 highlighted the following key guiding principles to ensure effective education reform:

- maintaining a clear focus on learning outcomes, noting that output goals (what children learn) must take priority over inputs and processes.
- promoting evidence-based, context-specific actions, while acknowledging the need for tailored solutions; to address local needs.
- ensuring effective communication between government, unions, schools, and parents to enhance broad understanding and participation.
- developing a technically sound accountability system, using reliable data to track progress.

The Plan introduced an accountability based long-term national programme for addressing the challenges of poor quality in schools and specified the key indicators and goals to be attained. In addition, these goals and indicators were also incorporated to the education ministers’ delivery agreement (DBE, n.d.) that is signed with the President of the country. The Plan comprises of 27 goals, of which the first 13 focus on key outputs and are measured by 18 indicators, while 14 goals focus on how the outputs are to be achieved, also measured by 18 indicators. The plan outlined 27 goals, divided into "output goals" (Goals 1-13, focusing on learning outcomes and coverage) and "how-to goals" (Goals 14-27, detailing implementation strategies). It emphasises a data-driven approach, drawing on sources like Annual National Assessments (ANA), General Household Survey (GHS), and the National Education Infrastructure Management System (NEIMS), to set realistic but ambitious targets and milestones. A key strategy proposed in the Action Plan is the implementation of the Annual National Assessment (popularly referred to as ANA) examination results as well as the regional and international assessments as a key mechanism to improve quality through assessment, monitoring and supervision. Twelve of the 18 indicators are based on assessment results: ANAs (6 indicators); Grade 12 National Senior Certificate Examinations (3); SACMEQ (2); and TIMSS (1)⁶. Of these 13 indicators, ten focus on performance in literacy and numeracy.

⁵ Updated to Action Plan 2019 (DBE, 2015) and. Updated to Action Plan 2024 (DBE, 2020)

⁶ SACMEQ refers to the Southern African Consortium for Monitoring Education Quality; TIMSS refers to the



Of note is that ANA indicators do not specify any performance targets but merely list “performance at the required level” (p. 173) with no information included on what is meant by ‘required level’. Addressing the lack of specificity, the performance targets were set at 50% in the country’s National Development Plan (NPC, 2012).

Regarding teacher and school incentives, no explicit specifications are provided across the different versions of policies reviewed. However, the Action Plan 2014 (DBE, 2012) argued for strengthening teacher accountability using a points-based system to monitor teacher development, leaving the issue of incentive unresolved. Notwithstanding these proposals listed, notions of a test-based accountability system for teachers still prevailed and remained unresolved. In the Action Plan to 2019 (DBE, 2015) the incentives linked to the ANA results are associated with the National Teacher Awards although no details are specified on how this will be implemented in practice. The Action Plan to 2014 set numeric targets for literacy and numeracy, while national assessments such as ANA and TIMSS heavily emphasised mathematics outcomes, making math a cornerstone of accountability metrics. The underlying approach in addressing the issue of accountability specified in the Action Plan to 2019 acknowledges that it is important to maintain a balance between monitoring and accountability and support and argues for accountability systems to consider issues of capacity as well as context within which education is provided including learner’s socio-economic backgrounds. However limited guidelines are provided on how the balance between accountability measures and provision of support will be addressed, and in particular, what and how support will be provided.

National Development Plan

The National Development Plan 2030 (NDP) is South Africa's comprehensive long-term vision and strategic framework designed to eliminate poverty and reduce inequality by 2030. The plan was developed by the National Planning Commission through extensive research, consultation, and public engagement, and adopted by the cabinet in 2012 (NPC, 2012). The NDP provides a holistic approach covering multiple sectors including economy, infrastructure, environmental sustainability, human settlements, education, healthcare, social protection, community safety, and state capabilities. Key implementation principles include promoting citizens to take responsibility for their own advancement while holding leaders accountable, developing a capable developmental state with strengthened accountability chains and improved coordination, adopting an integrated approach recognizing interdependence between different areas of development, and establishing a results-oriented framework of mutual accountability. The results-oriented mutual accountability approach advocated emphasises shared responsibility among all stakeholders—government, educators, parents, and learners—for achieving clearly defined education outcomes. This approach is based on three key pillars: (1) agreed-upon development goals aligned with national priorities, (2) measurable indicators to track progress, and (3) reciprocal accountability, where all actors are held answerable not



only for their roles but also for supporting each other to achieve these goals.

Chapter 9 of the NDP focuses on education, training, and innovation highlighting four critical capability-building mechanisms in: Early Childhood Development, Basic Education, Post-School Education, and the National Research and Innovation System. For the schooling sector, the Plan sets ambitious targets for education transformation that includes the following: establishing universal access to two years of quality early childhood development for all children aged 4-5 years before Grade 1; ensuring that 90 percent of learners in grades 3, 6, and 9 master the minimum competencies (defined as 50 percent and above) in language and numeracy, increasing the number of learners eligible for bachelor's programmes to 450,000 with strong mathematics and science qualifications, and improving South Africa's grade 8 TIMSS scores from 264 to 500 points by 2030 to be comparable with countries at similar development levels. In addition, the NDP also targets achieving secondary school completion rates of 80-90 percent, including learners in Further Education and Training colleges, with education becoming compulsory up to Grade 12 or equivalent vocational levels. Here too, the results-oriented mutual accountability model is performance-based with the aim to improve quality and equity. The NDP chapter advocates for routine testing, rigorous monitoring, and public reporting of learner outcomes as tools to drive improvements in the system. However, this emphasis also reinforces performativity, where the pressure to meet targets can lead to unintended consequences such as narrowing the curriculum, teaching to the test, and sidelining broader developmental goals.

Teacher Union Collaboration

The post-apartheid era also saw the development of new teacher unions that played a significant role in supporting national initiatives for improving teaching and learning. Notwithstanding the inherent employer-employee tensions and sometimes conflictual relationship between the Unions and the DBE, a key initiative launched in 2011 was the Teacher Union Collaboration (TUC). The TUC is a partnership programme developed in 2011 between the Department of Basic Education (DBE) and five major teacher unions: the National Professional Teachers' Organisation of South Africa (NAPTOSA), the National Teachers' Union (NATU), the Professional Educators' Union (PEU), the South African Democratic Teachers' Union (SADTU), the Suid-Afrikaanse Onderwysers' Unie/South African Teachers' Union (SAOU). The primary objectives of the TUC are to address teacher development issues, ensure ongoing professional development, strengthen the education system's delivery capacity for teacher development, and support the professionalisation of teacher unions (Govender & Cross, 2023).

Within the context of TUC, a key programme spearheaded by the largest trade union in 2012, SADTU, is the Assessment for Learning (AfL) Teacher Professional Development Programme. This Programme was implemented in collaboration with the Department of Primary Education at the Tshwane University of Technology where research on the use of AfL was being conducted to support teachers improve their pedagogical practices (Kanjee, 2012). The SADTU Programme focussed on the development of 450 "Lead facilitators" intended to provide training and support to other teachers across all the nine provinces in the country.



Phase 1, which focused on the Lead Facilitators was intended to be followed by the rolling out Phase 2 into schools. However, challenges in securing funding through the TUC delayed the implementation of the second phase.

Research on relevance of the AfL approach

Assessment for Learning (AfL) is a pedagogical approach based on the idea that significant improvements in learning take place when teachers and learners are able to obtain and use relevant assessment evidence for deciding where learners are in their learning, where they need to go and how best to close gaps in learning (Wiliam & Thompson, 2007). Research on AfL indicates that teachers who effectively use this approach see considerable learning gains among learners, and that the learning gains are noted for all learners, irrespective of their socio-economic backgrounds. However, teachers require high levels of expertise, experience, as well as appropriate tools and techniques, to successfully apply AfL approaches to benefit learners.

Given that, research on AfL at that time was primarily available from the north, it was critical to determine the applicability and relevance of this approach to the developing nations context. To determine the relevance of this approach, several research and intervention projects were undertaken in South Africa, including a randomised control trial (Kanjee, 2022; 2023; Kanjee & Bhana, 2022; Mthembu, Kanjee and Hill, 2023; Ramollo & Kanjee, 2023;). Given their critical responsibility of monitoring and supporting teachers' classroom practices, additional study, funded by UNICEF and the National Education Collaborative Trust, was also conducted involving subject advisors from three provinces. (Kanjee, 2023). Results from the independently conducted evaluation of the Formative Assessment Professional Development Programme for Teacher (FA-PDP), funded by the Zenex Foundation (Kanjee, 2018), implemented as a randomised control trial, demonstrated significant positive impacts on teachers' assessment knowledge and classroom practices across both fee-paying and no-fee schools (Kanjee & Bhana, 2022). The findings also revealed that effective implementation of the programme had a substantial positive impact on teachers in poorly sources, no-fee paying schools as well. Teachers from no-fee schools who participated in the intervention programme achieved higher levels of formative assessment competency than their better qualified colleagues from well-resourced, fee-paying schools who did not participate in the programme. The programme demonstrated a positive influence of the use of formative assessment on learner engagement during lessons, though further research is needed to establish the direct connection between improved pedagogical practices and enhanced learning outcomes.

Drawing on the results of the FA-PDP evaluation (Kanjee, 2018) as well as the Assessment for Learning in Africa (AFLA) project several key changes affected how the AfL approach was to be applied in the South African context. First, the conceptual and theoretical frameworks of the AfL approach applied in the UK and USA were adapted to suit the specific learning and teaching context of South African schools. Specifically, the five strategies proposed by Wiliam and Thompson (2007) were extended to seven strategies with planning and preparation and reflection and review added to account for the South African context. Second, to ensure uniformity in application and interpretation, a relevant theory of change, see Figure 6, as well



as appropriate definitions and understanding of concepts like AfL, AoL, formative assessment, summative assessment and formative assessment were agreed upon.

“If we support Lead Teachers to enhance their formative assessment knowledge, understanding and skills and to provide schools and teachers with appropriate support: (i) to enhance their capacity and use of formative assessment to obtain evidence of learners’ learning needs, and (ii) to develop and implement appropriate interventions that address the specific learning needs of learners; this will result in greater learning and improved learning outcomes.

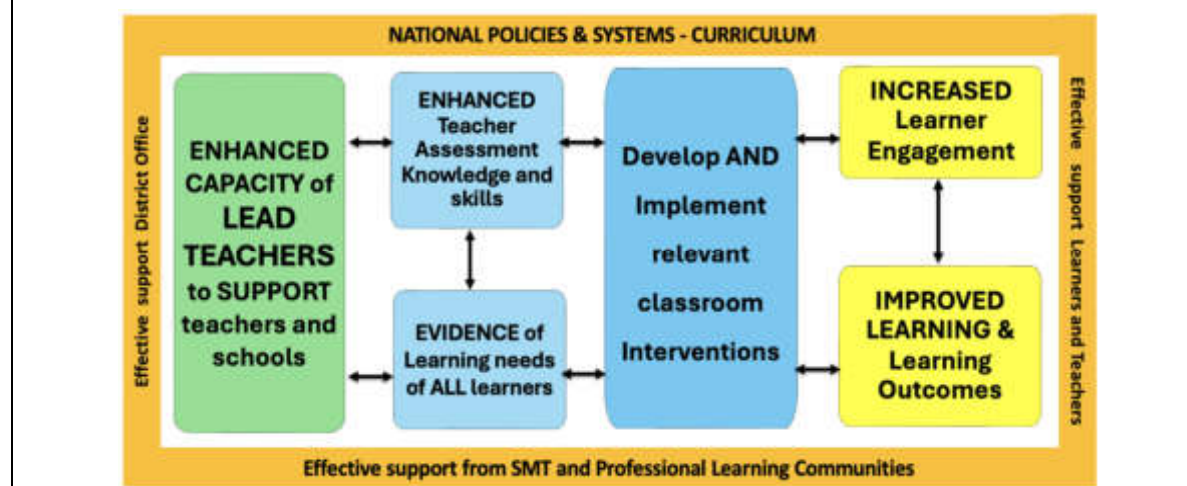


Figure 6: Theory of change for the Formative Assessment Lead Facilitator Programme (from Kanjee, 2018)

Third, an implementation model, referred to a ReMAPS (Kanjee, 2018), see Figure 7, was conceptualised and developed, with appropriate materials, to account for the specific context that define teaching and learning in South African schools across the different quintile categories (Kanjee & Bhana, 2022; Kanjee, 2022, 2023). The model aimed to support participants develop the required practical skills as well as theoretical knowledge of teachers and subject advisors to enhance pedagogical practices focussing on addressing specific learning needs of all learners in their classroom, especially learners from poor and marginalised communities. The ReMAPS model is based on a cyclical approach that (i) begins with workshops where participants collaborate to develop and acquire new knowledge, understanding and skills with facilitators and each other; and (ii) include applications where participants apply their new knowledge, understanding and skill in the classroom setting. The model comprises the following key activities applied in the process of developing new knowledge and skills:

- (i) **Reflect** on their current knowledge and skills,
- (ii) **Mediate** new knowledge and understanding during the workshops,
- (iii) **Adapt** this new knowledge and skills to their specific contexts,
- (iv) **Plan and prepare** for implementation and
- (v) **Present/Practice** their newly acquired FA skills in the classroom
- (vi) **Support** from professional learning communities, peers, facilitators, and online sessions underpins all activities.

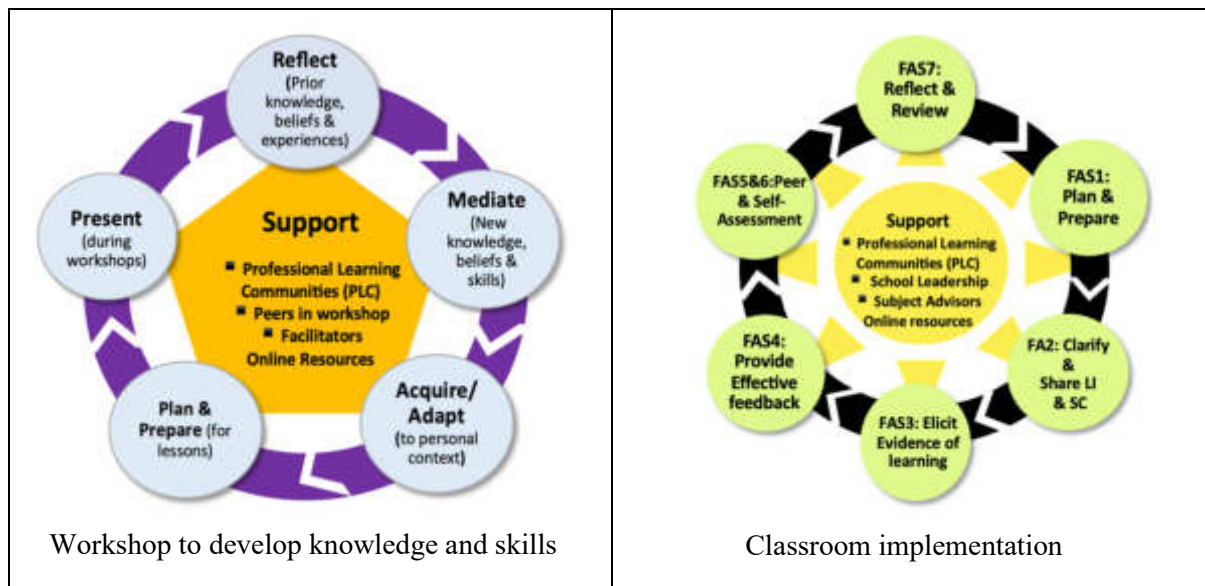


Figure 7: Overview of ReMAPS Model (from Kanjee, 2018)

Fourth, an appropriate framework for scaling up the professional development programme was developed, See Figure 8, that included support and capacity development for key decision makers at the national, provincial and district levels who are tasked with policy implementation and supporting schools and teachers as well as ensuring sustainable implementation of new knowledge and skills across the system. The framework for scaling up the programme comprised Four Phases⁷.

Phase 1: Selection and training of Lead Teachers.

Phase 2: Scaling up of programmes by Lead Teachers.

Phase 3: Supporting Lead Teachers to implement Assessment for Learning in schools.

Phase 4: Integrating Assessment for Learning approach in schools for new and current teachers.

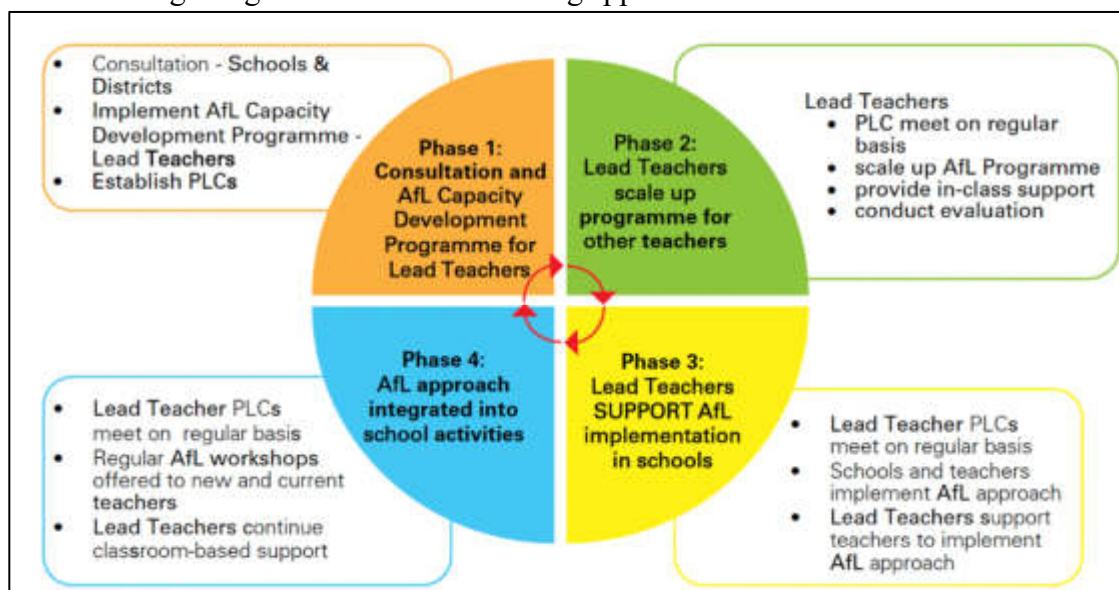


Figure 8: Key phases for scaling up the AfL intervention (from Kanjee, 2018)



New national assessment framework

In the aftermath of the cancellation of the Annual National Assessments, the DBE called for the implementation of a National Integrated Assessment Framework to address several limitations that hinder the effective use of assessment data for improving learning and teaching within the General and Further Education and Training Band (DBE, 2017; Mwelil 2018; Chetty 2019). These limitations included the dominant role of examinations, the ineffective use of assessment for identifying, and addressing the learning needs of learners, the poor quality of school-based assessment and the lack of a reliable indicator of system performance. The new framework proposed radical changes across the schooling system, that included: (i) implementing AfL as a pedagogical approach at the system school and classroom levels; (ii) introduction of General Education Certificate at the end of Grade 9; (iii) the implementation of Systemic Evaluation Studies in place of the ANAs and (iv) overhauling the school-based assessment. The renewed emphasis on improving the assessment system, and in particular, enhancing the implementation of AfL approaches, was a positive step to challenging the dominant performativity and accountability discourse and practices (Ramollo & Kanjee, 2023).

Decolonise education movement

The #RhodesMustFall and #FeesMustFall student campaigns in 2015 reignited the debate on the decolonisation of education in South Africa (Sayed, et. al., 2019). Students called for the elimination of Eurocentric curricula and institutional practices, and for the inclusion of African, indigenous, and other non-Western epistemologies. Students also linked their demands to broader social justice imperatives, arguing that educational decolonisation was intrinsically linked to demands for free, quality education for all. More importantly, the student demands called for comprehensive transformation that extended beyond policy changes to include fundamental shifts in teaching methods and institutional cultures to address the colonial-based systems governing knowledge production and dissemination within South African higher education institutions.

The impact of the decolonised education movement also extended to the schooling sector in South Africa. Compared to the substantial publications on the decolonising education within the higher education sector (Sayed, et. al., 2019; LeGrange, 2019), limited information is available on how these debates impacted on policy and practice within the schooling sector. Notwithstanding the limited number of publications, compared to the higher education sector, evidence from several sources indicates that these debates have influenced the thinking and practices of key policy makers within the schooling sector. In developing the new assessment framework, Chetty (2024) argued for a critical review of how standards from the global north were being used in the country and called developing local norms and standards for determining levels of performance by which children from Africa are judged as well as for rethinking the role and impact of international assessment in developing nations. Similar, in the strengthening of the curriculum process, issues of decolonising the curriculum to reflect the specific needs and aspiration of local populations as opposed to dominants Eurocentric views that impacted



on the current curriculum was highlighted as a key priority for curriculum specialists to consider as they fulfilled their tasks across the various subject areas (DBE official, 2024, personal communication). In addition, the Action Plan 2024 that was compiled in 2020 (DBE, 2020) explicitly presents decolonisation as one of the key issues impacting on effective transformation of the schooling sector. Findings from the third cycle of School Monitoring Needs survey (DBE, 2022) also revealed high levels of agreement among school principals and teachers regarding the need to ‘decolonise the curriculum’. Of the Grade 6, 9 and 12 teachers who had a view on ‘decolonising the curriculum’ (i.e., 81% of principals and, on average, 76% of Grade 6, 9 and 12 teachers, there was high level of agreement that: (i) issues of decolonisation should be taught at school for learners in primary school (81%); (ii) decolonising the curriculum in South Africa can help address key challenges in the country (87%); (iii) decolonisation of the curriculum can address how issues of inequality in South Africa are understood (89%)

The most extensive focus on the decolonisation of the curriculum in the South African schooling system was provided by the History Ministerial Task Team (HMTT) (DBE, 2018). The HMTT critiqued the current Eurocentric bias of the history curriculum and advocated for a more comprehensive and Africa-centred approach to adequately equip learners with the critical skills needed to understand a complex past. The recommendations called for: (i) an epistemological shift toward African ways of knowing that incorporates oral traditions and archaeological evidence; (ii) countering South African exceptionalism by situating the country within broader African contexts to promote Ubuntu philosophy and human solidarity; (iii) comprehensive teacher development and structural reforms including dual examination papers rather than superficial content adjustments. The report also provides an extensive discussion, and provides key proposals, on teacher professional development as a critical component in the decolonisation and strengthening of the History curriculum in South Africa. Additionally, the report notes that assessment practices in the GET band should be revisited, however, no details are provided on what the gaps need to be addressed, nor how this should be attained.

COVID-19 and post COVID period

This section discussed key policies as well as research initiatives and projects implemented post the COVID-19 pandemic, highlighting the impact on policy and practices regarding the issue of accountability and performativity on the schooling system (See Figure 9).



Key Assessment and Curriculum policies during and post-COVID

- School recovery plan in response to COVID-19 (DBE, 2020);
- Teacher Guidelines for Implementing Revised Annual Teaching Plans (ATPs) (DBE, 2020);
- Quality Assurance of the Assessment Programme in the GET Band, (DBE, 2020);
- Guidelines for implementing Formative Assessment in the GET Band (DBE, 2020);
- Action Plan to 2024 (DBE, 2020);
- Draft Policy for the General Education Certificate (GEC), (DBE, 2021);
- Adoption of Assessment for Learning as a national pedagogical strategy (DBE, 2021);
- Identification Management and Support of Underperforming Schools (DBE, 2023).

Figure 9: Key curriculum and assessment policies implemented post-2017

AfL as a pedagogical approach

While discussions on the value and use of Assessment for Learning as a national pedagogical approach have been ongoing since the cancellation of the Annual National Assessments (Chetty, 2017; DBE, 2016), and the DBE circulated detailed guidelines on the use of formative assessment in late 2022 (DBE, 2020) class in it was only in March 2021, in the middle of the COVID-19 pandemic, that the DBE announced that the adoption of Assessment for Learning as a national pedagogical approach for all schools (DBE, 2021). The implications of these changes were significant given that for the first time since the passing of the seminal assessment policy passed in 1998. Unlike past practices that focussed on summative assessment, explicit guidelines on the were made available for supporting teachers to implement formative assessment practices that moved beyond test scores and curriculum coverage and provided alternatives to the dominant summative assessment guidelines promoted in all the curriculum and assessment policies. Moreover, these changes also addressed long-standing critiques of the ‘assessment-focused but measurement-driven’ policies (Chisholm & Wilderman, 2013; , & Sayed 2013). Of particular significance is that the formative assessment guidelines adopted (DBE, 2020) were derived from research evidence reported from several studies undertaken in South Africa (Kanjee, 2018; Kanjee & Bhana, 2022).

In a critical initiative to enhance the implementation of the recently adopted AfL Pedagogical Strategy, the DBE in partnership with the National Education Collaborative Trust, implemented two AfL projects aimed at supporting policy makers and supporting district subject advisors who has the main responsibility for monitoring and supporting teaching and learning in the classroom. The programme for policy makers focussed on providing participants an overview of the AfL approach and its implications for policy as well as for key decisions regarding the scaling of the AfL to districts and schools under their jurisdiction. The second programme focussed on supporting district subject advisors to enhance their knowledge and skills in the use of FA so that they could improve the support provided to teachers. This



programme not only sought to empower subject advisors but also to demonstrate alternative approaches of support that extended beyond mere monitoring and reporting (Kanjee, 2023).

Systemic Evaluation and Early Learning National Assessment Studies

Following the cancellation of the Annual National Assessments in 2015, due primary to pressure from the Teacher Unions, the DBE signed an historic agreement with all Unions in the country to implement an integrated national assessment system that would entail: on: (i) moving away from high-stakes, annual testing to a more developmental, sample-based model; (ii) regular consultation with unions in the conceptualisation and implementation of the new assessment system; (iii) greater emphasis on using assessment data for system improvement rather than punitive measures against teachers or schools. A key component of this agreement was to implement a new model for the Systemic Evaluation Study (SES) ((Kanjee, et al, 2024a) and the Early Learning National Assessment (ELNA) (Kanjee, et al, 2024b). The studies comprise of sample-based assessments to monitor and evaluate the education system's achievement of broader social, economic, and transformational goals rather than focusing on individual student accountability. The seminal implementation of the SES model was undertaken in 2022, and ELNA in 2022.

Unlike the previous ANAs, the SES would be conducted every three years, assess performance of a sample of learners in Grades 3, 6, and 9, focussing on Reading and Mathematics, collect data from diverse stakeholders (learners, parents, teachers, principals, and district officials), and is based on representative sampling to reduce administrative burden while still generating statistically valid insights into system-wide performance and areas in need of improvement. Similarly, the ELNA is intended to be administered annually to a sample of learners in Grade 1 and focuses on numeracy and literacy. Of note is that the reporting of results for both the SES as well as the ELNA is based on the AfL approach (Kanjee, et al, 2024a; 2024b). Thus, no percentages or IRT scaled scores were used. Instead, the results were reported using performance levels and performance level descriptors (See Figure 10 and Appendix A) that provide detailed information on what learners know and can do and thus serve as a basis for using summative results in a formative manner (Molio & Kanjee, 2018).

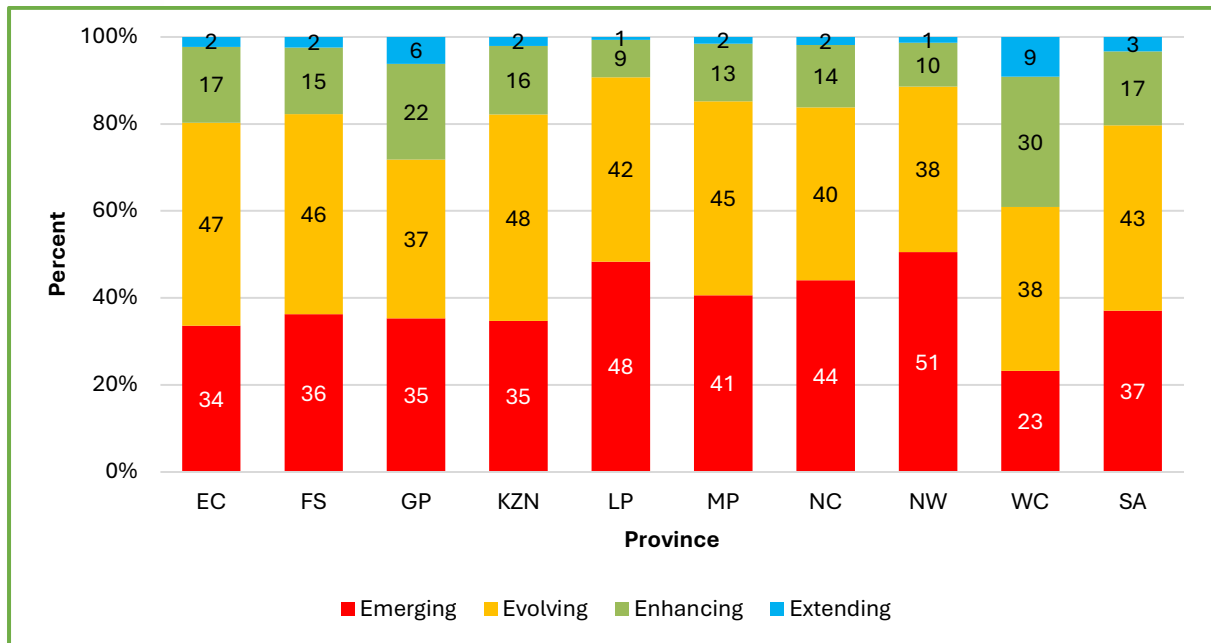


Figure 10: Percentage Grade 3 learners functioning at Mathematics Performance Level by Province (obtained from the DBE Systemic Evaluation Report (2024))

Proposed General Education Certificate

According to the draft policy gazetted in 2021, the proposed General Education Certificate (GEC) is intended to serve as NQF Level 1 qualification to formally recognise the completion of compulsory basic education in South Africa through Grade 9 (DBE, 2021b). The draft policy notes that the primary purpose of the GEC is to: provide a foundation certificate that acknowledges learners' skills and achievements, offers an earlier indicator of performance, serves as a quality assurance mechanism, and guides students toward appropriate further education pathways. Specifically, the Draft Policy notes that the main purpose of the assessment is:

- To set standards of the desired competencies required after 10 years of formal schooling (Grade R-9);
- To provide learners, parents and schools with information relevant to the learner on the pathway to follow;
- To provide formal recognition and certify learners based on their achievement at the end of ten years of schooling” (DBE, 2021, p. 34)

Key features of the GEC include a 120-credit structure requiring nine subjects (three worth 20 credits each and six worth 10 credits each), a three-component assessment approach (60% standardised external assessment, 40% school-based assessment, and aptitude assessments), and specific certification requirements including minimum achievement levels across subjects. While not an exit point from education, the GEC is designed to support learners in choosing among three pathways: academic (toward National Senior Certificate), vocational (through TVET colleges), or occupational qualifications, with appropriate concessions available for immigrant learners and those with barriers to learning. , The proposal policy still indicates that



the reporting of information will be based on the seven-point rating scale, notwithstanding its limitations for providing useful information for addressing learning needs of learners (Kanjee & Moloi, 2016). However, the recent piloting of the GEC learner assessment component is based on performance levels, with a clear intention to provide schools and teachers with more information improving learning and teaching rather than focussing on ranking and sanctions (Chetty, 2024).

The Curriculum Strengthening Process

The curriculum strengthening process initiated by the DBE in 2023 is aimed at revising the existing Curriculum and Assessment Policy Statements (CAPS) to equip South African learners with the competencies needed to thrive in a rapidly changing world. Weston (DBE, 2023a), the Director of Curriculum at the DBE leading the process noted that the intention of the strengthening process is not to rewrite the currently curriculum but rather to build on its strengths while addressing identified gaps and challenges. The draft Competency Framework (DBE, 2023b) lists four key components: Values (particularly democratic values underpinned by the Constitution), Knowledge (disciplinary content and skills), Skills (both foundational and transversal), and Character & Attitudes (social-emotional competencies) on which the curriculum strengthening process is based.. The framework notes that these components are deeply intertwined and should work together to improve curriculum policy, assessment practices, teacher development, learning and teaching support materials (LTSM), and the learning environment with the overall goal to enhance learners' holistic development.

Importantly, the Competency Framework (DBE, 2023b) addresses concerns raised in the decolonisation movement, arguing for a shift beyond knowledge paradigms from the Global North to foregrounding African philosophical thinking, social practices. Specifically, the framework focuses on the value and use of Indigenous Knowledge Systems (IKS), noting that the South African curriculum and education system have largely been inherited from European school systems and that this has resulted in the marginalisation of indigenous knowledge systems. The approach advocated in the Framework is intended to enhance diversity by valuing learners' cultural knowledge and promoting epistemic pluralism through the inclusion of multiple ways of knowing. By integrating Indigenous Knowledge Systems (IKS), it addresses the legacy of apartheid, fosters cultural awareness and critical thinking, and encourages practical strategies like translanguaging, traditional games, and engaging local knowledge holders in learning.

Accountability in the Competency Framework is presented as a core democratic value to support individuals to take responsibility for their actions, to work towards promoting democratic values that should underpin the transformation of the education system. However, the framework also critiques the performativity regimes embedded in current education practices. While the term “performativity” is not explicitly used, the Framework highlights how assessment practices in the schooling system often prioritises content recall over critical thinking and rewards individualism over collaboration, which undermines the important values like Ubuntu that schools should be promoting. However, the framework acknowledges the



difficulties of fully realising these ideals within entrenched systemic constraints, revealing the tension between values-based accountability and the realities of performance-focused education systems.

The Framework document also argues for Assessment for Learning (AfL) as central to the pedagogy of Sustainable Livelihoods, highlighting its value in promoting active learning and experiential approaches to develop an entrepreneurial mindset and track learners' progress across subjects. Aligned with these pedagogies, the DBE is equipping teachers with appropriate assessment strategies that foreground critical thinking, creativity, collaboration, communication, metacognition, and self-assessment to support holistic learner development. Importantly, the document also notes that these processes are also being piloted in the General Education Certificate (GEC).

The Action Plan to 2024

The Action Plan to 2024 was published in 2020 (DBE, 2020) before the onset of COVID-19 and the new regulations that significantly impacted only national policy and classroom practices across the schooling systems in South Africa (Kanjee & Ramollo, 2023). The latest iteration builds on the framework of the previous two Action Plans, with the focus on ensuring every young South African receives quality schooling and making continuous progress towards a clear vision for 2030, while maintaining the set of 27 goals and 36 indicators. The Action Plan to 2024, however adds several new sections that focuses on: the historical contexts, national and international commitments, theory of change for quality basic education, correspondence between the MTSF and Action plan elements, and corrections to the initially published PIRLS trend (Appendix C). In addition, Action Plan to 2024 updates the two innovation areas listed in the 2019, i.e. the Annual National Assessments and e-Education while adding a third area, i.e. early grade reading.

The section on historical contexts provides a more detailed contextualisation of South Africa's educational challenges by highlighting how colonialism and apartheid systematically damaged the education system, while acknowledging that past injustices continue to shape present realities. Specifically, the Action Plan to 2024 argues for a deeper understanding of the hard done by colonialism to the nation noting that this

understanding should guide a process of healing which affirms equality, undoes the marginalisation of African culture and values brought about by colonisation and apartheid, and moves beyond the confines of a Eurocentric worldview and curriculum" (DBE, 2020, p. 4).

The theory of change presented in the Action Plan to 2024 provides a comprehensive framework for understanding how different factors interact to produce educational improvement, positioning the learner at the centre while recognising the complex web of relationships between teachers, materials, assessment systems, school management, and community support. This theory of change in the 2024 plan explicitly acknowledges the interconnectedness of various system components, explaining how early childhood



development, teacher capacity, learning materials, class sizes, and school leadership work together to influence learning outcomes.

Critically, the addition of early grade reading as an important focus alongside assessments and e-Education draws on the latest evidence for improving Foundation Phase reading outcomes within the current context of teaching and learning in South African schools. The e-Education priority also evolved from focusing on strategy development to practical implementation, exemplified by the Data Driven Districts initiative and plans for providing digital workbooks on tablets to all learners. Moreover, the 2024 plan also presents compelling evidence of improvement across multiple assessments, drawing on the results from TIMSS results and the corrected PIRLS assessments. (DBE, 2020a). Most notably, Action Plan to 2024 explicitly argues for, and reframes its assessment strategy within the concept of "Results-Oriented Management and Accountability" approach advocated in the National Development Plan (NPC, 2012). This approach is aptly term 'ROMA' in the Actional Plan to 2024 (DBE, 2020a).

AfL programmes for policy makers and School Leaders

Another key innovative programme introduced by the DBE and the National Education Collaborative Trust (NECT)⁸ is the introduction of two additional capacity development programmes. The primary purpose of these programmes is to enhance the implementation of the AfL strategy across the entire education system by building capacity at multiple levels in order to create a coordinated approach to improving teaching and learning outcomes. These programmes are targeted at: (i) policy makers within the DBE at the national, provincial and district levels, as well key decision makers among education role-players (i.e. Teacher unions, NGOs, Donors); and (ii) school leaders. Implemented in collaboration with a local service provider, these programmes serve two purposes. First, they ensure a deeper understanding of the national AfL pedagogical strategy among all key role-players. This understanding is essential for achieving alignment across different system levels - national, provincial, district and school. Second, they enhance the support provided to teachers. This improved support helps teachers develop better pedagogical strategies to identify and address the learning needs of all learners. The goal is effective implementation of the AfL strategy across the entire education system. By building capacity at multiple levels, the programmes create a coordinated approach to improving teaching and learning outcomes.

Trends and Tensions: Entrenching and/or disrupting performativity regimes.

A reflection of the history of assessment in South Africa as well as a review of the key policies, programmes and project implemented in the post-apartheid era reveals key trends as well as the fundamental tensions in the implementation of performativity-based accountability systems impacting the schooling sector in South Africa. These trends and tensions are evident across key policy documents, different programmes and projects implemented within the schooling sector as well as across the practices of the different education role-players in the system. First,

⁸ The NECT was established as part of a public-private partnership programme with the South African government to support the Department of Education's to implementation strategy in the schooling sector.



colonial and apartheid-era assessment practices established a foundation for current performativity-based accountability systems that continue to disadvantage marginalised. Second, notwithstanding various reform efforts, the current schooling system is characterised by assessment focussed, and measurement driven' policies and practices that creates a paradoxical situation where the very mechanisms designed to improve education quality may actually impede genuine learning. Third, South African schools operate under multiple, accountability systems, that is bureaucratic, professional, results-based, and social accountability systems. These systems often overlap, and create complex environments in which schools must operate, and which has proven to be particularly challenging for under-resourced schools. Fourth, the primary focus on accountability has revolved around assessment of mathematics and language performance, often at the expense of other subjects, especially non-academic subjects. Fifth, the DBE's adoption of AfL as a national pedagogical strategy provided a renewed emphasis and options for different strategies and policies to address the challenge of equity and quality at the classroom and school levels as well as at the system (district, province, national) level. Sixth, several initiatives and actors have created the impetus for transformation, including the 'decolonise education movement', COVID-19 pandemic responses, research evidence on the use of AfL approaches in South Africa, the development of new national assessment framework as well as the streamlining of the curriculum.

Colonial and Apartheid-Era impact on Current Systems

The roots of the current performativity-based accountability systems within the schooling sector in South Africa can be directly traced to the educational and psychological systems and practices established during the colonial period, and later entrenched during the apartheid period. These systems and practices first manifested in the late 1800s, were based on standardised examinations that served as gatekeeping function to higher levels of the education system. During this time, teacher accountability mechanisms were introduced through the "payment-by-results" system where teacher salaries were directly tied to the performance of their learners. These practices were early examples of what Ball (2003) later theorised as performativity regimes—systems that fostered a culture where educational value was defined primarily by measurable outputs. This approach notably favours easily measurable content areas like mathematics and reading while overlooking less quantifiable content such as the social sciences that require examinations to focus on issues such as critical thinking, creativity, and cultural knowledge.

The introduction of psychological testing around 1916, heavily influenced by American eugenic practices, further entrenched the notion that standardised measurement could objectively classify learners according to their abilities. In practice, this system served a dual purpose: it appeared to provide scientific legitimacy to educational sorting while simultaneously justifying the systematic exclusion of African learners from quality education. Critically, the tests applied were culturally biased, designed to confirm predetermined notions of racial hierarchy rather than assess genuine learning potential (Foxcroft & Davies, 2008). The instruments were obtained from Europe and the USA and applied to Black South Africans without any adaptation to the local language and cultural context, further entrenching



Eurocentric epistemologies and reinforcing existing racial biases within the educational system.

The apartheid era transformed these colonial practices into a sophisticated system of racialised assessment that served the political economy of white supremacy (Nzimande, 1995). The 1953 Bantu Education Act represented a watershed moment in institutionalising assessment as a tool of oppression (Kallaway, 1984). This legislation not only enforced racial segregation in schools across the 18 different education departments but also segregated teacher training institutions as well as universities, ensuring that different racial groups would experience fundamentally different approaches to education and assessment (Kanjee & Makgamatha, 1998). Control over and entry into schools and the economy were regulated through an elaborate system of tests and examinations norm-referenced, primarily serving summative purposes to determine pass or fail (Kanjee and Sayed, 2013). These tests and examinations regulated access into further grades while promoting rote learning over critical thinking for both Black and White South Africans, reducing students to passive receivers of information rather than active participants in learning (Swartz, 1992; Kallaway, 1984). The matriculation examination, which was administered separately for various racial groups and based on different examinations, was maintained as an extremely high stakes assessment, used for selection into the higher education sectors as well as world of work (Reddy, 2008). These assessment regimes were misused to deprive the black population of access to resources and opportunities and intentionally stifled their intellectual development to meet the needs of the white minority for a cheap source of labour (Nzimande, 1996; Mathonsi, 1998).

Assessment-Focused and Measurement-Driven Policies

While the seminal assessment policy (DoE, 1998) advocated for an assessment for learning approach, promoting a shift from the high-stakes examinations-based system, subsequent assessment and curriculum policies since 1994, including different versions of the Action Plans, have systematically prioritised summative over formative assessment. Kanjee and Sayed (2013) noted that this prioritisation represents a fundamental contradiction between the assessment for learning practices advocated across different versions of curriculum and assessment policies, and the underlying summative and measurement driven practices promoted across these policies. These practices have led to what Kanjee and Sayed (2013) describe as an 'assessment focussed measurement driven' approach. In practice, this approach creates a paradoxical situation where the very mechanisms designed to improve education quality may impede genuine learning. This paradox manifests most clearly in the implementation of policy that promotes performativity regimes and practices across the different levels of the system, often having a differential negative impact on schools and staff serving learners from poor and marginalised communities.

The overt emphasis on standardised testing as prescribed in the use of the School-Based Assessment (SBA) system and the monitoring educational quality using national assessment results further entrenches testing cultures that privileges quantifiable outcomes over holistic learning maintains (Chisholm & Wilderman; 2013; Jansen, 2004). The current School-Based



Assessment (SBA) system, despite its formative intentions, retains the structural logic that promoted compliance-based approaches of the past by prescribing rigid assessment schedules and reporting frameworks that provide limited information on supporting teachers and learners to identify and address learning challenges. Thus, it's not surprising that teachers report spending disproportionate time on assessment administration and recording rather than on responsive teaching practices (Moloi & Kanjee, 2014). More concerning is that the implication of this paradox is particularly acute in under-resourced schools, where compliance with assessment requirements consumes limited teacher time and energy. Browsers (2021) demonstrates that "teachers from both the high and low performing schools experience similar accountability pressures although they experience these pressures in different ways," with teachers in poorly resourced schools spending disproportionate time on administrative compliance rather than addressing the specific learning needs of their learners.

The Annual Teaching Plans (ATPs) exemplify this paradox. Introduced to support teachers to more effectively implement the curriculum, the ATPs specify the content to be covered, and stipulate the sequencing and pace of coverage, effectively reducing teaching to mechanical content delivery. Bertram, Mthiyane and Naidoo (2021) note that the underlying rationale for this approach is based on well documented research in South African schools that has identified poor curriculum coverage as one of the key reasons for poor learner outcomes. In practice, the focus on coverage over learning results in perpetuating the fundamental tensions between curriculum completion and learning comprehension. The ATPs' weekly specification of content topics teachers must cover, measured by SBA tasks that specify what and when assessment must be conducted (DBE, 2011a; 2011b) transforms complex social processes and events that characterise the teaching and learning process into simple figures or categories of judgement (Ball, 2003). This mechanical approach contradicts pedagogical research demonstrating that meaningful learning requires flexible pacing responsive to learner needs (Alexander, 2008).

The proliferation of large-scale assessment studies further illustrates this paradox. The paper documents a total of 134 large-scale assessment studies, implemented at the district, provincial or national level since 1994 (Kanjee, 2024a). While limited information on costing is available, current estimated costs is approximately a billion rand. However, despite this massive investment in measurement, there has been limited use of this information to support data-driven decision-making aimed at identifying and addressing key challenges impacting quality and equity within the education system. The most critical challenge is the severe shortage of technical capacity within the education sector to analyse, interpret, and effectively use assessment results. A 2024 national audit revealed the lack of assessment or measurement programs at Masters or Doctoral levels, while two-thirds of organisations that identifying assessment as a core focus area of their work reported lacking essential measurement expertise (Kanjee, 2024a). Consequently, valuable assessment data remains underutilised, limiting its potential impact on classroom practice and systemic reform. Inadequate reporting and dissemination strategies further compound these challenges. Between 20-40% of school principals and teachers report being unaware of any assessment studies, while those who are aware often lack access to meaningful data interpretation (DBE, 2022). The absence of



performance level descriptors since 2002 means that results are presented in formats with limited practical value for teachers and policy makers seeking to understand and address specific learning deficits within the system (Kanjee & Moloi, 2016).

Multiple Overlapping Accountability Systems

Adding to the burden of testing and measurement, South African schools also operate under multiple accountability systems—bureaucratic, professional, results-based, and social accountability systems. In practice, these accountability systems often operate simultaneously within school systems, creating complex webs of responsibilities and reporting relationships. The balance between these approaches significantly impacts teaching practices, resource allocation, and ultimately learning outcomes. Over-reliance on any single approach can produce unintended negative consequences, such as (performance accountability) or rigid procedural compliance without innovation (bureaucratic accountability) (Flórez-Petour & Rozas-Assael, 2019). For instance, focusing resources on improving test scores (results-based accountability) may require reducing time spent on community engagement (social accountability) or professional development (professional accountability). Similarly, strict adherence to bureaucratic procedures may limit the professional discretion teachers need to address diverse learner needs effectively.

In the South African context, bureaucratic accountability, exemplified by the South African Schools Act (SASA) and the Whole School Evaluation (WSE) process, requires schools to demonstrate compliance with regulatory requirements and administrative procedures. Professional accountability, managed through the South African Council for Educators (SACE) and the Continuing Professional Teacher Development (CPTD) system, demands adherence to professional standards and ethics. Results-based accountability focuses on learner performance outcomes through mechanisms like the matriculation examination results, and the discontinued Annual National Assessments, while social accountability operates through School Governing Bodies (SGBs) and community oversight mechanisms.

This complexity is particularly problematic for schools serving marginalised communities. These schools must navigate all accountability systems while lacking the resources and capacity to meet their demands effectively. The reporting burden exemplifies this complexity. Schools must produce separate reports for different accountability systems: financial reports for SGBs, curriculum coverage reports for districts, professional development portfolios for SACE, learner performance data for provincial and national databases, and infrastructure compliance reports for various departments. Mestry, Moonsammy-Koopasammy, & Schmidt (2013) document how unrealistic reporting demands consumes significant school leadership time while providing little integrated insight into school improvement needs. In their study on teacher job satisfaction, Hofmeyr, Pampallis, Qvist, and Swelindawo (2024) found that 70% of teachers across both fee and no fee schools identified excessive paperwork as their primary



stressor. Data from interviews also revealed that teachers in poorly resourced schools derived greater fulfilment from the positive impact they have on disadvantaged children's lives, which compensated for frustrations caused by limited resources. The authors also found that teachers in mid-fee and high-fee schools report higher stress levels than those in no-fee and low-fee schools, primarily due to increased pressure from principals and parents to deliver excellent results.

Overemphasis on Mathematics and Language Performance

A review of the large-scale assessment studies implemented in South Africa reveals the predominant focus on the performance of learners in mathematics and language. This focus stems from a global emphasis on foundational skills as the primary indicators of educational quality and system performance. International large-scale assessments (ILSAs) such as the Trends in International Mathematics and Science Study (TIMSS), Progress in International Reading Literacy Study (PIRLS), and regional assessments like SACMEQ and PASEC, all emphasise reading and numeracy/mathematics as proxies for educational success. Similarly, national assessment programmes such as the ANAs, SES as well as the ELNA have adopted similar approaches, anchoring their indicators on reading and mathematics scores. Notwithstanding the value of these foundational competencies, the overwhelming attention to language and mathematics through policy targets, performance contracts, and school evaluation systems to raise achievement in these two subjects, has been the detriment of other subjects.

In practice the prioritisation of mathematics and language has several implications for curriculum delivery and learner experience. First, the teaching of other subjects such as the social sciences, creative arts, physical education, life orientation, and technology often receives less instructional time and attention. Teachers, under pressure to deliver measurable improvements in language and math scores, may reduce coverage or even exclude other subjects in daily practice (Spaull & Janem 2019). Second, learner development in essential skills such as critical thinking, civic responsibility, environmental literacy, emotional intelligence, and artistic expression is undermined. These skills are often cultivated in non-core subjects that fall outside the dominant accountability requirements. Third, the exclusion of subjects beyond language and mathematics in accountability metrics perpetuates inequality in access to quality learning experiences. Schools serving wealthier communities may still afford to offer additional co-curricular and interdisciplinary programmes, while under-resourced schools often feel compelled to focus narrowly on performance in tested subjects (Jansen, 2001; Spaull & Jansen 2019). This exacerbates existing disparities in educational outcomes and opportunities, undermining efforts to promote equitable and socially just education systems.



The focus on measurable outcomes in mathematics and language serves to reinforce performativity-based accountability systems. Teachers and schools are held accountable for learner performance on standardised tests, often with high-stakes consequences such as school ranking, promotion decisions, and public naming and shaming. This results in performative compliance—where teachers focus on teaching to the test rather than authentic learning—and encourages superficial improvements that may not reflect genuine learning gains. Further, performance data in math and language are used as primary indicators in national development plans, education sector reviews, and donor accountability frameworks. This institutionalises the privileging of these subjects in education planning and resourcing, creating a feedback loop that further marginalises non-tested domains. The risk is that education systems begin to conflate quality with test scores, losing sight of the broader aims of education, including values formation, social cohesion, and human development.

Assessment for Learning Pedagogical Strategy

The DBE's adoption of AfL as a national pedagogical strategy (DBE, 2021) provided a renewed emphasis and options for different strategies and policies to address the challenge of equity and quality at the classroom and school levels as well as at the system (district, province, national) level. Several key initiatives signal the DBE's intention to foreground teaching and learning over results and reporting. First, the publication of the National Assessment Circular No 3 (DBE, 2020), provided detailed guidelines for teachers' implementation of formative assessment in the classroom. The significance of this Circular marks the first attempt since 1994 at providing additional assessment support that extends beyond the summative use of information. Second, at the systems level, results of the 2022 Systemic Evaluation Study Learner Performance (Kanjee, et al, 2024b) as well as the Early Learning National Assessment results (Kanjee, et. al., 2024a) were reported using performance levels rather than percentage or IRT-based scale scores. Here too, the implications are profound, given the value of performance levels in enhancing the effective use of these results by district and provincial officials (Kanjee & Moloi, 2016). Third, notwithstanding the policy specification of the GEC to use the seven levels of reporting (DBE, 2022); the current piloting of the GET assessment component have been based on using performance levels for reporting results (Chetty, 2023). The clear signal is that the primary purpose of the GEC is to provide detailed and useful information for supporting learners enhance their selection of subjects as they move into the next phase of their education in the three-stream model, i.e. academic, technical or vocational. Fourth, the curriculum strengthening process, implemented to update the existing curriculum statements to better equip South African learners with the competencies needed to thrive in a rapidly changing world, also advocates for an AfL approach. Moreover, the Framework document guiding the strengthening process critiques the performativity regimes that promote assessment practices that prioritises content recall over critical thinking and rewards individualism over collaboration, arguing that this practices the important values like Ubuntu that schools should be promoting.



This shift had profound implications for accountability relationships across the education system. At the classroom level, AfL approaches emphasis teacher and learner involvement in the learning process and rather than compliance with reporting and recording test results. Significantly, the formative assessment guidelines adopted by the DBE (2020) were derived from research evidence reported from several studies undertaken in South Africa involving teachers as well as subject advisors (Kanjee, 2018; Kanjee & Bhana, 2022; Kanjee, 2023; 2024). These studies demonstrated that all teachers and district subject advisors, including those in poorly resourced schools and districts, could achieve substantial improvements in pedagogical practice through appropriate professional development. Importantly, the intervention model applied in the AfL professional development model, i.e. the ReMAPS (Kanjee, 2018) promotes pedagogical practices based on collaborative knowledge construction and contextual adaptation over curriculum coverage and test scores, challenging performativity based accountability regimes. Similarly, the scaling up model applied (Kanjee, 2018) created space for reconceptualising accountability relationships between different levels of the education system. Rather than hierarchical monitoring and sanctions, the approach is based on a distributed capacity-building approach involving collaboration between teachers, school leaders as well as district officials.

However, despite the transformative potential of AfL adoption, and intentions of key role-players, the tension between AfL principles and existing performativity-based accountability regimes creates practical challenges for effective implementation of the curriculum. Teachers report feeling caught between the professional judgments emphasised by AfL and the compliance requirements of existing systems (Kanjee, 2023; 2024). Specifically, teachers reported that they must still provide marks to meet SBA requirements, complete work schedules and reports for school principals while implementing their newly developed assessment formative assessment skills (Kanjee, 2018; Kanjee & Ramollo, 2023). School managers struggle to protect teachers' time for collaborative assessment discussions while meeting reporting requirements while district officials trained in AfL approaches reported that they must still conduct traditional compliance monitoring visits and have little time for providing effective support for improving teaching and learning (Kanjee, 2023).

Role-players' challenging performativity regimes

Several initiatives, role-players and events have created the impetus for challenging current dominance of performativity-based accountability regimes in South Africa. The decolonise education movement, sparked by the #RhodesMustFall and #FeesMustFall campaigns in 2015, fundamentally challenged the epistemological foundations of South African education systems. These movements went beyond demands for access to question the very nature of knowledge production and validation within educational institutions, with students calling for the elimination of Eurocentric curricula and institutional practices, and for the inclusion of African, indigenous, and other non-Western epistemologies. In their review of assessment practices in South Africa, Chetty (2024), director of assessment in the DBE notes that they used decolonial arguments to challenge the use of northern standards in South African assessment, calling for developing local norms and standards for determining levels of performance by which children



from Africa are judged. Similarly, Weston (2024), Director of the DBE Curriculum Unit reported that issues of decolonisation was a key consideration as they embarked on the national process to streamline the curriculum (DBE, 2023b). Another likely impact of this movement was the establishment of the History Ministerial Task Team (DBE, 2018) that was established by the DBE to address concerns about the state of History education in South Africa, to investigate making History a compulsory subject and to Africanise and decolonise its curriculum.

The COVID-19 pandemic created unprecedented disruption that opened space for reconsidering established accountability practices. As documented in the paper, the pandemic period saw the "adoption of Assessment for Learning as a national pedagogical approach for all schools" in March 2021. This represented a significant shift from the measurement-driven approaches that had dominated since 1994. In their study on the impact on COVID-19 on teachers, Kanjee and Ramollo (2023) report that the limited monitoring of schools during this period, and greater flexibility allowed for regarding curriculum implementation, created new spaces that allowed teachers to experiment with new and different pedagogical practices. Most teachers in their study reported that these opportunities allowed them to foreground learning over curriculum coverage while promoting greater collaboration with colleagues across subject and grades. The authors argue that the overall effect of these practices challenged prevailing cultures of accountability and performativity regimes impacting on their daily lives.

Research evidence on Assessment for Learning effectiveness in South African contexts also provided relevant empirical support for its adoption to support the education transformation process in schools. The independently conducted evaluation of the Formative Assessment Professional Development Programme (Kanjee & Bhana, 2022) demonstrated the significant positive impacts on teachers' assessment knowledge and classroom practices of the locally developed intervention programme, particularly in poorly resourced schools. This evidence challenges deficit-based assumptions about teacher capacity in schools serving marginalised communities and point to alternative approaches to addressing issues of accountability as well as professional development programmes that foreground support and capacity over monitoring and evaluation.

The development of the new national assessment framework also serves as a critical step toward challenging existing results-based accountability approaches. This framework proposed long awaited changes across the schooling system that includes implementing AfL as a pedagogical approach, introducing the General Education Certificate, implementing Systemic Evaluation Studies in place of the ANAs, and overhauling school-based assessment (DBE, 2017). Already, the impact of these challenges is being felt with the reporting of results from two national assessment studies, the SES and ELNA, based on performance levels that aim to provide more information to support next steps for interventions instead of scores that focus on comparisons of performance. These changes point to a positive step to challenging the dominant performativity and accountability discourse and practices (Kanjee & Ramollo, 2023).



Teacher union initiatives to support their members to improve their pedagogical practices, particularly through the Teacher Union Collaboration (TUC) established in 2011, created represent another mechanism for transforming accountability relationships. The SADTU Assessment for Learning programme implemented in 2012 (Kanjee, 2014) as well as the adoption of a resolution by all Unions to implement the AfL pedagogical strategies, demonstrate how organised labour could shape accountability and performativity policies and practices. Similarly, the contribution of the NECT to provide further support to education officials and school leaders to enhance their understanding of the AfL Pedagogy and thereby promote effective support to teachers for its implementation also attests to the role of the NGO sector in supporting professional development programme that challenged punitive accountability approaches.

CONCLUSION: WAY FORWARD

This paper highlights key tensions between entrenched performativity-based accountability regimes and emerging transformative possibilities in South African schooling system. The paper argues that colonial and apartheid accountability systems and practices continue to shape current systems through policy prescriptions, institutional arrangements and epistemological assumptions. In its most crude form, this is manifested in the matriculation examinations which continues to shape pedagogical practices in schools and the career trajectories of all learners existing the schooling system. The transformative agenda implemented in the post-apartheid schooling system has made substantial strides in addressing issues of access and quality within the schooling system. However, the different iterations of curriculum and assessment policies as well as the Action Plans have resulted in institutionalising dominant performativity-based accountability systems in the school sector. In practice, these systems manifest in the paradox of assessment-focused measurement-driven policies where mechanisms intended to improve quality actually impede genuine learning, particularly in schools serving vulnerable communities. Beyond theoretical concerns, the practical challenges of multiple, overlapping accountability systems create environments where bureaucratic compliance often displaces meaningful teaching and learning, with under-resourced schools bearing the heaviest burden."

However, the evidence also indicates substantial efforts within the DBE as well as among key role-players in challenging dominant performativity-based accountability discourses creating tensions and contradictions within policies and its implementation at the different levels of the system. The decolonisation movement has challenged fundamental assumptions about knowledge creation, current curricular and pedagogical practices. The COVID-19 pandemic disrupted established patterns and created space for alternatives. Research evidence has demonstrated the effectiveness of Assessment for Learning approaches in South African contexts. Institutional innovations and teacher union collaboration have created new possibilities for professional rather than bureaucratic accountability. The DBE's adoption of AfL as national pedagogical strategy, represents the most concrete manifestation of these transformative initiatives.



These transformation initiatives converged around several key insights that challenge existing accountability paradigms. First, they demonstrate that meaningful improvement requires addressing professional capacity rather than simply intensifying measurement and sanctions. Second, they show that context-sensitive approaches could achieve better outcomes than standardised compliance requirements. Third, they reveal that sustainable transformation required authentic stakeholder engagement rather than top-down policy implementation. What these tensions reveal is that changing assessment practices that foreground learning and which address learning needs of all learners, especially learners from poor and marginalised backgrounds requires not just policy reform but sustained efforts from all role-players to bring about the cultural transformation in how educational quality is understood and how the challenge of equity is addressed.

The way forward requires sustained commitment for developing a more integrated approach that balances accountability with genuine support and capacity building, considering the diverse realities of South African schools and communities. The evidence presented here suggest that while performativity remains entrenched, spaces for transformation are expanding, with increasing involvement of key role-players providing alternatives to current dominant discourses. Critical to this process is acknowledging that accountability is, on its own, neither good nor bad. Rather, the effects of any accountability system depend on the policy and political agendas to which it is attached, how it is used, the goals, values, and purposes it serves."

However, realising these transformative possibilities will require not just policy reform but fundamental reimagining of what accountability means in a context of extreme inequality and historical injustice, and its implication for realising a truly democratic society. At the policy level, there is an urgent need to review current policies, goals, and objectives, while at the practice level, greater emphasis is required on capacity development and sharing, with a specific focus on implementing the AfL pedagogical strategy. The advocacy for the AfL represents not just a pedagogical shift but a potential transformation in how the relationship between teaching, learning, and accountability is conceptualised. Ultimately, the tension between performativity and transformation reflects deeper contests over the purposes of education in post-apartheid South Africa. The critical question to address is whether an education system can simultaneously pursue standardised accountability measures designed primarily for monitoring and evaluation while also addressing the specific learning needs of learners in vastly different contexts. The South African experience suggests that with the concerted efforts of all key role-players, including government, unions and civil society, fundamental transformation of accountability paradigms is possible, and that the contradictions that continue to undermine efforts to achieve quality education for all can be adequately addressed.



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Appendix A: Grade 3 Numeracy Performance Level Descriptors (from DBE SES Report)

EMERGING	EVOLVING	ENHANCING	EXTENDING
Learners at this level are able to	Learners at this level are able to	Learners at this level are able to	Learners at this level are able to
Represent numbers and their relationships, and to count, estimate, calculate and check with competence and confidence Complete simple patterns. Recognise, name and describe characteristics of two-dimensional objects in a variety of orientations and positions. Measure standard measures in a variety of contexts. Read and write time	Represent numbers and their relationships, and to count, estimate, calculate and check with competence and confidence in solving problems. Extend and complete simple patterns. Recognise, name and describe characteristics and relationships between two-dimensional shapes and three-dimensional objects in a variety of orientations and positions. Estimate and measure standard measures in a variety of contexts. Read and write time and solve problems. Sort, order and organise data to answer questions.	Recognise numbers and their relationships, and to count, estimate, calculate and check with competence and confidence in solving problems. Extend and complete patterns. Recognise, name, describe and compare characteristics and relationships between two-dimensional shapes and three-dimensional objects in a variety of orientations and positions. Estimate, measure and compare standard measures in a variety of contexts. Calculate, read and write time and check with competence and confidence in solving problems. Interpret and analyse data to answer questions.	Describe numbers and their relationships, and to count, estimate, calculate and check with competence and confidence in solving problems. Extend and complete more advanced patterns. Describe and critically compare characteristics and relationships between two-dimensional shapes and three-dimensional objects in a variety of orientations and positions. Estimate, measure and critically compare standard measures in a variety of contexts. Calculate, read and write time and check with competence and confidence in solving Advanced problems. Critically analyse data to draw conclusions and make predictions.



RELATIONSHIPS BETWEEN TEACHER PRACTICES, ABILITY GROUPING, EQUITY AND LEARNERS' MATHEMATICAL IDENTITIES

Aarifah Gardee

University of the Witwatersrand

In this paper, I explore the mathematical identities of two higher-performing Grade 10 learners in the context of ability grouping in South Africa. Both learners were taught by the same mathematics teacher but placed in different mathematics classrooms, one designated as high-performing and the other as mixed-ability. I compare their mathematics teacher's practices across these two classrooms and examine how his practices relate to each learner's mathematical identity. The findings highlight how ability grouping practices contributed to inequities, as these learners were offered different opportunities to construct mathematical identities and learn mathematics. Although both learners expressed enjoyment of mathematics and took extra initiative in their learning, their experiences in their mathematics classroom and the mathematical identities that they constructed differed. The learner in the high-performing mathematics classroom experienced greater pressure to perform well and had more negative experiences of learning mathematics, than the learner in the mixed-ability classroom. This study also shows how ability grouping reinforces the stratification of learners, intersecting with race, gender, and class status, to inform learners' mathematical identities and learning experiences in inequitable ways.

INTRODUCTION

Over the past few years, there has been a growing body of research focused on the concepts of identity and equity in mathematics education (Gutiérrez, 2012), with particular attention to ensuring access to quality learning opportunities for diverse learners (Vithal, Brodie & Subbaye, 2023). Equity, as defined by Gutiérrez (2012), is characterised by fairness, not sameness, with the concept of identity providing insights into exploring the experiences of learners who are underrepresented in research (Radovic, Black, Salas & William, 2017).

Gutiérrez (2012) described equity in mathematics as the “inability to predict mathematical achievement and participation based solely on student characteristics such as race, class, ethnicity, sex, beliefs, and proficiency in the dominant language” (p. 19). In the South African context, most of these characteristics remain closely correlated with mathematical achievement. For example, the findings from the Trends in International Mathematics and Science Study (TIMSS) highlight strong links between learner performance in mathematics and their home socio-economic status, use of the language of learning and teaching, and school affluence (Kanjee, Chetty, Shoji, et al., 2024). Additionally, the Department of Basic Education reports have consistently shown that boys outperform girls in the matric mathematics examinations (DBE, 2025), and Spaul (2019) has shown that in 2018, more Indian and White learners passed mathematics in matric, and passed it well, compared to Black



learners.

These patterns highlight the ongoing relevance of equity-focused research in South Africa, where inequalities in access, experience, and achievement continue to shape learners' mathematics learning, their mathematical identities, and their future life chances. As Vithal et al. (2023) argue, addressing issues of equity is essential for meaningful transformation in education. Studies have also shown that equity-focused mathematics education research (e.g., Vithal et al., 2023) and mathematical identity research (e.g., Graven & Heyd-Metzuyanim, 2019) are more commonly conducted in countries with strong economies, infrastructure, and technological advancement. This presents a significant gap, as South Africa differs markedly from these countries in terms of economic strength, infrastructure, and levels of inequality.

One contributor to educational inequality in South Africa and beyond is the widespread educational ideology concerning fixed mathematical ability, that is, the ideology that learners have innate mathematical abilities, thus undermining the importance of effort in mathematical learning (Boaler, 2020; Du Plooy, 2019). In some schools, learners are grouped according to their perceived ability based on assessment scores, which is a practice called ability grouping. While proponents of ability grouping argue that the practice allows teachers to better meet learners' needs, by providing extra support to those who are struggling and additional enrichment activities to those who excel, research shows that such practices exacerbate existing inequalities, rather than mitigate them (Boaler, 2020; Du Plooy, 2019).

Ability grouping tends to reinforce hierarchies across mathematics classrooms (Boaler, 2020; Du Plooy, 2019). Learners grouped as higher performing are often given challenging and engaging tasks. Boaler et al. (2000) found that one-third of learners in their study placed in high-performing classrooms were disadvantaged. These learners could not cope with fast-paced lessons, which were characterised by teachers placing high pressure on learners to keep up. They found that the learners designated as lower-performing experienced even more damaging repercussions, by experiencing work that was too easy, with their opportunities for learning being minimised. These inequitable learning opportunities can inform learners' mathematical identities, by influencing how they see themselves as mathematics learners and how they engage with mathematics.

To contribute to this under-researched area in South Africa, this paper explores the relationship between teacher practices, ability grouping, equity and learners' mathematical identities. I focus on one Grade 10 mathematics teacher who taught mathematics in a South African school, called Mr Banza (all names are pseudonyms). Mr Banza taught two groups of learners, one grouped as high-performing and another as mixed-ability. I will explore how his teaching practices informed the mathematical identities of two learners, who identified as female and performed better than others, in each classroom.

The research questions are:

1. How do teacher practices, in the context of ability grouping, contribute to Grade 10 learners' mathematical identities?



2. How do ability grouping practices relate to the mathematical identities of learners who performed better than others in their classroom communities?

A FRAMEWORK FOR LEARNERS' MATHEMATICAL IDENTITIES

In this paper, I use a critical realist perspective to define learners' mathematical identities. Drawing on the work of critical realists Marks and O'mahoney (2014) and Archer (2002), I define mathematical identity as a social construct that emerges through a structured relationship between personal identity, social identity, both offered and constructed, and agency (see Fig. 1) (see Gardee & Brodie, 2021).

Personal identity refers to what "people care about in the world" (Archer, 2002, p. 15), and is characterised by internal processes, encompassing people's emotions, interests and memory (Marks & O'mahoney, 2014). Archer (2002) emphasises the centrality of emotions in shaping personal identity, as emotions reveal a person's priorities and motivations. Positive emotions reinforce self-worth and encourage continued engagement, while negative emotions can undermine self-worth and lead to disengagement.

Social identity is distinct from but intricately related to personal identity. Social identity refers to the roles that communities make available to people and how people take up those roles (Marks & O'mahoney, 2014). So, there are two kinds of social identities. The first kind of social identity is the social identity offered to learners by teachers, which are the opportunities provided to learners to become members of the mathematics classroom community and engage with mathematics (Gardee & Brodie, 2021). The second kind of social identity is the social identity constructed by learners, which is shaped by what learners "care about in social roles" (Archer, 2002, p. 17). While these two social identities are not the same, they are related (see Fig.1).

Learners can construct their social identities differently from what is offered because of agency. Agency functions as the intermediary between personal identity and social identity, both offered and constructed (see Fig. 1). Reflexivity, innovation, and intentional action are indicative of agency (Burkitt, 2015), as are decisions not to act. Personal and social identities inform agency, as what learners care about in the world and in social roles influences their participation in mathematics classrooms. Agency also shapes learners' personal and social identities, as through participation or non-participation, learners learn more about themselves in context.

Gardee and Brodie (2021) developed a framework to examine the relationship between learners' mathematical identities and the social identities offered by teachers in classroom communities (see Fig. 1). Mathematics teachers offer learners two kinds of social identities: affiliation or marginalisation. Learners who are offered social identities of affiliation are provided with full access to learning mathematics to become full members of the classroom community. The practice of teachers offering learners social identities of affiliation is usually associated with equitable practices when all learners are given opportunities to learn mathematics deeply, a point that I will return to in the discussion of the findings. Conversely,



some teachers may offer learners social identities of marginalisation by restricting their access to learning mathematics. The practice of offering learners social identities of marginalisation is associated with inequitable practices, as these learners are not given the same opportunities to learn and succeed as those offered identities of affiliation. In response to the social identities offered, learners exercise agency through different kinds of participation.

According to Wenger (1998), participation can be categorised as full participation, peripherality, or marginality. He explains that full participation differs from peripheral and marginal participation, as the latter two reflect varying levels of non-participation. Peripheral participation includes an element of non-participation “to enable a kind of participation that is less than full”, whereas marginal participation represents a more distinct form of non-participation (Wenger, 1998, p. 165). Each type of participation corresponds to a different kind of membership in the community. Full participation enables full membership, peripheral participation allows for peripheral membership or potential full membership over time, while marginal participation leads to exclusion and non-membership.

Building on Wenger (1998), Marks and O’mahoney (2014), and Archer (2002), Gardee and Brodie (2021) showed that learners can identify with the social identities offered or construct their mathematical identities differently. Learners who identify with the social identity of affiliation offered by teachers can construct mathematical identities of affiliation, by participating fully in the mathematics classroom. Similarly, learners who are offered social identities of marginalisation may construct mathematical identities of marginalisation by participating in limited ways, or not at all. Learners can construct mathematical identities differently from what is offered, by complying with the social identity offered and participating peripherally, rather than fully or not at all when offered social identities of affiliation or marginalisation. Peripheral participation leads to the construction of a mathematical identity of compliance. Alternatively, learners may resist the social identities offered, by participating marginally even when offered affiliation, or striving for full participation even when marginalised, thus constructing identities of resistance.

In later work, Gardee (2024) expanded this framework and proposed a reciprocal relationship between what she calls the micro-level, focusing on interactions between the social identities offered to learners and the mathematical identities they construct, and the macro-level, which includes broader social influences such as political perspectives, educational ideologies, like those related to ability grouping, and social class structures (see Fig.1).

According to Louie and Zhan (2022), a focus on the macro-level is key for equity, as broader influences like educational ideologies contribute to what they call the “culture of exclusion” (p. 366). Gargroetzi (2024) similarly argues that mathematics is “steeped in racialised, gendered and ableist hierarchies of intelligence and mathematical abilities” (p. 128), so it is essential to account for educational ideologies. While racialised, gendered, and ableist ideologies of mathematical ability are distinct, these ideologies are all related to a political perspective at the macro-level called neoliberalism.

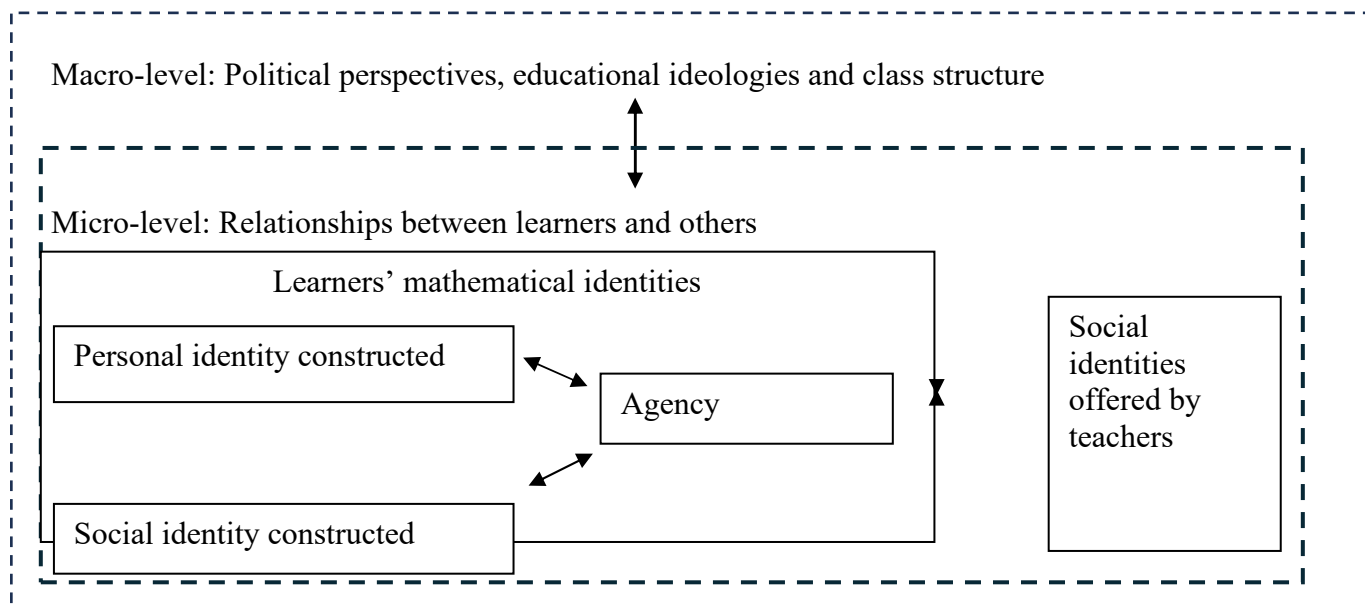


Figure 1: Mechanisms responsible for the emergence of learners' mathematical identities

Neoliberalism is a political ideology associated with market capitalism, which emphasises individualism and competition as key to human relations. From this view, learning mathematics is framed as an investment in skills and knowledge, positioning the learner as future human capital (Sengupta-Irving, 2021). Neoliberalism is associated with the stratification of learners through labels such as “smart” or “dumb”, which are linked to ideologies of ability (Sengupta-Irving, 2021). These ideologies are enacted in practices like ability grouping, which aligns with neoliberal thinking by using educational success or failure to sort learners according to perceived economic potential. Sengupta-Irving (2021) also argues that neoliberalism intersects with an economic system that privileges patriarchy, whiteness, and class elitism while excluding poor and Black youth.

Therefore, research must focus on the relationships between the micro- and macro-levels to understand how learners' mathematical identities and issues of equity are shaped both by immediate interactions, such as teacher practices, and by broader forces, such as ability grouping ideologies.

METHOD

This study forms part of a broader research project that explores the mathematical identities of Grade 10 girls in two public secondary schools in South Africa. In this paper, I focus on the teaching practices of Mr Banza, a mathematics teacher at one of the participating schools. Mr Banza taught two Grade 10 mathematics classrooms: one designated as mixed-ability (June exam average: 41%) and another as high-performing (June exam average: 78%).

The school was co-educational, with both boys and girls in the school. In the high-performing classroom of 24 learners, 5 were Black learners, 2 were Coloured learners, with the remainder Indian. In the mixed-ability classroom of 28 learners, there were 12 Black learners, 1 Coloured



learner with the rest Indian. The high-performing group included 8 girls, while the mixed-ability group included 11 girls. There were no White learners in both of these classrooms.

From each classroom, I selected one of the higher-performing female learners, identified by Mr Banza, for further discussion. Damisha, an Indian learner, who achieved 94% in the June mathematics exam, was selected from the high-performing classroom, while Kajol, an Indian learner, who scored 57%, was selected from the mixed-ability classroom. I focus on these learners because they were considered by Mr Banza as being more successful than others in their mathematics classrooms, which allowed me to examine both the similarities and differences in their learning experiences.

Data collection

Data were collected over three weeks during the third school term of 2022 (July–October) at each of the participating schools. The data included videotaped lessons, field notes, photographs of learners’ notebooks, and audiotaped semi-structured interviews with teachers and learners. Approximately 200 minutes of lessons were recorded in each of Mr Banza’s classrooms. Field notes and photographs of learners’ notebooks were used to supplement the recordings and provide additional contextual information.

All interviews were conducted after the lesson observations. Mr Banza participated in one interview that focused on his pedagogical approach and social interactions with learners across both classrooms. Sample questions included: “Can you please describe your pedagogy?”, “What strategies do you use to include all learners in your lessons?”, and “How do you identify (name of learner) as a learner in your classroom?”

The learner interviews focused on exploring learners’ mathematical identities and their experiences of learning mathematics in Grade 10. Sample questions included: “Please describe how you see yourself as a mathematics learner” and “How do you feel when doing mathematics in your classroom?” All teacher and learner interviews included follow-up probing questions to elicit deeper reasoning and richer responses.

Data Analysis

A qualitative approach was used to analyse the data. All lesson recordings were transcribed, and the analysis focused on three core aspects: 1. Mr Banza’s teaching practices in both classrooms, 2. his interactions with Damisha and Kajol, and 3. Damisha and Kajol’s participation in their mathematics classrooms. Mr Banza’s pedagogy was coded based on the types of questions he asked in terms of whether they required lower-order reasoning, such as simple recall, or higher-order reasoning, requiring deeper engagement with mathematical concepts. I also examined any other differences in how he taught the high-performing and mixed-ability classrooms, such as how he explained concepts to learners in each classroom. His relationships with the learners were analysed in terms of whether he interacted with and supported them during lessons.



To analyse how Damisha and Kajol exercised agency and participated in lessons, I focused on their interactions with Mr Banza and their tasks, in terms of whether they asked or responded to questions, completed their tasks, and engaged in self-directed activities such as taking additional notes or doing extra exercises.

Mr Banza's interview was analysed to explore: (1) how he described Damisha and Kajol as mathematics learners, and (2) how he explained his teaching methods and social relationships with learners, including these two focal participants. These insights, combined with the classroom observation data, helped to analyse how he offered social identities to learners, as either affiliation or marginalisation (see Gardee & Brodie, 2021).

The interviews with Damisha and Kajol were analysed with a focus on: personal identity, social identity constructed, experiences of social identities offered, and agency. For personal identity, I focused on how each learner described herself as a mathematics learner, including her emotions and interests in relation to mathematics (Archer, 2002; Marks & O'mahoney, 2014). For social identity constructed, I examined how each learner positioned herself within the classroom community, and the extent to which she felt comfortable in the classroom community (Archer, 2002; Marks & O'mahoney, 2014). I also analysed their experiences of the social identities offered by Mr Banza, which provided additional insight into how each constructed her mathematical identity. For agency, I examined how both learners described their participation in the mathematics lessons and the extent to which they took additional initiative in their learning, along with their reasoning for doing so.

Following this process, I then coded how Damisha and Kajol constructed their mathematical identities in relation to the social identities offered by Mr Banza, either as affiliation, compliance, resistance, or marginalisation. Throughout the analytical process, I remained attentive to how broader ability grouping ideologies influenced the social identities offered to each learner and the mathematical identities the learners constructed. Data from classroom observations and interviews were continuously compared to strengthen the credibility of the findings.

FINDINGS

Mr Banza's teaching practices

In both mathematics classrooms, Mr Banza usually began his lessons by providing learners with classwork that they were expected to complete with their peers. As learners worked on the tasks, he moved around the classroom, checking their progress and assisting them. Once they completed the tasks, he would go over the corrections, sometimes by inviting learners to present their solutions on the board, and at other times by giving lengthy explanations himself. While he did occasionally ask questions during these explanations, these questions typically required lower-order reasoning in both classrooms.

In his interview, Mr Banza described his teaching style as predominantly learner-centred. He explained that he preferred learners to explore the content themselves and to struggle with the



tasks before he intervened:

“Most of the times, it’s more learner-centred. Yes, that is my teaching style... I want learners to discover more of things that are happening than me, like feeding them what they have to. I hope you observed my lessons mostly in these strong classes. I will let them struggle. First, they have to like, push the boundaries before I intervene. I will let them struggle. And then when I see okay, now I have to step in, then I step in. It's more about learner-centred. They have to do stuff themselves. I only assist the learning process”.

However, there were differences in the ways he taught each group of learners, as he explained that his approach to allowing them to struggle was predominantly for the “strong” classroom. He explained:

“Yes, there's a big difference between the two. Even my teaching style. When I am in the other class, I need to also shift in my teaching style”.

According to Mr Banza, the key difference was his “involvement” in the learning process.

“That would be the key difference, my involvement. In that other class (mixed ability), I'm more involved in the process of learning. So it's like you are working together with them so that they don't miss any step. But in this other class (high-performing) here you are behind the steps, they are working by themselves. You're just there guiding where you see you have to intervene. Sometimes you don't even need to explain other things because you notice that they already know it. You need to move, or you need to provide something more challenging than what you gave them, even in terms of questions. Some of the questions that I, I work with these ones. I cannot do it in the other class. I cannot like what we did today. I cannot like work with it in the other class. It will take me a double or even two days just to do that worksheet”.

The tasks given to learners in the high-performing group were more challenging than the tasks given to learners in the mixed-ability group, with Mr Banza providing more guidance to learners on how to complete tasks in the mixed-ability group, a claim that aligned with what I observed during the lessons. In addition, the learners in the higher-performing group covered more work than the mixed-ability group. In one lesson with the high-performing group, Mr Banza told learners that they completed a section that others in the same grade did not, so the section might not even be assessed because learners in the other classrooms have not done it.

Mr Banza held high expectations of learners in the high-performing classroom. While he assigned homework to the mixed-ability group, which he sometimes called the “lower stream”, Mr Banza did not expect them to do the work, highlighting his lower expectations of them. He explained:

“The lower stream is like they don't like working. They don't like working, that is the most challenging part of teaching that cuts through, that is the most challenging. You can give a task to be done like homework. Don't expect more than 50 percent of them completing the work. Don't expect it. So, when you give homework, you need to follow up, otherwise you won't get



it done, which is not the case in that other class (high-performing). You might not give homework, but they bring you the work done”.

I observed that Mr Banza did not assign homework to the high-performing group. He explained that learners in the high-performing group typically brought work to him on their own. He also explained that these learners themselves were aware that they were expected to work harder and perform better than others:

“They also know. Those that are here also know that we are better than them, those other kids. So, they have to, like, work harder”.

While with the mixed-ability group, he explained that these learners knew that they were “weak”: “They know that we are weak, that’s why we are not in that other class”.

In terms of his relationships with learners, Mr Banza came across as supportive and willing to assist all learners in both classrooms. He was frequently seen helping learners without being asked and made a point of checking in on their work. In his interview, he expressed a belief that all learners can succeed in mathematics and said he tried to cultivate a “love” for the subject by drawing connections to everyday life. He also offered free extra lessons after school, further demonstrating his commitment to supporting all learners. In this way, he offered all learners in both classrooms a social identity of affiliation, even though learners in the higher-performing group had more opportunities to develop their reasoning through challenging tasks, in comparison to the mixed-ability group.

Damisha

Social identity offered to Damisha

In his interview, Mr Banza described Damisha as “a good girl”. He spoke positively about Damisha, explaining:

“When you like explaining something, they get it. They start working. They're not waiting for you to come and guide again...they start working”.

From my observations, I noted that Mr Banza was willing to support Damisha, even though she seldom initiated interactions with him. He was often seen standing quietly behind her and her peer, observing their work and only intervening when he noticed an error. These interactions suggested that Mr Banza offered Damisha a social identity of affiliation with his classroom community.

In her interview, Damisha expressed a more critical view of both his pedagogy and their social interactions. She appreciated the many opportunities that Mr Banza offered them to practice mathematics, describing it as beneficial for assessments:

“Um I think it's a nice way of preparing for like tests and stuff. Like to just keep on practising and doing them while the teacher’s around. So when you don't understand how to do something,



you can ask”.

Despite this, she found Mr Banza only “sometimes” helpful. She experienced his explanations as moving “too fast” and said that he did not “understand” when she struggled with mathematics:

“(He) doesn't understand like, where... why you're struggling with something. So he'll just kind of try to help. But he doesn't really understand that you don't understand... he will just like assume that like, you know how to do some things and stuff when you need help, then he doesn't start from like where you need the help. He's like, he skips a few steps”.

Damisha explained that he did engage with her, but she experienced him as making assumptions about what learners in her classroom could do, and as concentrating his attention on a select group of learners whom he believed were more capable than others:

“It makes me a little bit irritated that like he doesn't understand that, like not everybody can do, like what some people can do... I feel it's kind of unfair because he knows that they... he wants them, it's like he wants them to do better and he wants... he's more worried about them”.

While Mr Banza did interact with Damisha, I noticed that he spent more time engaging with two learners who outperformed Damisha in the exams. Unlike her, these learners frequently initiated interaction with him and asked for help. So while Mr Banza offered Damisha an identity of affiliation, she did not experience the social identity that he offered her as being equal to that of the learners who performed better than her.

Personal identity

When describing her personal identity, Damisha described herself as “school-focused”, explaining that “a lot of my life revolves around school work”. She described mathematics as an important subject that came easily to her: “it comes naturally. Like I understand pretty easily what's going on”. She also said that she was currently enjoying mathematics: “it's all been making sense so far”. Reflecting on her experience of mathematics more broadly, she explained:

“I feel fine, like I've never struggled with anything specifically in math. Even if I do struggle, it's just for a little while. But like by the time it's time to write the test, I understand”.

Social identity constructed

In terms of her social identity, Damisha said she felt “comfortable” in her mathematics classroom, but also admitted to experiencing stress, particularly due to Mr Banza's classroom practice of calling on learners unexpectedly:

“Mr Banza likes to pick on some people sometimes to answer questions and stuff”.

She expressed mixed feelings about being in a high-performing classroom. On one hand, she



appreciated being surrounded by peers who could support her learning. On the other, she was critical of the high expectations and kinds of assessments that accompanied being in this classroom:

“It can be sometimes like nicer because like when you need help, like somebody knows what to do, but also like there is like higher expectations also, like he sets harder tests and stuff like that.... I think that is unfair. I think we should all be writing the same tests and writing on the same level of difficulty”.

She was particularly uncomfortable with the pressure that came with these higher expectations:

“I don't like having those higher expectations... Even in the class I feel like they put high expectations on me”.

Damisha explained that she was placed in the high-performing classroom because she received “high marks”, but she expressed concern about how Mr Banza compared learners across classrooms:

“They (learners in the mixed-ability group) also, they say that, like he'll (Mr Banza) say a lot of the time, like his other class (referring to her high-performing classroom) is better. And his other class (referring to her high-performing classroom) gets higher marks, he does bring it up. They also. I don't think, they like it so much. Like it's not really nice, like having your teacher, like, constantly comparing it to, like another class that's like he knows just like naturally does better”.

So, she felt that learners in her classroom, including herself, performed “naturally” better than learners in the mixed-ability classrooms.

Agency

Damisha's personal and social identity were related to how she exercised agency in the classroom. She was often observed working closely with her peer on tasks, but she did not initiate interactions with Mr Banza. She said that her participation with him was limited to instances when he selected her to respond to a question:

“If there's a question, if he asks something and sometimes, he'll pick on people, just to answer. So that's how I participate”.

She was often observed marking her own work and taking additional notes when Mr Banza explained concepts. She was also seen discussing her tasks with her peers during break time. While she did not say if her parents assisted her, she said they thought she was good in mathematics. She also said that both of them are accountants, which indicates that she came from a middle-class family.

Summary



Taken together, Damisha's case illustrates how her experiences of the social identity offered by Mr Banza, along with her personal and social identities, were related to the way she participated in the mathematics classroom. While Mr Banza offered her a social identity of affiliation, Damisha developed her mathematical identity as compliance, by participating with her peer and completing her tasks, rather than engaging with Mr Banza.

Kajol

Social identity offered to Kajol

Mr Banza described Kajol as a learner who worked hard, although he could not understand why her performance in assessments did not reflect her effort:

“She works hard. She really works hard. But she's not getting the level. It's not paying. The outcome is not what really she needs to get from the hard work, but she likes working hard”.

Based on her hard work, Mr Banza believed she had the potential to pursue mathematics in the future.

He further explained that Kajol participated actively in lessons, which I observed as well. He also said that her participation was different from others in the mixed-ability classroom:

“(She) is not shy of asking. They like... they interact with the teacher, with me personally. They are free to ask. They feel free to ask and they want you to be also involved in checking their work, whatever. What is not the case with others all the time. With these, if you give them something, you see them working. You see them working like trying to find out, trying to solve this. They are involved in the process. Not like others”.

During lesson observations, Mr Banza was often seen interacting with her as he moved around the classroom. These interactions suggested that Mr Banza offered Kajol a social identity of affiliation with his classroom community.

Kajol responded positively to the social identity offered by Mr Banza. She said she enjoyed his lessons “a lot”, particularly because of the emphasis on practice. She appreciated the way he helped them when they struggled:

“If you don't understand it, he'll often sometimes walk us through half the question, half the answer. And then he says, work it out yourself because then you'll understand it more”. Kajol also said she appreciated his encouragement for learners to work together during lessons.

When speaking about her relationship with Mr Banza, she described him as supportive and helpful: “He's always there, like to help us if we need help”. She experienced him as inclusive, treating all learners fairly and giving everyone the chance to participate: “He tries to involve all the learners, even the ones struggling”.

Like Damisha, Kajol was aware that learners were grouped according to ability. She explained



that Mr Banza told her she had the potential to move to the high-performing group the next year:

“He says that he thinks I can achieve higher marks than I get and he believes that next year I can go into the higher achieving class because there's two separate classes, one that learns faster than the other. So he thinks the faster class gets high marks. So he says last time we went to our parents meeting, he told me that next year he does believe I can get put into the faster learning class because...(He told me) I have potential to do good in maths”.

While she found this motivating, she also had some reservations:

“I feel like it's very motivating... But at the same time, I don't know if it would be too fast or overwhelming for me, because in this class I do feel very confident as I'm one of the learners who's getting a lot of the answers right and understanding it. So I feel like in the faster class, a lot of kids will understand it faster than me. And then when they do, I'll feel like almost like dumb because I'm not understanding it as fast as they are”.

So, Kajol felt confident in her current mathematics classroom because she understood the work and was achieving the correct answers, unlike others in her classroom. Her hesitation about moving into the high-performing group stemmed from the fear that she might no longer feel confident, as she believed that others would grasp the work faster and make her feel “dumb”.

Personal identity

When describing her personal identity, Kajol explained that mathematics was her favourite subject. She said:

“I feel like I'm very eager to learn maths because I do enjoy it and I'm doing around average in maths. It's like I understand it and that's what makes me love it”.

She explained that she had enjoyed mathematics since primary school, found mathematics relevant to her life, and said that it was “intriguing” that she could understand mathematics, while others could not.

Social identity constructed

In terms of her social identity, Kajol said that she felt both “comfortable” and “confident” in her classroom. She explained:

“I feel nice. It's not like you're tense because the class is allowed to... It mustn't be like, it's not like it has to be pin drop silence. Like, obviously there has to be a level of control, but it's like you can ask for help and it's like a very comfortable environment to me”.

She added that she felt confident particularly because she often got the correct answers in the classroom.



Agency

In terms of agency, Kajol explained that she didn't interact much with Mr Banza directly because she experienced anxiety when engaging with teachers in general. However, I observed her often engaging with him as he walked around the classroom to assist learners. She said that she preferred to work with her peer and was often seen discussing tasks with her peer during lessons. If she did not understand, she said she would look through her book or use YouTube to assist her when preparing for exams. Both Mr Banza and Kajol confirmed that she attended the free extra classes that he offered after school. She also said that her parents did not assist her. Her mother was a hairdresser, and her father was a truck driver, which indicates that she came from a working-class family.

Summary

Taken together, Kajol's case illustrates how the social identity offered by Mr Banza and her personal and social identities, related to how she participated in the mathematics classroom. She constructed her identity in affiliation with the social identity offered by Mr Banza.

DISCUSSION AND CONCLUSION

This paper makes two key contributions, each emerging from the research questions. The first contribution relates to understanding how teacher practices, in the context of ability grouping, contribute to Grade 10 learners' mathematical identities. In line with findings from other studies (e.g., Boaler, 2020; Du Plooy, 2019), learners in Damisha's high-performing classroom were afforded more opportunities to engage deeply with mathematics through challenging tasks and covered more content, compared to learners in Kajol's mixed-ability classroom, where tasks were simpler and lessons progressed at a slower pace. While learners in the high-performing classroom were given opportunities to struggle productively, Mr Banza provided more guidance to the mixed-ability group. So, even though both learners were taught by Mr Banza, they experienced different pedagogies and opportunities to learn mathematics, based on their placement in different classrooms.

Unlike research showing that teachers who teach mathematics in the context of ability grouping are often unwilling to support all learners (e.g., Boaler et al., 2000), Mr Banza was willing to support all and offered everyone opportunities to learn mathematics, thereby offering all learners an identity of affiliation. However, affiliation meant different things for learners in different groups, which is where the concept of equity becomes key. As a consequence of how he spoke about and taught learners, mathematical success in the high-performing classroom meant completing challenging tasks, while in the mixed-ability group, mathematical success meant completing easier tasks. These differences presented learners with inequitable opportunities to learn mathematics and become successful mathematics doers and thinkers. The June mathematics results reflect these disparities: learners in the mixed-ability group achieved an average of 41%, while those in the high-performing group achieved an average of 78%. So the study shows how ability grouping ideologies are closely related to the kinds of social identities offered to learners.



Further, neoliberalism and ability grouping practices often marginalise Black and working-class learners (Sengupta-Irving, 2021). Kajol's mixed-ability classroom was more racially diverse (12 Black learners and 11 girls in a classroom of 28), while Damisha's classroom had more Indian and male learners (5 Black learners and 8 girls in a class of 24). Kajol came from a working-class background, while Damisha came from a middle-class family. These findings highlight how ability grouping practices not only reflect but also reproduce inequalities, particularly along lines of race, gender, and class, as learners in Damisha's classroom had opportunities to learn mathematics more deeply than learners in Kajol's classroom. These inequalities are critical concerns in the South African context, where race, gender and social class status continue to correlate with performance in mathematics, as shown by the TIMSS and DBE reports.

The second contribution relates to the second research question, which entailed understanding how ability grouping practices shape the identities of learners who perform better than others in their classrooms. Consistent with Boaler et al.'s (2000) findings, Damisha experienced high expectations, a fast-paced curriculum, and pressure from her placement in a high-performing classroom as sources of stress. She also felt that Mr Banza paid more attention to learners who performed better than her, which perhaps informed why she chose not to engage with him. This resulted in her constructing a mathematical identity of compliance, as she participated with tasks and collaborated with her peer, but avoided interaction with Mr Banza. While many learners' enjoyment of mathematics is linked to the social identities offered by their teachers (Gardee & Brodie, 2021, 2022), Damisha's enjoyment of mathematics was related to her personal identity: mathematics made sense to her.

Damisha's case also reveals how ability grouping ideologies shaped her identity. She referred to mathematics as something that "comes naturally" to her and believed that learners in her class "naturally" outperformed others in the mixed-ability classrooms. While her view of ability aligns with dominant ideologies suggesting that mathematical ability is innate (e.g., Gargroetzi, 2024), her success was underpinned by effort. This finding shows how ability grouping ideologies inform the mathematical identities of learners, as Damisha misattributed her success to ability, instead of effort, and so she viewed herself as being different and naturally capable, unlike others in the mixed-ability classrooms.

Like Damisha, Kajol also enjoyed mathematics and demonstrated extra effort in her learning. While much research highlights the negative effects of ability grouping on learners in lower groups (Boaler et al., 2000), this was not the case for Kajol. Even though she was given less challenging tasks than those in the high-performing group, her success supported her identity especially since she identified herself as one of the learners achieving the correct answers in her classroom. She was hesitant about joining the high-performing classroom, despite being told she had the potential, because she feared that moving could shift her identity from being successful to being "dumb" in comparison to others. So her mathematical identity was shaped by her relative success in her current classroom. This finding reveals that learners' mathematical identities are not only shaped by teacher practices, such as pedagogy and social relationships, but also by how their success compares to that of their peers.



In conclusion, this paper contributes to the growing body of research on the relationships between teacher practices, ability grouping ideologies, equity, and learner identity, particularly in the South African context. My findings show how ability grouping practices continue to reinforce inequalities, stratifying learners by race, class, and gender, and offering unequal opportunities for mathematical learning. While mixed-ability grouping is often framed as more equitable, this study reveals how deficit labels like “lower stream” or “weak” can still operate within these settings, limiting learners’ access to learning mathematics deeply. Importantly, this paper highlights the complexity of learners’ mathematical identities, showing how ability grouping ideologies, peer dynamics, teacher practices, learners’ personal identities, their social identities and agency, all relate to learners’ mathematical identities.

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EXPLORING GRADE 11 LEARNERS' CONCEPTUALISATION OF THE NATURE OF ROOTS OF QUADRATIC EQUATIONS

Sive Ndluzonke and Benjamin Tatira

Walter Sisulu University

This study examined how grade 11 learners in Eastern Cape Province in South Africa perceive the connection between a quadratic equation's discriminant and its roots' characteristics. It also looked into the difficulties learners encounter while trying to grasp this concept and the factors that play a role in causing these challenges. Understanding quadratic equations, distinguishing between real and complex roots, and representing them graphically pose challenges for many learners. This study employed a qualitative research methodology to explore learners' comprehension of the characteristics of the roots of quadratic equations. The research utilised qualitative data gained from content analysis of 57 learners' written responses to a test. Purposive sampling technique was used to interview ten grade 11 learners and their teacher. The findings of this research indicate that teaching quadratic equations in a way that prioritises relational or conceptual understanding at the appropriate level can help learners utilise correct procedures alongside comprehension.

INTRODUCTION

Comprehending quadratic equations is crucial in high school mathematics education since it provides a base for more complex mathematical ideas and practical problem-solving. Quadratic equations in the form of $ax^2 + bx + c = 0$ can have either real or complex roots, and this scenario significantly impacts learners' understanding of algebraic concepts.

A study by Ncube and Luneta (2024) reveals that learners in rural and underprivileged regions often encounter obstacles that affect their comprehension of mathematics, such as restricted availability of educational materials and inconsistent levels of educator readiness. These elements may impact on how learners understand and use mathematical ideas, like the properties of roots of quadratic equations.

Understanding quadratic equations, distinguishing between real and complex roots, and representing them graphically pose challenges for many learners. Common misunderstandings regarding these concepts have been thoroughly studied in educational research. For instance, Alajmi (2020) points out that learners commonly misinterpret the graphical meanings of complex roots and often do not have a unified conceptual structure for solving quadratic equations. Additionally, Ndlovu and Malan (2021) point out that misunderstandings in algebra, especially with quadratic equations, can cause major challenges in learners' mathematical progress.

This study aims to investigate how Grade 11 learners perceive and comprehend the origins of



quadratic equations, such as real and complex roots, and to determine any common misunderstandings or challenges they might face. To investigate Grade 11 learners' understanding of the roots of quadratic equations in the Eastern Cape, the following research questions have been developed:

1. *How do Grade 11 learners conceptualise the nature of the roots of quadratic equations?*
2. *What common misconceptions do Grade 11 learners have regarding the nature of roots of quadratic equations?*

The main goal of these research questions was to explore how Grade 11 learners perceive quadratic equation roots, the obstacles they encounter, and the impact of current teaching methods. The initial inquiry aims to offer effective suggestions derived from the study results directed towards improving teaching approaches, correcting misunderstandings and enhancing educational materials. The second inquiry focuses on investigating learners' comprehension of real and imaginary roots, as well as their skill in distinguishing between them and utilising this awareness to solve quadratic equations problems with improved understanding.

LITERATURE REVIEW

Challenges faced by learners when solving algebraic equations

A fundamental concept in algebra is quadratic equations, which science, technology, engineering and mathematics (STEM) majors need to be fluent in. Nevertheless, quadratic equations appear so complicated that school learners struggle in solving them (Kusi, Boateng & Teku, 2025). In their study, Kusi, Boateng and Teku (2025) posited that the use of technology can help learners understand quadratic equations. The challenges in quadratic equations stem from lack of understanding of the concept of equations in general. Studies by Falker, Levi and Carpenter (1999), Ketterlin-Geller et al. (2007) and Knuth et al. (2008) have indicated that both young and old learners struggle to grasp the concept of the equal sign. Learners do not realise that equality is a concept that shows when two mathematical expressions have the same value. Mamba (2012) stated that a lack of comprehension of the purposes of the equal sign and its characteristics in equations was a significant obstacle for learners, impeding their ability to solve equations accurately.

Studies show that learners frequently encounter difficulties with different aspects of quadratic equations, such as understanding the properties and significance of the roots. For instance, Alajmi (2020) discovered that learners often struggle with comprehending the visual display of intricate roots and how they affect the solution of quadratic equations. The difficulty frequently stems from a limited grasp of complex concepts and in picturing abstract mathematical ideas (Alajmi, 2020). Similarly, Kieran and Meyer (2017) found that learners often struggle to understand how the coefficients of a quadratic equation relate to the characteristics of its roots. They pointed out that although learners may grasp the method of determining roots with the quadratic formula, they frequently lack a clear comprehension of the reasons behind roots being real, complex, or repeated. Kotsopoulos (2007) implies that



difficulties in solving quadratic equations through factorisation might stem from not remembering basic multiplication facts, which can degrade learners' problem-solving skills.

The literature thoroughly documents misunderstandings related to quadratic equations. For example, research conducted by Ndlovu and Malan (2021) revealed that typical misunderstandings involve mistaking the discriminant for the nature of the roots or not fully grasping the importance of complex roots in practical situations. Such misunderstandings may impede learners from accurately solving quadratic equations and applying their knowledge in different mathematical problems (Ndlovu & Malan, 2021). Papageorgiou and Fokeides (2021) reinforced the idea that learners frequently have difficulties in understanding the discriminant and how it influences the characteristics of roots. Papageorgiou and Fokeides (2021) highlighted the importance of teaching approaches that specifically target these misunderstandings and offer learners a deeper grasp of quadratic functions. Tendere and Mutambara (2020) classified errors in solving quadratic equations as conceptual, procedural, and technical. The conceptual errors found were learners' misconception on like terms, incorrectly applying factorisation in a non-standard quadratic equation and mishandling signs and radicals. The procedural errors involved memorising and following the algorithms in solving without completely understanding the principles behind these.

Luneta and Makonye (2010) described mistakes as errors and difficulties shown by learners in their mathematics responses. They also distinguish between two categories of errors: systematic errors and unsystematic errors. As Lukhele et al. (1999) stated, unintentional errors are displayed by learners and are unlikely to be repeated. On the other hand, systematic errors can persist over time as they are consistently built or reconstructed due to adopting incorrect ideas when tackling a specific problem. Some learners opt for quick solutions in mathematics, leading to potential errors (Idris, 2011).

Pedagogical strategies

Effective teaching strategies are crucial for improving learners' understanding of quadratic equations. Research by Gutiérrez and Boero (2022) suggest that employing visual aids, dynamic software and interactive learning environments can significantly enhance learners' grasp of complex mathematical concepts. Their study highlights the importance of using multiple representations of quadratic functions to help learners visualise and understand the nature of their roots (Gutiérrez & Boero, 2022). In order for learners to grasp the process of finding the roots of a quadratic equation, educators must create a learning environment that supports learners and encourages critical thinking, incorporating both memorisation and algebraic problem-solving (Lithner, 2008).

Different methods of teaching can be employed when teaching quadratic equations for comprehension, such as arithmetic or numerical representations, algebraic or symbolic representations, and visual or geometric representations to solve for the roots of quadratic equations (Katz & Barton, 2007). Furthermore, Herawaty et al. (2021) demonstrated that squares, rectangles and unit square cards are effective tools for teaching quadratic equations through factorisation techniques. Educators may utilise conceptual visuals to instruct quadratic



equations, aiding in enhancing their teaching by gaining a deeper understanding of their learners (Kabar, 2018). Nonetheless, Sosa-Moguel and Apricio-Landa (2021) discovered that educators struggle to select a suitable approach for teaching quadratic equations in a way that helps learners grasp the concept of determining roots. According to Prima Sari and Jailani (2019), certain educators were discovered to be teaching quadratic equations too quickly, resulting in decreased learner engagement and, ultimately, a lack of comprehension of the topic. Vaiyavutjamai and Clements (2006) discovered that instruction on quadratics tended to focus on educators, emphasizing procedural comprehension rather than explaining the symbols' meanings, which could lead to challenges for learners in finding the roots of quadratic equations.

THEORETICAL FRAMEWORK

This study is guided by Skemp (1976) and Kilpatrick et al.'s (2001) theoretical frameworks of mathematical proficiency, which are pivotal in understanding the difficulties learners are faced with when solving quadratic equations. Mathematical proficiency is defined through four interrelated strands: procedural fluency, conceptual, relational and instrumental understanding. These strands provide a lens through which learners' challenges with the roots of quadratic equations can be examined.

Procedural knowledge

Kilpatrick et al. (2001) define procedural knowledge as the concentration on processes, the proper timing and method of their usage, and possessing the capability to execute them with flexibility, precision, and efficiency. Procedural knowledge entails a sequence of actions that need to be adhered to in order to address mathematical issues. This understanding encompasses algorithmic abilities, strategies, and approaches. However, Skemp (1976) believes that simply knowing how to solve mathematics problems is not enough. Learners must be able to explain why certain techniques are used for specific tasks, known as relational understanding. According to Pegg (2010), limited working memory capacity may cause learners to use incorrect methods when solving problems. Hiebert (1999) posits that when learners rely on memorised procedures, they may struggle to grasp the underlying concepts of the mathematical problem due to limited knowledge to support the procedures they are using. Learners would commit to memory steps to find the solutions of quadratic equations without understanding the significance of those solutions. To effectively solve mathematical problems, it is essential to combine procedural knowledge with conceptual knowledge.

Conceptual understanding

Conceptual understanding involves a practical comprehension of mathematical concepts. Learners who possess a thorough understanding of concepts have knowledge beyond just memorised facts and techniques. They comprehend the significance of a mathematical concept and its practical applications. They have structured their information in a cohesive manner, allowing them to absorb new concepts by linking them to existing knowledge. Kilpatrick et al. (2001) explain that conceptual knowledge is the understanding of learners regarding concepts,



operations, symbols, diagrams, procedures and relationships between ideas in the field of mathematics. This kind of comprehension leads to blending mathematical understanding with both content and context. Grade 11 learners demonstrate conceptual understanding by connecting the roots of quadratic equations in various scenarios. Kilpatrick et al. (2001) identify conceptual understanding as a key component of mathematical proficiency.

Instrumental understanding

Skemp (1976) referred to instrumental and relational understanding as the two elements of conceptual understanding. Instrumental understanding involves the application of procedures and formulas without understanding their origins. For instance, a learner can frequently arrive at the correct solution faster and with more reliability through instrumental thinking rather than relational thinking (Skemp, 1976). Rote learning is an isolated form of knowledge that encourages memorising ideas without truly understanding them. It motivates learners to apply processes in order to solve mathematical problems without understanding the reasoning behind those problems. The ability to find factors of quadratic equations and determine their roots through procedural memorisation is key to instrumental understanding, regardless of grasping the meaning behind those roots.

Relational understanding

Skemp argued that, while instrumental understanding may have short-term benefits, such as taking less time to learn procedures and skills, relational understanding is more adaptable to new tasks and is an appropriate goal. Relational understanding allows learners to build their own schema or mental maps to enable them to find their own solution processes to achieve a variety of mathematical goals. Learners who have a relational approach to mathematics understand concepts that come from cognitive schemas, a system of interconnected ideas (Machaba, 2016). The concept of relational understanding involves interpreting the origins of quadratic equations, deducing quadratic equations with specified roots, finding the second root given one root and calculating the coefficients when two roots are known.

METHODOLOGY

A qualitative research methodology was used in this study, which involves gathering, studying and deciphering detailed, story-based, and visual data to understand the development of algebraic problem-solving skills in Grade 11 learners. Qualitative researchers frequently utilise various types of data sources in a single study. They could utilise observations, interviews, written documents, or any other means to address their research question (Leedy & Ormrod, 2005). In qualitative research, the researcher depends on participants' perspectives, poses overarching questions, gathers primarily verbal data from participants, interprets and examines these words for patterns (Creswell, 2012).

The case study research design was used which saw data being gathered from learners and teacher in the natural settings, which was their classroom. This prevented learners from feeling comfortable, which could impact the reliability of the collected data. This study was conducted



in a senior secondary school located in Eastern Cape Province, where the first author was doing teaching practice. The educational setting in the Eastern Cape Province district is due to a mix of socio-economic challenges and varying educational experiences. The test was administered during the normal class period and learners had half an hour to finish it. A class of fifty-seven Grade 11 learners who were learning Mathematics in 2024 were involved as participants. It was deemed that 57 learners would be adequate to identify descriptive values, errors and thought processes of the learners. These learners were familiar with quadratic equations to answer the study questions, as the topic was covered that same year. The participants were assigned a test consisting of four problems to be solved. The learners were required to evaluate the value(s) of x in the following items: (i) $x(2x - 3) = 5$; (ii) $-x^2 - 2x + 3 = 0$; (iii) $2x^2 - 3x = 5$; and (iv) $2x^2 - 4x = 0$. The four items were taken from an examination question bank, some of which mirrored questions found in a Grade 11 mathematics textbook. The learners' scripts were carefully checked for possible mistakes and misunderstandings that learners might have made on the items. The test data responded to research question 1.

After completing the tasks, interviews were conducted individually for thirty minutes, based on the responses provided. Purposive sampling was chosen to select ten learners and one teacher because the learners were required to solve these equations using their existing knowledge. The selected teacher was the mathematics teacher for this Grade 11 class, who could give insight into learners' misconceptions regarding the nature of roots of quadratic equations. Interviews were necessary as we could not completely understand the learners' thoughts from their written responses. It was more effective to hear it directly from the participants. The interviews allowed the researchers to gain a thorough comprehension of the reasons behind learners' errors and the source of their misconceptions. Interview data responded to research question 2. Thematic analysis was employed for analysing data since the researchers adopted a qualitative methodology. This involved transcribing interviews, followed by categorising and identifying patterns in the data.

FINDINGS AND DISCUSSION

Written responses

The data provided outlines the performance of learners on the four equations, which were categorised into wrong answers (WA), right answers (RA) and unfinished attempts (UF). Table 1 illustrates the learners' performance in the test.



Table 1. Summary of learners' answers to the class test

Questions	Wrong answers (WA)	Right answers (RA)	Unfinished (UF)
1. $x(2x - 3) = 5$	22 (38%)	32 (57%)	3 (5%)
2. $-x^2 - 2x + 3 = 0$	17 (30%)	36 (63,157)	4 (7, 01%)
3. $2x^2 - 3x = 5$	24 (42%)	26 (45,614)	7 (12,28)
4. $2x^2 - 4x = 0$	27 (47%)	25 (44)	5 (8, 77%)

The equation in item 1 demonstrates above-average comprehension, as 57% of participants provided the correct answer. The number of incorrect responses is below-average, which shows that some learners' responses indicated a lack of comprehension of quadratic equations presented in non-standard form, as proposed by Hu et al. (2022) and Kotsopoulos (2007). Figure 1 illustrates Learner 3, who lacked procedural or instrumental understanding of solving non-standard quadratic equations.

Figure 1. Lack of comprehension of non-standard quadratic equation

In interviews with learners, it appears that remembering procedures played a role in finding roots through factorisation. Like the initial equation, item 2 also registered a high success rate, accurately answering 63.16% of the time. All these learners managed to change the quadratic equation to an equivalent with positive coefficients. The percentage of incorrect responses is less than the previous calculation, showing an improved understanding of this idea. Failure to reduce the quadratic equation to standard a positive coefficient of x^2 was the common error, as shown in Figure 2.

Figure 2. Incorrect manipulation of the negative signs by Learner 10

The results showed that learners did not understand how to proceed when only one root was provided, and they needed to find the other root. The reason for their adherence to incorrect



procedures is uncertain. Skemp (1976) and Kilpatrick et al. (2001) suggest that without understanding the reasons behind the procedures they use, learners do not have a conceptual understanding of the concept.

Item 3 presents a tougher challenge for learners, as only 45.61% answered correctly, while 42% provided incorrect responses. The incomplete attempts were also significant at 12.28%. This learner takes out x instance of transpose and solves the equation. This shows a lack of conceptual understanding of factor zero. Tendere and Mutambara (2020) and Vaiyavutjamai and Clements (2006) found that a small number of learners can solve greatest common factor equations, such as $z^2 - 2z = 0$ and $b^2 - b = 0$. In the same way, this response indicated that the learner utilised improper methods to find the roots of the equation, revealing a superficial understanding as they chose factors without considering the signs of the terms in the centre and at the end. Others employed the method of common factors $x(2x - 3) = 5$, finding the roots of the equation using the incorrect approach (shown in Figure 3).

$$\begin{aligned}
 2x^2 - 3x &= 5 \\
 x(2x - 3) &= 5 \\
 x &= 20 \quad \text{or} \quad 2x - 3 = 0 \\
 x &= -\frac{3}{2}
 \end{aligned}$$

Figure 3. Incorrect approach to item 3

Item 4 had only 44% accuracy and 47% incorrect answers. There were fewer incomplete tries than before, but the comprehension seems to have decreased overall. Many learners transposed $-4x$ and divided by 2, as shown in Figure 4.

$$\begin{aligned}
 &2x^2 + 4x = 0 \\
 &2x^2 = -4x \\
 &x = -2
 \end{aligned}$$

$$\begin{aligned}
 &2x^2 + 4x = 0 \\
 &\frac{2x^2}{2x} = \frac{4x}{2} \\
 &2x = 4x \\
 &x = 2
 \end{aligned}$$

Figure 4. Incorrect solution to item 4 by Learner 12 and Learner 18 respectively

The same observation was found in the studies conducted by Kim et al. (2021) and Tendere and Mutambara (2020), where learners were seen to have memorised methods for solving quadratic equations. According to Herawaty et al. (2021), for learners to comprehend the origins of quadratic equations, they must be familiar with the precise definition of the quadratic equation. For instance, those learners were unable to determine the solutions to the equations such as $2x^2 - 4x = 0$.



4.2 Interview with the teacher

The transcription of a semi-structured interview with the grade 11 mathematics teacher is shown below.

Researcher: *I am researching why numerous learners find it challenging to identify the roots of quadratic equations. Based on your experience, what are some typical challenges that learners face?*

Educator: *The greatest challenge I noticed was that learners were oblivious to the fact that all quadratic equations produce two roots, whether real or non-real. Hence, some divided both sides by a variable to remain with only one root. Also, where factorisation was expected to be used, learners did not apply it, and to the few who did so, they obtained incorrect factors. Incorrect factors led to incorrect roots, obviously.*

Researcher: *That is reasonable. Can you provide more details on why factorisation could be especially difficult for learners?*

Educator: *Definitely. A profound comprehension of the connection between the coefficients and the factors of a quadratic expression is necessary for factorisation. Learners must understand that the solutions of the quadratic equation are connected to the factors of the equation. At times, they may not understand this connection, particularly if they have only been exposed to factorisation without linking it to the process of solving equations.*

Researcher: *That is logical. Can you explain why factorisation could pose specific challenges for learners?*

Educator: *Another obstacle is connected to the challenge of managing negative numbers and signs. For instance, learners frequently have difficulty finding the right pair of factors involving negative numbers when factoring expressions such as $x^2 - x - 12$. They could wrongly believe that every factor must be positive or they might misplace the operators.*

Researcher: *Do you notice any common misconceptions that learners regularly have?*

Educator: *One common misunderstanding is that learners frequently think that every quadratic expression can be factored into whole numbers.*

In the interviews with learners, the participants consistently displayed errors and misconceptions in both their written responses and verbal answers. We found that some learners recognised their mistakes and misunderstandings after we asked them to elaborate on their solutions. This study aided in prompting participants to contemplate their responses more thoroughly, ultimately bringing attention to their flawed reasoning. While the factorisation is the preferred method of solving quadratic equations where factors exist, the interview with the teacher revealed a relational misunderstanding of the method of factorisation. There were challenges in evaluating the factors in an equation by learners, and at the same time, some regard every quadratic expression $ax^2 + bx + c$ to be factorisable. These flaws are evidence of learners' lack of conceptual understanding of solving quadratic equations. Learners also lack conceptual understanding of the factoring method because of their limited understanding of the zero-product property as illustrated in Figure 3 (Didiș, Baș, & Erbaş, 2011).



What we learned from this study is that participants displayed numerous mistakes in their answers to the test. This aligns with Luneta and Makonye (2013) who reveal that learners commit various errors in completing a single mathematics problem. Learners struggle with mathematics because they are overwhelmed by the variety of concepts and ideas. They are unable to determine which schemas should be used and which should not be used (Skemp, 1976). Frequently, they integrate new ideas into existing frameworks instead of updating them; they adjust and expand their frameworks in order to absorb fresh information. This was the case in finding the roots of quadratic equations. The learners demonstrated a limited understanding of how to factorise quadratic expressions. Some learners in this study failed the initial steps of solving by factorisation. This involves not recognising the various forms the equations can have, thus making it difficult to progress in solving the problem. Furthermore, learners also struggled to identify the correct factors of the given quadratic equation. It has been discovered that learners tend to retain existing schemas rather than expanding them. Many mistakes made by learners were caused by not grasping the importance of creating new schema. Generally, learners were unsure of how to proceed and found it challenging to incorporate new concepts into their existing knowledge. Frequently, learners demonstrated difficulty in comprehending the requirements of the question and tended to approach the items based on their existing knowledge, like reducing a quadratic equation to a simple equation.

CONCLUSION

The theoretical framework applied in the analysis of this study has directed attention towards comprehending the rationale behind utilising specific methods for resolving quadratic equation challenges. The research utilised two linked elements of comprehension, combining relational and conceptual understanding, and combining instrumental and procedural understanding, in order to explore Grade 11 learners' grasp of quadratic equations. Problems posed in this study contained elements of both categories of conceptualisation, but it takes an individual learner's solution process to determine the category he or she operates in. They may be in the same class and taught by the same teacher, but diverse conceptualisations were observed.

This study indicated that learners mostly handle processes without improving their understanding of solving quadratic equations by making connections. Meaning in the learner's mind is linked to connections they make (Stein & Smith, 1998). This study's findings showed problems and misunderstandings among learners when working on quadratic equations, supporting the literature sources cited (Prima Sari & Jailani, 2019; Tendere & Mutambara, 2020; Vaiyavutjamai & Clements, 2005). One possible cause of the challenges and misunderstandings unveiled could be the struggle to articulate the synopsis of a quadratic equation, seen as the solution of the standard form equation $ax^2 + bx + c = 0$. The research indicates that teaching quadratic equations in a way that prioritises relational or conceptual understanding at the appropriate level can help learners utilise correct procedures alongside comprehension. When teachers accumulate many misconceptions and errors, they develop diverse strategies to help students overcome them (Hu, Son & Hodge, 2022).



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ANALYSING 11th GRADE LEARNERS' LEVEL OF UNDERSTANDING OF EXPONENTIAL, HYPERBOLIC, AND PARABOLIC FUNCTIONS

Amokelo Given Maweya and Kereng Gilbert Pule

University of South Africa

This study investigates Grade 11 learners' level of understanding of exponential, hyperbolic, and parabolic functions within the South African mathematics curriculum. Grounded in the Action-Process-Object-Schema (APOS) theory, the research employs a qualitative content analysis of 58 learners' responses to a standardized examination. The aim was to determine learners' cognitive levels when engaging with functional tasks and identify persistent conceptual challenges. The findings reveal a pronounced reliance on procedural reasoning, with most learners operating at the Action and Process levels of the APOS framework. Learners demonstrated difficulty transitioning to higher-order cognitive levels, particularly when interpreting asymptotic behaviour, solving inequalities, or integrating symbolic and graphical representations—tasks aligned with Object and Schema levels. These patterns indicate a systemic gap between rote procedural fluency and deep conceptual understanding. The study underscores the need for pedagogical reforms prioritizing conceptual development through technology-enhanced visualization, commognitive discourse, and structured error analysis. By mapping learner performance onto APOS levels, the study offers critical insights for curriculum developers, educators, and policymakers aiming to improve instructional design and foster cognitive growth in functional learning, ultimately preparing learners for success in advanced STEM fields.

Keywords: APOS theory, Grade 11 learners, exponential functions, hyperbolic functions, parabolic functions

INTRODUCTION

Mathematical functions are critical in developing learners' problem-solving abilities, logical reasoning, and conceptual understanding. The function concept, widely regarded as foundational in mathematics, underpins key topics across algebra, calculus, and mathematical analysis (Niss, 2020). In secondary school mathematics, particularly at the 11th-grade level, learners must engage with complex function types, including exponential, hyperbolic, and parabolic functions. These functions are integral to the curriculum and serve as prerequisites for advanced topics in calculus, physics, and engineering (Corrêa & Haslam, 2021). Ayalon and Wilkie (2019) emphasize that cultivating a multifaceted conceptualization of functions is a central aim of mathematics education, requiring learners to understand procedures and the deeper structural and graphical relationships within and across function types.

The South African high school mathematics curriculum's functions are foundational, serving as a unifying theme across various mathematical topics. Within the Further Education and



Training (FET) phase, the Curriculum and Assessment Policy Statement (CAPS) outlines clear expectations for learners to understand the concept of a function and its inverse, identify critical features such as domain, range, intercepts, asymptotes, intervals of increase and decrease, and symmetry, and fluently move between multiple representations—algebraic, graphical, numerical, and verbal (DBE, 2011). CAPS includes a range of function types such as linear, quadratic, exponential, hyperbolic, trigonometric, and cubic, reflecting the centrality of this concept in mathematics education.

Despite this structured curriculum, national diagnostic reports from the DBE (2019–2024) consistently reveal that learners underperform in the functions section of Mathematics Paper 1, which accounts for approximately 30% of the examination marks in Grades 10 to 12 (DBE, 2024; Mpofu & Pournara, 2018). Specifically, Grade 11 learners face difficulties analysing exponential growth and decay, interpreting the asymptotic behaviour of hyperbolic functions, and understanding the graphical features of parabolic functions. Research suggests that these challenges stem from a limited conceptual understanding and an overemphasis on rote procedures (Mbhiza & Sempe, 2024). Learners often struggle to make meaningful connections between symbolic expressions and their graphical or contextual interpretations, which limits their ability to reason abstractly and solve problems effectively (Buthelezi, 2021). These difficulties hinder not only their academic performance but also their ability to develop higher-order mathematical reasoning skills.

To address this persistent learning gap, this study employs the Action-Process-Object-Schema (APOS) theory as a theoretical lens to analyse Grade 11 learners' levels of understanding of exponential, hyperbolic, and parabolic functions, through a detailed analysis of learners' written responses to examination questions.

Research questions

The study aimed to answer the following questions:

- What cognitive levels (Action, Process, Object, Schema) do Grade 11 learners exhibit when solving questions involving exponential, hyperbolic, and parabolic functions?
- Which components of these function questions present the most significant challenges to learner understanding?

Significance of Study

This study holds significant value in improving mathematics education by addressing learners' persistent challenges in understanding functions, particularly exponential, hyperbolic, and parabolic functions. A significant contribution of this research is its role in informing teaching strategies that emphasize conceptual understanding rather than rote procedural knowledge. Many students struggle with functions due to an overemphasis on memorization, which limits their ability to apply mathematical concepts meaningfully in different contexts. By identifying specific areas of difficulty, this study provides valuable insights that can help educators refine their instructional approaches to enhance learners' comprehension and problem-solving abilities.



The findings of this study are crucial for curriculum developers and policymakers in designing more effective instructional materials and assessment strategies. Given the prominence of functions in high school mathematics assessments, particularly in standardized examinations, a lack of conceptual understanding could significantly impact overall mathematics performance. A well-structured curriculum fostering procedural fluency and deep conceptual understanding is essential in ensuring learners develop the necessary competencies for success in mathematics and related disciplines. Beyond secondary education, this study serves a broader purpose in preparing learners for higher education and professional fields. Functions are foundational in STEM-related careers, including engineering, physics, economics, and computer science. This ensures learners develop a robust understanding of functions and equips them with essential analytical and problem-solving skills critical for academic and professional success.

APOS THEORY

This study is grounded in the Action–Process–Object–Schema (APOS) theory, which provides a comprehensive framework for analysing students’ cognitive structures as they learn mathematical concepts. Formulated initially by Dubinsky (1984) and expanded by Dubinsky and McDonald (2001), APOS theory draws from Piaget’s theory of reflective abstraction, emphasizing that understanding a mathematical concept involves the construction and coordination of mental structures—actions, processes, objects, and schemas—based on internal cognitive development (Dubinsky & Wilson, 2013; Arnon et al., 2014). The four levels of APOS are explained below:

Action

An action is a transformation executed in response to an external stimulus, often under the guidance of an instructor or explicit instruction. It is procedural and step-by-step, relying heavily on external cues. For instance, a student can substitute numerical values into a function expression and solve the equation without internal reflection functions at this level (Dubinsky & Harel, 1992; Makonye, 2017). Recent studies show that learners at the action level perceive functions as static relationships, typically between two sets, and follow algorithmic procedures without holistic understanding (Matindike & Makonye, 2023; Mukuka & Tatira, 2025). Such learners often cannot manipulate function representations unless prompted by structured instruction mentally.

Process

As students internalize repeated actions, they form a process—a mental mechanism that allows them to reflect on and mentally carry out transformations without explicit step-by-step execution. At this level, learners start recognizing a function as a dynamic entity that takes input and produces output, and they are capable of reversing or modifying steps mentally (Breidenbach et al., 1992; Tziritas, 2011). Empirical findings support the development of process understanding as critical for conceptualizing functions as operators rather than static entities. For example, Madzore (2023) discovered that learners who participated in organized



vocabulary training could conceptually describe transformations of functions without carrying out complete calculations, suggesting internal understanding.

Object

A mathematical object emerges when a student encapsulates a process, perceiving it as a whole entity that can be manipulated or acted upon. At this stage, functions are understood not just as actions or operations but as reified entities, such as entire equations or graphs, that can be inputs to other operations (Dubinsky & McDonald, 2001). Research by Matindike and Makonye (2023) indicates that many students struggle to make this shift, especially when encountering composite or inverse functions, suggesting a gap between procedural proficiency and object conception. Mastery at this level involves recognizing a function graph as an object with properties like domain, range, and continuity, without relying on computational processes.

Schema

A schema is a coherent and dynamic collection of actions, processes, and objects that form a flexible structure in the learner's cognitive system. Schemas enable learners to apply their understanding across diverse mathematical contexts. The function schema, for example, integrates algebraic, graphical, and tabular representations and can interpret, predict, and generalize functional behaviour (Arnon et al., 2014; Şefik et al., 2021). According to Longe and Maharaj (2023), schema-level understanding distinguishes expert learners from novices. Students at this level can approach unfamiliar problems by invoking and coordinating internalized structures of functional behaviour, leading to deeper conceptual insight and problem-solving flexibility. Following the theoretical framework, this paper reviews literature based on functions and APOS theory.

LITERATURE REVIEW

This literature review focuses on studies that apply APOS theory to secondary school learners' comprehension of parabolic (quadratic), hyperbolic, and exponential functions.

Parabolic Functions: Challenges in Understanding Quadratic Concepts

Research indicates that secondary school learners often struggle with the conceptual aspects of quadratic functions. Hlangwani et al. (2023) explored learners' difficulties with the vertex concept in quadratic functions, revealing that many students lacked a deep understanding of the function's structure and relied heavily on procedural methods. Using APOS theory, the study identified that learners predominantly operated at the action level, performing calculations without grasping the underlying processes or conceptualizing the function as an object. Similarly, a study by Ndlovu (2019) found that Grade 11 learners exhibited limited progression beyond the action level when dealing with algebraic functions, including quadratics. The analysis of written tasks demonstrated that learners often failed to make connections between different representations of functions, such as symbolic and graphical forms, indicating a lack of schema development.



Common Misconceptions in Function Learning

Misconceptions about functions hinder learners' ability to develop a deep understanding of mathematical relationships. Matindike and Makonye (2023) conducted an APOS-based analysis and found that learners frequently confuse hyperbolic functions with parabolic functions due to similarities in their graphical appearances. This confusion affects their ability to determine function behaviour and transformations accurately.

Another significant misconception involves exponential growth and decay. Mohammad (2019) found that many learners incorrectly assume that exponential functions have a constant rate of increase instead of an increasing rate of change, leading to errors in real-world applications such as population growth and financial modelling. Additionally, Madzore (2023) found that learners view function transformations as simple translations rather than understanding the algebraic effects of changes in coefficients and exponents. Adebayo (2023) highlighted that inadequate algebraic foundations contribute to challenges in learning functions, particularly when dealing with the inverse of exponential and logarithmic functions. The study emphasized that learners who struggled with algebraic manipulation in earlier grades found it challenging to transition into more advanced function topics in Grade 12.

Exponential Functions: Procedural Focus and Conceptual Gaps

Exponential functions represent another area where learners show a strong procedural focus but lack sufficient conceptual understanding. While specific studies applying APOS theory to secondary school learners' written assessments on exponential functions are limited, related research offers insights into common difficulties. For example, studies have highlighted that students often perceive exponential functions merely as procedures for calculation, without understanding the underlying processes or the function as a mathematical object (Mudaly & Mpofu, 2019; Díaz-Berrios & Martínez-Planell, 2022). This procedural approach indicates a dominance of action-level thinking, with minimal progression to higher APOS levels. The lack of schema development hinders learners' ability to connect exponential functions to real-world contexts or other mathematical concepts, such as logarithms or growth models.

Despite the valuable contributions of prior studies, a notable gap persists in the literature regarding applying APOS theory to analyse secondary school learners' written responses, particularly in parabolic, hyperbolic, and exponential functions. While many existing studies rely on tasks, interviews, or technology-mediated interventions, fewer have systematically examined learners' written examinations to determine their cognitive development stages as defined by APOS theory. This gap is significant given learners' persistent misconceptions about functions and the need for diagnostic tools that align with cognitive learning models. To address these challenges, several pedagogical strategies have been recommended. One such approach is the commognitive method, which emphasizes structured mathematical discourse to promote deeper reasoning. Mpofu and Mudaly (2020) found that discussions focusing on function properties improved learners' recognition of asymptotes and interpretation of transformations. Additionally, technology-enhanced instruction has proven effective; tools like GeoGebra and Desmos enable learners to manipulate function parameters in real-time,



fostering conceptual understanding and reducing reliance on rote procedures (Ngubane, 2023; Moeti, 2015). Another promising strategy is error analysis, where learners engage with and reflect on their misconceptions. Malahlela (2017) demonstrated that incorporating structured error reflection into instruction clarified misunderstandings and reinforced correct mathematical reasoning. Collectively, these approaches offer practical solutions, but a focused APOS-based analysis of learners' written assessments remains underexplored and essential for informing targeted instructional interventions.

RESEARCH METHODOLOGY

This study employed an analytic qualitative approach, grounded in the APOS theory developed by Dubinsky and McDonald (2001). The APOS framework was a cognitive lens to analyse learners' conceptual development in understanding exponential, hyperbolic, and parabolic functions. The study comprised 58 Grade 11 learners, purposively sampled from schools under the Tshwane West District, who completed a standardized Mathematics Paper 1 in November 2024.

Data Analysis

Learner responses were systematically classified using the APOS theoretical framework, which facilitated identifying their cognitive engagement with function concepts. Tasks at the Action level required learners to perform algorithmic procedures with minimal conceptual understanding, such as substituting values into standard function equations. At the Process level, tasks involved internalized operations where learners demonstrated emerging conceptual insight and an ability to manipulate functions mentally. Object-level tasks assessed the learners' capacity to treat functions as holistic mathematical entities by integrating symbolic, graphical, and contextual representations. Finally, tasks at the Schema level required combining multiple mathematical ideas and flexible application across various representations and contexts. To support the classification, a detailed content analysis matrix was developed (refer to Tables 1 and 2), aligning examination items with the corresponding APOS levels. The Curriculum and Assessment Policy Statement (CAPS) for Mathematics designed the test instrument, and a subject specialist moderated the paper to ensure content validity. Reliability was further enhanced through standardized marking protocols and a double-coding verification process, which minimized subjectivity in categorizing cognitive levels.

Data collection instruments

The primary data source consisted of learners' written responses to two structured examination questions, 5 and 6, targeting specific function concepts. Question 5 (refer to figure 1) assessed learners' understanding of parabolic and exponential functions, focusing on formulating equations, identifying asymptotes, determining intervals of inequality, and analysing gradient changes. Question 6 (refer to figure 2) evaluated learners' ability to interpret and graph exponential and hyperbolic functions, deduce asymptotic behaviour, and perform function translations.



QUESTION 5

Drawn below is the graphs of $f(x) = ax^2 + bx + c$ and $g(x) = k \cdot m^x$. The parabola has intercepts $(-5; 0)$ and $(-1; 0)$. The exponential graph passes through the points $(0; 2)$ and $(1; 6)$.

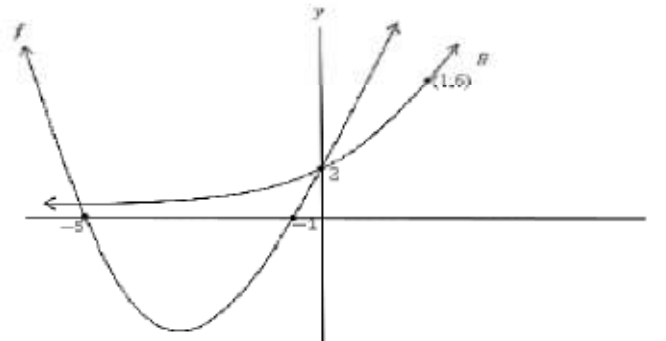


Figure 1: Extract from the Tshwane West exam, Mathematics Paper 1 (November 2024)

Conversely, the purpose of question 6 (refer to Figure 2) was to evaluate the learners' understanding of several crucial areas concerning exponential and hyperbolic functions. Specifically, it aimed to assess their ability to determine the value of a variable within an exponential function, identify the equations of asymptotes, and determine the range of these functions. Furthermore, learners were expected to find the x - and y -intercepts of hyperbolic functions and to transition from symbolic representations to their graphical counterparts. The question also requires an understanding of the axis of symmetry for functions with a positive gradient and the skill to calculate the value of b after the function's translation.

QUESTION 6

Given: $f(x) = \frac{-3}{x+2} - 1$ and $g(x) = -a^x$. The point $(1; -3)$ lies on g .

- 6.1 Determine the value of a (2)
- 6.2 Write down the equations of the asymptotes of f . (2)
- 6.3 Write down the range of g . (1)
- 6.4 Determine the x - and y -intercepts of f . (4)
- 6.5 Draw neat sketches of f and g on the same set of axes on the DIAGRAM SHEET. Clearly show all asymptotes and intercepts with the axis. (6)
- 6.6 Determine the axis of symmetry for f with a positive gradient. (2)
- 6.7 Given that $h(x) = f(x) + b$, determine the value of b so that the horizontal asymptote of f will be $y = 1$. (1)

Figure 2: Extract from the Tshwane West exam, Mathematics Paper 1 (November 2024)



Application of APOS Theory in Function Concepts

The study applied the Action-Process-Object-Schema (APOS) theory to analyze learners' understanding of function concepts based on responses to selected exponential, hyperbolic, and parabolic function questions. This framework enabled classifying learners' cognitive levels, providing insights into their conceptual development. The analysis was structured according to the four APOS levels: Action (A), Process (P), Object (O), and Schema (S) (Dubinsky & McDonald, 2001). The following section discusses the APOS classification based on learners' engagement with the function concepts, incorporating content analysis from Tables 1 and 2 to illustrate how different function-related tasks align with APOS levels.

Action Level

At the Action level, learners demonstrate a fundamental understanding of functions by performing procedural tasks without deep conceptual reasoning. Their responses are based on memorization, rule application, and a step-by-step execution of predefined methods rather than an internalized grasp of function relationships (Dubinsky & Wilson, 2013). Table 1 presents several items classified under the Action level, including 5.1: Determine the equation of a parabola in the form $y = ax^2 + bx + c$, 5.2: Determine the values of m and k , and 5.3: Write down the asymptote of an exponential function. Similarly, Table 2 reflects the Action level with 6.1: Determine the value of a , 6.2: Write down the equations of the asymptotes of the hyperbola, and 6.3: Write down the range of an exponential function. Learners functioning at this level can substitute values into given functions, solve basic equations, and recall function definitions, but they find it difficult to establish conceptual connections between different function representations. Their understanding is rule-driven and procedural, with limited reasoning ability beyond calculations (Maharaj, 2013).

Table 1: Content analysis using APOS in Exponential and parabolic functions items.

Items number	Concepts of items	Items Marks	APOS level
5.1	Determine the equation of f in the form $y = ax^2 + bx + c$.	4	A
5.2	Determine the values of m and k .	3	A
5.3	Write down the asymptote of $g(x)$.	1	A
5.4.1	Determine the values of x for which $2 \leq g(x) \leq 6$.	2	P
5.4.2	Determine the values of x for which $f(x) > 0$	2	P
5.4.3	Determine the values of x for which $f(x) \cdot g(x) < 0$	2	P
5.5.1	Determine the values of p for which $g(x) = p$ has a positive root.	2	O
5.5.2	Determine the values of p for which $f(x - p) = 0$ has one positive and one negative root.	2	O
5.6	Determine the average gradient between the points $(-5; 0)$ and the y -intercept of g .	2	O

Process Level



At the Process level, learners start to cultivate an internalized understanding of functions, enabling them to explain mathematical relationships and identify patterns. Rather than simply executing procedures, they can justify their reasoning and perceive functions as dynamic entities. From Table 1, tasks requiring Process-level reasoning include 5.4.1: Determine the values of x for which $2 \leq g(x) \leq 6$, 5.4.2: Determine the values of x for which $f(x) > 0$, and 5.4.3: Determine the values of x for which $f(x) \cdot g(x) < 0$. In Table 2, the Process level is illustrated in 6.4: Determine the x - and y -intercepts of the hyperbola, 6.5: Draw neat sketches of the hyperbola and exponential functions on the same set of axes on the DIAGRAM SHEET. Clearly show all asymptotes and intercepts with the axis, and 6.6: Determine the axis of symmetry for the hyperbola with a positive gradient.

At this level, learners internalize mathematical processes and attempt to explain function properties, such as critical points (intercepts, asymptotes, and intervals of positivity/negativity). They view curve sketching as a holistic process rather than a step-by-step calculation (Weller et al., 2011). However, while they demonstrate a firmer conceptual grasp than those at the Action level, their reasoning is still incomplete, often lacking higher-order abstraction and generalization.

Object Level

The Object level represents a significant cognitive leap, where learners view functions as complete entities rather than a sequence of isolated procedures. At this stage, they can connect symbolic and graphical representations, extract key function characteristics, and understand transformations in a broader mathematical context (Dubinsky & Wilson, 2013).

Table 1 illustrates Object-level thinking in 5.5.1: Determine the values of p for which $g(x) = p$ has a positive root; 5.5.2: Determine the values of p for which $f(x - p) = 0$ has one positive and one negative root; and 5.6: Determine the average gradient between the points $(-5, 0)$ and the y -intercept of an exponential function. Similarly, Table 2, 6.7: Given that $h(x) = f(x) + b$, determines the value of b so that the horizontal asymptote of the hyperbola will be $y = 1$. Learners at this level generalize function behaviours, recognize the interplay between equations and graphs and understand how function modifications affect their properties. Unlike Process-level learners, they can analyse and manipulate functions flexibly, demonstrating a deeper conceptual awareness of mathematical structures (Maharaj, 2013).

Schema Level

At the highest cognitive level, the Schema level, learners integrate multiple functional concepts into a cohesive and interconnected framework. They synthesize function properties, switch between graphical and algebraic representations effortlessly, and solve unfamiliar problems using deep conceptual understanding (Dubinsky & Wilson, 2013). At this level, learners do not see functions in isolation but rather as part of a broader mathematical system. They can transition fluidly between equations and graphs, understand function behaviours across different contexts, and apply function principles to solve complex, non-routine problems. This level was assessed based on the extracted questions.

Table 2: Content analysis using APOS in Exponential and hyperbolic functions items.



Items number	Concepts of items	Items Marks	APOS level
6.1	Determine the value of a .	2	A
6.2	Write down the equations of the asymptotes of f .	2	A
6.3	Write down the range of g .	1	A
6.4	Determine the x - and y -intercepts of f .	4	P
6.5	Draw neat sketches of f and g on the same axes on the DIAGRAM SHEET. Clearly show all asymptotes and intercepts with the axis.	6	P
6.6	Determine the axis of symmetry for f with a positive gradient.	2	P
6.7	Given that $h(x) = f(x) + b$, determine the value of b so that the horizontal asymptote of f will be $y = 1$.	1	O

FINDINGS

Table 3 presents the results of 58 learners assessed on various aspects of parabolic and exponential functions. The table provides an overview of learners' performance across different cognitive levels, indicating how well they could engage with the mathematical concepts tested. It includes the maximum marks per item, which vary depending on the complexity of the question, and the total marks obtained by all learners, which serve as an indicator of overall achievement. Additionally, the table details the number of learners who obtained full marks for each item, offering insight into how many students demonstrated complete mastery of the task. Conversely, it also presents the number of learners who scored zero, further dividing them into those who attempted the item but provided incorrect responses and those who did not attempt the question. The latter group may indicate a lack of confidence, inadequate preparation, or unfamiliarity with the tested concept.

Each item in Table 3 is classified according to APOS levels, which categorize the cognitive demands of the task. The analysis reveals that most learners struggled with the items, particularly those requiring higher-order thinking. At the Action level, learners exhibited some proficiency in executing procedural tasks, but the results indicate substantial gaps in fundamental understanding. Item 5.1, which requires determining the equation of $f(x)$ in the form $y = ax^2 + bx + c$, had four learners obtaining full marks (See Figure 3: learner L7 response). In comparison, 50 learners scored zero, suggesting that most students struggled formulating quadratic equations.



Table 3: Learners' results of question 5, exponential and parabolic functions items.

Parabolic and exponential functions	No. of learners wrote	Total marks per item	Marks obtained by all learners per item	Number of learners who got the total marks for the item	Number of learners who got 0 per item	No. learners who attempted the item but got 0	Number of learners who did not attempt the item
Item number							
5.1	58	4	22	4	50	8	42
5.2	58	3	34	8	44	14	30
5.3	58	1	10	10	48	12	36
5.4.1	58	2	20	10	48	10	38
5.4.2	58	2	16	7	49	12	37
5.4.3	58	2	10	5	53	19	34
5.5.1	58	2	0	0	58	25	33
5.5.2	58	2	5	1	54	20	34
5.6	58	2	22	11	47	10	37

In Figure 4, it was observed that learner L19 substituted various points from the graphs into both the quadratic and exponential functions. However, this action appeared to be carried out without clearly understanding the task's requirements or the intended outcome. This lack of comprehension indicates that the learner may not have fully grasped the expectations associated with the problem. Similarly, in Item 5.2, only eight learners determined the correct values of m and k , while 44 learners received zero marks, demonstrating significant difficulty in parameter identification. Moreover, Item 5.3, which required learners to state the asymptote of $g(x)$, was attempted by most learners, but 48 students scored zero, reflecting a poor understanding of asymptotic behaviour.



$y = ax^2 + bx + c$
 $= \frac{2}{5}x^2 + \dots$
 $y = a(x - x_1)(x - x_2)$
 $2 = \frac{2}{5}(x + 5)(x + 1)$
 $= \frac{2}{5}(x^2 + x + 5x + 5)$
 $y = \frac{2}{5}x^2 + 2x + 2$ ✓

Figure 3: learner L7 response

5.1 $y = (x + 5)(x + 1)$
 $= x^2 + 6x + 5$
 $2 = k^0$
 $k = 2$
 Sub k in y . cm²

Figure 4: learner L19 response

Moving to the Process level, performance remains weak, with a notable number of learners unable to interpret function relationships conceptually. For instance, in Item 5.4.1, where learners had to determine the values of x for which $2 \leq g(x) \leq 6$, 10 learners obtained full marks, while 48 scored zero. Similarly, in Items 5.4.2 and 5.4.3, which assessed functions positivity and product inequalities, only 7 and 5 learners achieved full marks (refer to Figure 5), while 49 and 53 learners failed the item, respectively. In Figure 6, it was observed that learner L46 tended to either substitute numbers randomly or use them precisely as they appeared when answering the questions. This behaviour indicates a possible deficiency in strategic thinking or comprehension of mathematical principles, as their responses lacked a consistent pattern or logical reasoning. These results suggest that most students struggle to internalize functional behaviour and apply mathematical reasoning beyond mechanical calculations.

5.4.1 $0 \leq x \leq 1$ ✓
 5.4.2 $x < -5$ ✓ or $x > 1$ ✓
 5.4.3 $-5 < x < -1$ ✓

Figure 5: learner L21 response

5.4.1 $2 \leq g(x) \leq 6$
 $= 2 \leq g(0) \leq 6$
 5.4.2 $f(x) > 0$
 $= f(1) > 0$
 5.4.3 $f(x) \cdot g(x) < 0$
 $= f(1) \cdot g(0) < 0$

Figure 6: learner L46 response

Performance was notably poor at the Object level, which requires learners to connect algebraic and graphical representations. Item 5.5.1, assessing the determination of the values of p for which $g(x) = p$ has a positive root, saw all 58 learners failing to obtain any marks (See both figures 7 and 8), demonstrating extreme difficulty in handling function roots. Item 5.5.2, which required learners to determine the values of p for which $f(x - p) = 0$ has one positive and one negative root, was equally challenging, with only one learner achieving full marks and 54 scoring zero. Finally, in Item 5.6, learners had to determine the average gradient between two points. Despite its complexity, 11 learners obtained full marks, and 47 scored zero. These findings suggest that most learners do not yet perceive functions as whole objects and struggle to generalize their properties beyond isolated procedures.



S.S.1. $q(x) = p$
S.S.2. $f(x-p) = 0$

Figure 7: learner L46 response

S.S.1. $p = 1$
S.S.2. 1 and 0

Figure 8: learner L31 response

The central area of difficulty was quadratic equations and inequalities. The nature of the root question saw zero correct responses, and 25 learners received a score of zero despite attempting it. This suggests learners might have fundamental misconceptions about factorization, quadratic formula applications, or inequality manipulation. A similar pattern emerged in inequality-related questions, where many learners scored zero or did not attempt the question. This highlights the need for reinforcement in algebraic reasoning and equation-solving techniques.

Table 4 outlines learners' performance in exponential and hyperbolic function questions, following the same structured approach as Table 3. It shows 58 learners' engagement with each question, showing their mark distribution and identifying performance trends across different cognitive levels. The table includes the maximum marks allocated per item, reflecting the relative weight of each question in the assessment. Additionally, it presents the total marks obtained by all learners, offering insight into the overall level of achievement in exponential and hyperbolic functions. The table also distinguishes between learners who achieved full marks, demonstrating complete mastery of the task, and those who scored zero.

The zero-score category is further divided into learners who attempted the item but provided incorrect answers and those who left the question blank. Learners who attempted the item but received zero marks may have misapplied formulas, misunderstood concepts, or made computational errors. Meanwhile, learners who did not attempt the question may have difficulty recalling relevant concepts, lack confidence, or do not understand the mathematical properties being assessed. Identifying these distinctions provides a clearer picture of where instructional interventions may be needed to support learners in developing their functional knowledge.



Table 4: Learners' results of question 6, exponential and hyperbolic functions items.

Exponential and hyperbolic functions	N o. of le ar ne rs wr ot e	To tal m ar ks pe r ite m	Mark s obtai ned by all le ar ne rs per ite m	Number of le ar ne rs who got the total marks for the ite m	Numb er of le ar ne rs who got 0 per ite m	Number of le ar ne rs who attempt ed the ite m but got 0	Num ber of le ar ne rs who did not attempt the ite m
Item number							
6.1	58	2	58	28	28	11	17
6.2	58	2	49	19	28	14	14
6.3	58	1	10	8	49	14	35
6.4	58	4	134	26	20	12	8
6.5	58	6	81	2	30	15	15
6.6	58	2	9	4	53	21	32
6.7	58	1	3	3	55	35	20

Table 4 presents learners' performance in exponential and hyperbolic function items, highlighting similar difficulty trends across APOS levels. The analysis reveals that most learners struggled with the items, particularly those requiring higher-order thinking. At the Action level, performance varied across different tasks. Item 6.1, which required determining the value of a , was the best-performing item, with 28 learners obtaining full marks and only 28 learners scoring zero. In Figure 9, it is clear that learner L38 exhibits a thorough understanding of the item. This learner not only comprehends the concepts presented but is also actively engaging with the content on an actionable level. Additionally, L38 is one of the 28 learners who have attained this level of proficiency, demonstrating a robust command of the subject matter.

$$\begin{aligned}
 6.1. \quad y &= a^x \\
 3 &= -a^1 \\
 -a^1 &= -3 \\
 -1 &= -1 \\
 a &= 3
 \end{aligned}$$

Figure 9: learner L38 response

Similarly, Item 6.2, which asked learners to state the equations of asymptotes, had 19 learners achieving full marks, while 28 failed the question. However, Item 6.3, which involved writing down the range of an exponential function, was significantly more challenging, with 49 learners



scoring zero and only 8 obtaining full marks. This suggests that while learners can handle basic function tasks, they struggle with abstract function properties such as range determination.

At the Process level, performance improved slightly. Item 6.4, which requires determining the x - and y -intercepts of the hyperbola, had 26 learners obtaining full marks, making it one of the better-performing questions. This suggests that a significant portion of the learners could apply function concepts at a procedural level. In Figure 10, it is demonstrated that learner L26 can substitute various alternative values for the intercepts of hyperbolic functions. This skill reflects an active engagement with the material at a hands-on level, showcasing a practical approach to mathematical concepts. However, despite this proficiency in manipulating values, the learner struggles to fully grasp the underlying simplification necessary to understand the task and its intended outcome. This gap in comprehension underscores the need for further development in the learner's problem-solving abilities and conceptual awareness related to the task.

6.4. x -intercept: $y=0$
 $f(x) = \frac{-3}{x+2} - 1$

y -intercept: $x=0$
 $f(x) = \frac{-3}{0+2} - 1$
 $y = \frac{-3}{2} - 1$
 $y = -2$
 $y(0, -2)$

Figure 10: learner L26 response

However, performance dropped in Item 6.5, where learners had to sketch graphs of the hyperbola and exponential functions. Only two learners scored full marks, while 30 received zero, indicating a significant weakness in visual representation and function interpretation. Similarly, Item 6.6, which assessed the axis of symmetry, had only four learners achieving full marks, while 53 failed the task, reinforcing the difficulty in understanding the geometric properties of functions. At the Object level, performance was abysmal, as seen in Item 6.7, where learners were required to determine the value of b so that the horizontal asymptote of $f(x)$ would be $y = 1$. Only three learners obtained full marks, while 55 learners failed the task. This reinforces the earlier observation that learners struggle with advanced function manipulation and generalization.

In summary, learners performed better in exponential functions than in other mathematical topics. In the "Determine the variable of an exponential function" question, all 58 learners attempted it, with 28 achieving full marks. Similarly, 49 learners correctly identified the asymptote of a hyperbolic function, indicating a stronger familiarity with this concept. However, asymptote-related questions on translated functions still presented difficulties, with many learners failing to recognize how transformations affect function properties. The results indicate that most learners operate at the lower APOS levels (Action and Process) and face significant challenges at the Object and Schema levels. The most difficult concepts involved asymptotic behaviour, function transformations, and function roots, with particularly low



performance in items requiring object-level abstraction and higher-order problem-solving. The high failure rates in conceptual and graphical representation tasks suggest that many learners rely heavily on memorization rather than developing a deep understanding of function relationships.

DISCUSSION

The findings of this study reveal a pronounced reliance on procedural knowledge, with the majority of learners demonstrating cognitive engagement at the Action and Process levels of the APOS framework. In contrast, only a small fraction reached the Object level, and virtually no evidence of Schema-level understanding was observed. This trend underscores a critical gap between procedural fluency and conceptual mastery in learners' understanding of exponential, hyperbolic, and parabolic functions. At the Action level, learners displayed competency in routine operations such as substitution and solving standard equations (e.g., Items 5.1, 5.2, and 6.1). However, they struggled to interpret deeper functional properties such as asymptotic behaviour or domain constraints. This pattern aligns with the findings of Mbhiza and Sempe (2024), who observed that learners, particularly in under-resourced contexts, often rely on rote memorization with limited conceptual transfer. Dubinsky and Wilson (2013) similarly noted that students functioning at the Action level tend to perceive functions as static, procedural tasks rather than dynamic mathematical objects.

A subset of learners exhibited emerging conceptual reasoning at the Process level, particularly in tasks involving graphical interpretation, intercept identification, and inequality analysis (Items 5.4.1–5.4.3; 6.4–6.6). Nonetheless, their performance remained inconsistent and fragmented, suggesting partial internalization without generalized understanding. This finding supports Buthelezi (2021), who found that learners often engage superficially with inequalities and graphical representations, lacking the ability to abstract function behaviour across varying contexts. Tasks that required Object-level thinking, such as function transformations, interpretation of roots, or algebraic-graphical integration (Items 5.5.1, 5.5.2, and 6.7), presented a significant challenge. Success rates in these items were near-zero, indicating learners' difficulty in encapsulating processes into coherent mental objects. These findings resonate with Matindike and Makonye (2023) and Adebayo (2023), who reported persistent misconceptions regarding transformations and symbolic equivalence, further exacerbated by weak algebraic foundations. Learners' inability to manipulate functions as integrated entities reflects a critical gap in progressing beyond procedural manipulation.

No substantial evidence of coherent, flexible understanding was identified at the Schema level. Learners could not coordinate multiple function representations or apply known concepts to novel, non-routine problems. This observation reinforces the argument of Longe and Maharaj (2023) and Malahlela (2017), who emphasized that Schema-level reasoning is seldom achieved in secondary mathematics classrooms without intentional pedagogical scaffolding and discourse-rich instruction. Taken together, the results corroborate national diagnostic reports (DBE, 2024) and existing literature on APOS-based analysis (Arnon et al., 2014; Weller et al., 2011), which emphasize that the lack of schema-oriented instruction leads to a systemic



disconnect between procedural competence and conceptual insight. The data also supports findings by Mpofu and Pournara (2018), who found that many high school learners fail to connect algebraic expressions and graphical representations, particularly in asymptotic behaviour and functional transformations.

CONCLUSION AND RECOMMENDATIONS

The study concludes that Grade 11 learners demonstrate a limited conceptual understanding of exponential, hyperbolic, and parabolic functions, with the majority functioning at the APOS framework's lower cognitive levels—Action and Process. Learners predominantly rely on algorithmic procedures and symbolic manipulation, yet struggle to interpret, generalize, or transfer mathematical relationships to unfamiliar contexts, limiting their progression to higher-order reasoning. This deficiency is primarily attributed to instructional approaches emphasizing procedural fluency over conceptual discourse. Existing literature supports this view, indicating that learners often encounter conceptual barriers when instruction neglects multiple representations and cognitive strategies necessary for deep understanding (Mpofu & Pournara, 2018; Dubinsky & Harel, 1992). Addressing these challenges requires a pedagogical shift. Educators are encouraged to integrate dynamic visualization tools such as GeoGebra and Desmos to facilitate the exploration of function transformations (Ngubane, 2023; Moeti, 2015).

Additionally, employing commognitive strategies that foreground mathematical discourse, and the structural features of functions can enrich learners' conceptual development (Mpofu & Mudaly, 2020). Incorporating error analysis and fostering metacognitive reflection are critical for surfacing and correcting misconceptions (Malahlela, 2017). Moreover, instruction should be designed to guide learners progressively through the APOS hierarchy—from Action to Schema—through cognitively sequenced tasks. These classroom strategies must be supported by targeted professional development programs that enable teachers to accurately diagnose learners' cognitive stages and implement APOS-aligned methodologies effectively. Without such systemic and pedagogically informed interventions, the disconnect between procedural competence and conceptual insight will persist, constraining learners' preparedness for advanced STEM disciplines.

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COMMON ERRORS THAT POINT TO MORE PROFOUND LEARNER MISCONCEPTIONS IN THEIR SOLVING OF ALGEBRAIC EQUATIONS

Dumisani Mgidi and Viren Ramdhany

University Of Johannesburg

South African learners' mathematics performance was poor across all topics, but especially in algebra. There are several reasons for this, including the extensive use of variables. Learners commit many different types of errors when solving algebraic equations, and the teacher is responsible for identifying and addressing these errors. This article reports on the findings of a master's study in which grade 9 learners from a Johannesburg high school were asked to answer an assessment task that contained different types of algebraic equations. The learners' responses were inductively analysed, and the errors committed were categorised into groups. Some misconceptions have also been inferred. The errors included (among others): learners' difficulties with addition and subtraction of like/unlike terms; challenges when dividing terms and when solving for x ; incorrect application of the distributive rule (including operation error); poor conceptual understanding of the lowest common denominator (LCD) when simplifying fractions; errors when transposing terms; equals sign error; and misapplication of exponential laws. Two mathematics teachers were interviewed and their knowledge of learner errors when working with algebraic equations was elicited. The researcher sought to determine the relationship between learners' errors and teachers' declarative mathematical knowledge for teaching.

A STATEMENT ABOUT THE FOCUS OF THE PAPER

This paper reports on the findings of a master's study that sought to understand Grade 9 learners' common errors, pointing to more profound misconceptions in their solving of algebraic equations. Accordingly, this study addresses the following research question: *Which of these common errors points to more profound learners' misconceptions?* This study is important because it provides insight into how learners' errors and misconceptions develop and continue. They inform mathematics teachers of learners' errors and misconceptions to enhance their teaching strategies. Furthermore, as mathematics teachers, it is important to know which common errors lead to deep learner misconceptions. In doing so, one will be able to overcome or minimise learners' frustrations in algebra. Finally, the investigation of learners' errors and misconceptions helps teachers create effective teaching and learning strategies to address learners' errors.

THEORETICAL FRAMEWORK

Constructivism is a cognitive learning theory that focuses on the role of the human mind in constructing knowledge (Piaget, 1968) and explaining how learners learn. According to Makhubele and Luneta (2014) and Ncube (2016), constructivism is the process by which an individual actively learns by constructing and reconstructing knowledge and reflecting on it to gain an understanding. Constructivism is a learning theory that helps us to understand how



learners learn. Learners' prior knowledge is vital; therefore, if many learners make errors, it could suggest that teachers do not elicit learners' prior knowledge before going on to new work. Furthermore, learners learn more effectively when they connect new knowledge to prior knowledge. Thus, misconceptions are strengthened, and errors continue. Thus, constructivism is related to learning and thinking. Mbewe (2013) added that for learners to understand mathematical concepts completely, they must rediscover the mathematical principles, thus emphasising the role of prior knowledge in learning.

LITERATURE REVIEW

Algebra

Algebra is a foundational skill that prepares learners for other areas of mathematics. (Stemele & Jina Asvat, 2024). Çetin (2021) pointed out that algebra plays an important role in the development of other topics of mathematics. Algebra is recognised as one of the areas in school mathematics that learners often struggle to understand because of its abstract nature. Algebra is a part of mathematics that uses letters and other symbols to represent quantities and situations. For example, we represent unknown numbers with variables, and variables are letters from the alphabet, such as x and y . The sum of a number and 5 can be written as $x + 5$, and any letter of the alphabet can be used as an unknown number. In algebra, 5 multiplied by x (i.e., $5 \times x$) is written as $5x$, as per convention, multiplication symbols are often not written.

Errors

To determine learners' learning difficulties, teachers need to analyse their work to know exactly which errors they are committing and why. There are many different definitions of these errors. An error is a mistake, blunder, and miscalculation or misjudgement in solving a mathematical problem, which leads to an incorrect response to a question (Aygör & Özdağ, 2012; Mbewe, 2013; Mulungye, 2016). Stemele and Jina Asvat (2024) described errors as systematic and rooted in a deeper conceptual understanding. According to Abakah and Brijlall (2024), errors originate from misconceptions. Learners make errors when their answers are incorrect. According to Brodie (2014), when learners make multiple errors and gain confidence from them, misconceptions can develop. Consequently, these misconceptions resulted in additional errors.

Misconceptions

As with the errors, various definitions of misconceptions abound in the literature. Misconceptions are systematic errors in interpreting, understanding, or applying mathematical concepts (Agustyaningrum et al., 2018) or "the misapplication of a rule, or an over- or under-generalisation of the situation" (Bowers, 2021, p. 9). Tooher and Johnson (2020) defined misconception as a weak understanding of a concept, causing repeated errors. Misconceptions can also be regarded as incorrect formation of an idea, which results from incorrect reasoning and information. Misconceptions are one of the key reasons why learners perform poorly in mathematics. Learners with misconceptions are frequently unaware that their ideas are



incorrect. Whenever they are informed or made aware of their misconceptions, it is difficult for them to abandon them.

Causes of errors and misconceptions

It was also observed that students' errors in simple visual and algebraic expressions were caused by choosing the wrong algebraic operation, ignoring parenthesis in algebraic expressions, misinterpreting the shape pattern, and arithmetic operation errors (Kabadaş & Mumcu, 2024). According to Ebenezer and Fraser (2001), misconceptions are caused by learners when they develop concepts in their minds in line with their understanding. They are generally defined as ideas that are not scientifically correct but can be explained by learners in their way. Misconceptions are usually caused by the results of learners' experiences and incorrect knowledge in the lower grades. Abakah and Brijlall (2024) noted that misconception serves as a barrier to learners developing conceptual understanding as it does not invoke learners' rational reasoning.

METHODOLOGY

This study employed a qualitative approach to this descriptive study, with the aim of gaining insights into learners' errors in solving algebraic equations. Qualitative methodology is characterised as being descriptive and focuses on understanding phenomena in a more profound or individualised manner. Seventy-two Grade 9 mathematics learners from two different classes, along with two mathematics teachers at a local high school, were purposively and conveniently selected to participate in this study.

Data collection included learner-written assessments as well as semi-structured interviews with two teachers. After learners had written an assessment, the two mathematics teachers were invited to participate in the interviews. These semi-structured interviews aimed to gain a deeper understanding of the teachers' Mathematical Knowledge for Teaching (MKT) in relation to identifying and addressing learners' errors. Qualitative interview data were inductively analysed.

FINDINGS

Types of errors identified from the analysis of learners' answers. Some errors were repeated because learners were unaware of them.

Error type 1: Computation errors

Addition and subtraction errors include incorrect addition/subtraction of like and unlike terms. In this type of error, learners added or subtracted variables and numerical coefficients to obtain a single-term answer. Learners also added coefficients, constants, and coefficients of different terms.



$$1710 = 3x + 4$$

$$= 10 = 7x$$

Dividing incorrectly when solving for x . In this error type, the learners typically completed most steps correctly, but in the final step, when they are required to divide by the coefficients of x , they committed errors. Learners did not seem to understand that they must divide both sides using the same coefficient, which must be the coefficient of the variable.

$$12. 10 = 3x + 4$$

$$10 - 4 = 3x$$

$$6 = 3x$$

$$\frac{6}{6} = \frac{3x}{6}$$

Incorrect application of distributive rules. In this error type, learners incorrectly and inconsistently apply the distributive rule. Most learners were unable to operate the brackets well. They did not seem to know the terms to be multiplied.

$$11. 3(x+4) = 15$$

$$= 3x + 4$$

Calculation of the lowest common denominator when simplifying fractions. The learners did not know when and how to remove the LCD. The learners showed with a pencil that they were using distributive law, which shows that they have a poor conceptual understanding of the LCD.

$$1. \frac{x-6}{2} + \frac{2(x+6)}{4} = x+3$$

$$4(x-6) + 6(2x+6) = 4(x+3)$$

$$4x - 24 + 12x + 36 = 4x + 12$$

$$4x + 12x - 4x = 24 + 24 - 36$$

$$\frac{9x}{9} = \frac{12}{9}$$

$$x = \frac{12}{9}$$

Error type 2: Errors when transposing terms



In most examples, the learners knew that they needed to transpose terms across the equal sign in both directions; however, when transposing the terms, the signs of the terms were not changed correctly and consistently.

$$1) 4x - 10 = 6$$
$$4x = 6 + 10$$
$$4x = 16$$

Error type 3: Equals sign error

In this type of error, many learners started their answers with an equal sign next to the margin. Technically, this is incorrect, but does not always affect the answer.

$$1, 4x - 10 = 6$$
$$= 4x = 6 + 10$$
$$= 4x = 16$$
$$4 \quad 4$$
$$= 4x$$
$$= 10x + 15(x + 5) = 0$$
$$= 10x + 15x + 75 = 0$$
$$= 25x + 75 = 0$$
$$25 \quad 25$$
$$x = -3$$

Error type 4: General Algebraic error

1. Exponents/exponential laws.

There are several different exponential laws and rules. In the following learner's answer, the learner incorrectly interprets x^3 as $x + x + x$ instead of $x \cdot x \cdot x$, they used addition instead of multiplication.

$$q1 \quad x^3 = 125$$
$$= x + x + x = 125$$

2. Working with variables

Learners ignored these variables. This speaks to learners being confused and not knowing what to do with the variables, so they choose to ignore them. Learners lacked knowledge of variables and needed to understand their meaning. From the analysis of learners' answers, there is clear evidence that the variable concept is poorly understood.

$$12 \quad 10 = 3x + 4$$
$$= 10 - 3 + 4 = 11$$



Summary of all the errors

No.	Type of errors	Explanation
1	Computation errors	Computational errors can be explained as errors in addition, subtraction, multiplication, and division of numbers (Agustyaningrum et al., 2018). In counting, learners made errors such as incorrectly adding and subtracting, multiplying, and dividing. Here are a few examples: • Addition and subtraction errors, including incorrect addition/subtraction of like and unlike terms • Dividing incorrectly when solving for x • Incorrect application of the distributive rule (including operation error) • Poor conceptual understanding of the calculation of LCD when simplifying fractions
2	Transposing of terms error	In algebra, transposing a term is a process in which any term of an equation is brought to the other side of the equals sign. For example, when given $x + y = z$ when solving for x, you will get $x = z - y$, then y is said to be transposed.
3	Equals sign error	Vermeulen and Meyer (2017) explained that the '=' symbol can represent the mathematical equals sign, and it is an important concept when learners solve numerical and algebraic equations. This error is due to how learners see that the answer comes after the equals sign symbol.
4	General algebraic error	(Dhlamini, 2023) mentioned that learners see exponents as unnecessary, challenging, and complicated concepts. Misapplication of exponential laws indicates that the learners applied the law of exponents without studying the circumstances under which the law can be used (Dhlamini, 2023). Challenges working with variables occur when learners substitute letters with numbers, such as when learners' responses suggest that the letter has been given a numerical value. Küchemann (1978) stated that learners assign numerical values to variables. This is possible because learners use their prior knowledge of numbers in the algebra framework.

CONCLUSION AND RECOMMENDATIONS

In this study, I found that learners experienced misconceptions when solving equations that involved fractions when they needed to find the Lowest Common Denominator (LCD). I also found that errors that point to more profound learner misconceptions are learners' difficulties when dealing with the addition and subtraction of like and unlike terms and difficulties dealing with variables. Adding and subtracting like and unlike terms should be revisited to eliminate



the errors and misconceptions in linear equations. Linear equations are challenging because learners do not clearly understand how to add or subtract terms from variables and constant terms. Most learners had difficulties finding the LCD, and these learners had a limited understanding of multiplicative facts (Pitsi, 2016). Therefore, I recommend appropriate teaching of the concept of identifying LCD.

Learners' errors and misconceptions can extend beyond careless mistakes. They often arise from prior knowledge learned in the earlier grades. As a result, many errors are due to an incomplete or incorrect understanding of the concepts that need to be applied correctly. Teachers need to understand the errors and misconceptions that learners commit to effectively preventing and addressing them. The curriculum at the beginning of teacher training should show how common errors and misconceptions arise, are prevented, and corrected to ensure that teachers are well equipped to recognise and address them. Additionally, teachers should strive to understand the origins of their learners' errors and misconceptions by deeply exploring their understanding. Understanding common learner errors in algebra provides a pathway to improving instruction. Finally, by addressing misconceptions at their roots, teachers can promote a deeper mathematical understanding and minimise persistent errors.

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UNLOCKING MATHEMATICS AND ACCOUNTING POTENTIAL: EXPLORING EDUCATORS TEACHING STRATEGIES THAT SPARK LEARNER MOTIVATION

Mmathapelo Phungo

North-West University

Mathematics and Accounting are essential subjects in secondary school curricula, providing Learners with vital knowledge and skills for future careers. Lack of learner motivation in these subjects remains a concern. A qualitative approach was employed in this study. The research sampled a total of ten (10) schools and purposefully selected ten (10) participants in Tshwane North District. The study comprised a purposefully selected group of 10 educators, consisting of 5 Accounting teachers and 5 Mathematics teachers, who provided valuable insights into their teaching experiences and strategies in secondary schools. This study is underpinned by Behaviourism theory. Data was collected using semi-structured interviews with educators allowing for in-depth exploration of their perspectives on effective teaching strategies. An interpretative paradigm was employed to gain a deeper understanding of the experiences and perceptions of educators. Thematic data analysis was used to identify patterns and themes in the data, providing rich insights into the complexities of teaching and learning in Mathematics and Accounting. The research contributes to a deeper understanding of teaching and learning in Mathematics and Accounting subject in secondary schools. It provides understanding for educators and researchers of teaching strategies that spark learner motivation. The result of the research informs teacher professional development in education to enhance the quality of Mathematics and Accounting education in secondary schools.

INTRODUCTION

Mathematics and Accounting subjects are crucial for learners' future careers in business, finance, management, and other fields. Narain, (2023) that these subjects equip learners with the ability to solve problems, think critically, and make informed financial decisions Many Learners struggle with these subjects due to low engagement and motivation. Kassim, Marfo and Abu, (2025) indicated that low learner engagement can lead to poor academic performance and decreased motivation which leads to limited career opportunities. Mirzajonova and Parpiyeva, (2022) pointed out that to enhance learner motivation in these critical subjects, effective teaching strategies must be implemented in secondary schools. Innovative and learner-centred teaching approaches can create a supportive and challenging learning environment for the learners. Nwosu and Matashu (2022) explained that the educators should encourage active participation and ownership of learning in their classrooms. When learners are motivated, that will improve academic outcomes and foster a positive attitude towards these subjects, which can have long-term benefits for learners' career prospects.



PROBLEM STATEMENT

This study investigates the opportunities and challenges associated with teaching Mathematics and Accounting in secondary schools, with a particular focus on the Tshwane North District. Traditional teaching methods, such as lectures and textbooks, have been criticized for their inadequacies in fostering deep learning and engagement (Harefa, 2024). These methods frequently rely on rote memorization and repetitive exercises, which may not adequately develop the critical thinking and problem-solving skills necessary in the complex work environment of today (Yao et al., 2024). Additionally, the growing complexity of Mathematics and Accounting concepts, as well as the necessity for real-world applications and technology integration, present significant challenges for educators in effectively teaching these subjects (Ling & Mahmud, 2023). The objective of this study is to investigate effective teaching strategies and learner engagement in Mathematics and Accounting skills among secondary school students.

In the field of Commerce, Mathematics and Accounting are inseparable and indispensable. Accounting applies Mathematics concepts to financial management, analysis, and reporting, while Mathematics provides quantitative skills for problem-solving and decision-making (Letshwene, 2014). The two subjects have an unbreakable connection, as mathematical skills are essential for the interpretation of data, statistical analysis, and financial calculations in Accounting. Manfreda and Hodnik, (2021) reveals that equipping learners with a thorough comprehension of financial concepts and practices, the combination of Mathematics and Accounting equips them for careers in Commerce and related disciplines. Through the study of both subjects, learners acquire critical thinking, analytical, and problem-solving abilities, which render them more employable and adaptable in the industry.

This study investigates the relationship between teaching strategies and learner engagement in Mathematics and Accounting among secondary school Learners in Tshwane North District. It aims to identify effective teaching strategies that promote learner engagement, develop financial literacy, mathematical skills, and accounting knowledge, and prepare Learners for success in their future careers (Verma, (2024).

The study's significance lies in its potential to inform evidence-based practices for high-quality Mathematics and Accounting education, leading to improved learner outcomes, increased motivation, and better preparation for the 21st century.

RESEARCH QUESTIONS

1. What teaching strategies effectively promote learner motivation in Accounting subject?
2. How do these teaching strategies influence learner motivation in pairing Mathematics and Accounting subjects in their subject choice?
3. How does educators' perception impact learner motivation in Accounting and Mathematics?



RESEARCH OBJECTIVES

1. To explore and describe teaching strategies that foster learner motivation in Accounting classes.
2. To examine the influence of teaching strategies on learner motivation in Mathematics subject.
3. To evaluate the impact of an educator's perception on learner motivation in the subject Accounting and Mathematics?

LITERATURE REVIEW

The South African Department of Basic Education (DoBE) faces significant challenges in promoting interest and engagement in Mathematics and Accounting among secondary school learners (Verma, 2024). Despite educators' efforts to employ innovative teaching methodologies, various obstacles hinder effective lesson delivery. Inadequate resources, insufficient teacher training, language barriers, negative perceptions and attitudes, and curriculum challenges are among the obstacles that the South African Department of Basic Education encounters in its efforts to promote Mathematics and Accounting (Daly & Weber, 2021). The delivery of effective lessons is hindered by these challenges, which also contribute to poor learner performance and discourage engagement in these subjects. It is important to address these issues to enhance the quality of education and cultivate an interest in Mathematics and Accounting among secondary school learners in commercial stream. This literature review aims to explore the challenges facing Mathematics and Accounting education in South African secondary schools (Schunk & DiBenedetto, 2020).

Challenges encountered in Accounting Education

South African secondary schools face a discouraging challenge in Accounting education and a severe shortage of qualified and inexperienced educators (Daly, & Weber, 2021). This scarcity not only compromises the quality of teaching but also deteriorates learner confidence, creating a disturbance on their understanding and performance (Beatson, De Lange, O'Connell, Tharapos, & Smith, 2021). Complex concepts are glossed over, nuance is lost, and learners are left struggling to grasp the fundamentals. The consequences worsen poor performance, and a widening skills gap in the commercial field. Karlsson and Noela, (2022) emphasised that learners often develop negative attitudes towards the subject due to various factors. Poor resource allocation can limit access to essential learning materials and technology, hindering learners' ability to engage with the subject in a meaningful way. Jordaan (2022) explained that the lack of motivation among learners can stem from uninspiring teaching methods or a failure to connect the subject to real-world situation.

Kaliris, Issari, and Mylonas (2023) revealed that insufficient career guidance can also leave learners uncertain about the relevance and potential of Accounting in their future careers.



Challenges encountered in Mathematics Education

Mathematics education plays a vital role in learners' cognitive development, problem-solving skills, and future career opportunities. Aini and Ridwan, (2024) pointed out that literacy and numeracy skills are fundamental building blocks of education, empowering individuals to navigate complex global challenges, solve problems, and make informed decisions. Grimmond, (2024) explained that by mastering these essential abilities, people can better understand the world around them, from natural phenomena to social dynamics. Literacy and numeracy skills unlock opportunities, foster critical thinking, and enable individuals to reach their full potential (Hazin, Rahmamawati, Yani & Ladiqi, 2025). However, research highlights several challenges that hinder learners' ability to excel in mathematics. Montesdeoca, (2023) confirms that one major issue is the lack of relevance and engagement, where learners often perceive mathematics as abstract and disconnected from real-life applications. This can lead to disinterest and demotivation among learners. Insufficient teacher training and inadequate resources can result in ineffective instruction and limited access to essential learning materials (Kabir & Jalali, 2021).

According to Szabo-Reed, Willis, Lee, Hillman, Washburn and Donnelly (2019), Accounting learners' academic achievement is significantly influenced by their performance in Mathematics classes. When learners perform well in their Mathematics subject, it builds their ability for critical thinking which is a critical skill in Accounting subject. For the learner to be able to read and analyse Accounting transactions, critical Mathematical critical skill is essential. Nwosu and Vorster, (2021), Bobe and Cooper, (2019) further explained that inadequate foundational skills in Mathematics and limited language proficiency can create significant barriers to learning, making it difficult for learners to comprehend and apply Mathematics and Accounting concepts effectively. Velasco (2019) found that learners in South Africa who study Accounting have been found to be inadequate in critical writing, Mathematics skills, and capacity for critical thinking. It is crucial to address these challenges for unlocking learner potential and fostering a positive and productive learning environment in commercial streams.

To address these challenges, several strategies need to be considered to improve mathematics education in secondary schools. Connecting mathematics to real-life scenarios and applications can increase relevance and engagement will be other effective strategies. Technology integration, such as utilizing mathematics apps and games, can make learning interactive and fun. Sikhombo, (2020) pointed that differentiated teaching can help cater to the diverse needs of Learners, ensuring that all learners receive effective support. Hands-on learning activities and manipulatives can also help learners develop a deeper understanding of mathematical concepts. Moreover, focusing on conceptual understanding and problem-solving skills can provide learners with a strong foundation in Mathematics.

Manfreda Kolar and Hodnik, (2021) pointed out that a deep understanding of mathematical concepts enables learners to apply their knowledge to solve problems and interpret mathematical information effectively. Purba and Azis, (2022) revealed that learners who lack conceptual understanding often struggle with problem-solving, particularly in areas like



algebra, where abstract thinking is crucial. To bring solutions to this dire challenge, educators can employ strategies such as collaborative learning, real-world applications, and regular assessments.

Teaching Strategies and Interventions

Cronin, (2017) explained that the use of study groups, as employed in teaching practice, allows learners to engage in peer-to-peer learning, share perspectives, and develop a deeper understanding of mathematical concepts. O'Neill and Short, (2025) confirmed that relating curriculum concepts to real-world scenarios can also enhance learners' motivation and interest in the subject. Regular topic tests provide opportunities for learners to apply their knowledge and receive feedback, helping to identify areas where further support is needed. Dare, Keratithamkul, Hiwatig and Li, (2021) indicated that by emphasizing conceptual understanding and using these strategies, educators can help learners develop a strong foundation in mathematics and foster a positive attitude towards the subject. Promoting a growth mindset by encouraging learners to ensure that mathematics is accessible and achievable can help reduce Mathematics anxiety and improve overall performance (Yildirim, 2017).

THEORETICAL FRAMEWORKS

BEHAVIOURISM

This study is underpinned by behaviourism. Behaviourism is a psychological approach that focuses on observable behaviours and the environmental factors that influence them. Ertmer and Newby (2013) indicate that behaviourism can be characterised as a theoretical framework that incorporates the acquisition of all behaviours through interactions with the environment as its fundamental principle. According to the learning theory inherited features exert minimal influence on behaviour since behaviours are predominantly acquired by environmental influences. Behaviourism, a theory of learning, may be traced back to the late 19th and early 20th century (Stryker & Serpe, 1982).

Behaviourism focuses significant emphasis on the way individuals interact with their environment. According to the principles of behaviourism, a branch of psychology that focuses on behaviour, all behaviours are acquired through interactions with the environment in a process known as conditioning. Letswene (2014) asserts that the notion that learning is influenced by a stimulus and subsequent response can be attributed to the contributions of B.F. Skinner. This approach is grounded in the principles of classical conditioning and operates under the assumption that learners will exhibit specific behaviours in response to a given stimulus and behaviours are acquired via the process of learning. Behaviour can subsequently be altered through the implementation of reinforcement or punishment techniques (Grusec, 1994). Operant conditioning, a theory by B.F. Skinner, involves using reinforcement and punishment to modify behaviour. It suggests that learners will repeat reinforced behaviours and avoid punished ones. In Mathematics and Accounting education, it can be applied through rewards for positive achievements like praise or stars, and gentle corrections or redirection for negative behaviours like lack of participation.



Additionally, theoretical frameworks Social Cognitive Theory can provide valuable insights into Mathematics and Accounting education (Bussey & Bandura, 1999) . Social Cognitive Theory emphasizes observation and reinforcement, which can help educators design strategies that promote positive learning environments and behaviours. This approach can help educators model positive attitudes and behaviours towards Mathematics and Accounting, leading to more effective teaching strategies (Bandura, A. (2001). When educators understand the psychological needs driving learner behaviour, then educators can design effective teaching strategies that promote intrinsic motivation and engagement. Combining these frameworks with can provide a comprehensive understanding of the complex factors influencing teaching and learning in Mathematics and Accounting. This approach can help develop effective teaching strategies that cater to diverse learner needs, ultimately improving learning outcomes.

METHODOLOGY

This study employs a qualitative research approach to explore teaching strategies that unlock learner potential and motivation in Mathematics and Accounting education (Kostere & Kostere, (2021). Through in-depth interviews with educators in selected secondary schools in Tshwane North District, this research aims to gather rich insights into the challenges and opportunities in teaching these subjects. The qualitative data collected will be thematically analysed to identify key patterns and themes that can inform effective teaching strategies (Braun & Clarke, 2023).

Population and sampling

The population of this study comprised ten (10) public secondary schools in Tshwane North District (Stratton, 2021). The Tshwane North District is one of the fifteen (15) school districts within the Gauteng Province. The researcher intentionally selected ten (10) educators (Andrade, 2021). The participants in this research study were deliberately chosen to be five mathematics educators and five accounting educators. The researcher opted for purposive sampling due to its ability to select cases that effectively demonstrate the desired features and processes.

Data collection and instrument

In this study, I employed semi-structured interviews with 10 secondary school teachers (5 Accounting and 5 Mathematics) from selected schools that offered these subjects to obtain in-depth insights from educators (Knott, Rao, Summers and Teeger, 2022). Each educator participated in a 30–45-minute face-to-face interview at their workplace, after working hours which allowed for open discussion. The interviews were audio-recorded with consent and enriched with notetaking to capture key points and observations. Data collection encompassed 12 weeks, ensuring a comprehensive exploration of educators' perspectives and experiences (Chriatian, 2023 & Makahane, 2019).



Data Analysis and Ethics

The process of data analysis was conducted after completion of all interview schedule. To initiate a procedure, the qualitative data collected was arranged using participant interview schedules Braun and Clarke, (2023) pointed out that the thematic analysis involve coding and interpreting the data to extract meaningful insights into educators' experiences and perceptions. The field of research ethics is concerned with distinguishing between right and wrong actions and establishing guidelines for researchers conduct in their endeavours. Ensuring the integrity of the research, ethical considerations such as informed consent, confidentiality, and anonymity were strictly adhered to. The findings provided a nuanced understanding of the teaching strategies that promote learner motivation and engagement in Mathematics and Accounting subjects, contributing to the development of practical recommendations for educators and policymakers.

RESULTS AND DISCUSION

Teaching strategies

Teaching strategies emerged as a significant theme in the thematic analysis. A significant proportion of the participants elucidated several pedagogical approaches employed in the teaching of Mathematics and Accounting as subject. The sub-theme originated from pedagogical approaches that incorporated collaborative discussions. According to the feedback provided by educators during the interviews, it was found that group discussions are perceived as very successful teaching methodologies for imparting knowledge in the field of commerce where Mathematics and Accounting are majors.

The finding is supported by the remarks provided by participants.

“Strategies that are being used when teaching Mathematics and Accounting are as follows: we have group discussions where learners are grouped according to their abilities, and performing learners or highflyers may assist other learners (Participant 3).”

“What I usually do is group these learners and make sure that each group has underperforming and well performing learners to balance the knowledge they have and so that they assist each other. This group allocation brings competition, whereby each group wants to perform better than the others (Participant 6).”

The study sub-theme revealed that educators simplify topics like Mathematics and Accounting by explaining the basics and allowing learners to apply their knowledge through class activities. This approach helps learners fully comprehend the subject and enhances their understanding of the subject matter.

In support to the above finding, the following comments made by one of the participants:

“Normally, what I would do to make my learners fully understand Accounting as subject, is to first give the theoretical concepts of Accounting, I then give them a short exercise to



demonstrate their understanding (Participant 2). ”

Due to the nature of the schools that were visited, the use of technology emerged as a sub-theme when it came to teaching methodologies utilised by educators. With that assertion, participants indicated that learners are different, and they have different learning skills, therefore, the use videos, audio, and games to teach the subject was pointed out as additional ways to promote effective teaching and learning.

Some of the participants commented as follows to corroborate the above finding.

“The types of learners we are teaching these days are exposed to phones, the internet, and technology. Although, phones and the usage of technology were forbidden in classrooms during those times, what we usually do for subjects like Mathematics and Accounting, we send videos to learners to watch at home, and the next day, when they address the topic, they discover that learners have some background on the subject, and the educators now emphasise that specific topic, giving learners a clear understanding of what they did not understand (Participant 9). ”

Participant 10 said:

“To improve the understanding of the learners, what I usually do is to create a WhatsApp group where they explained and clarify anything learners may not grasp in class, allowing them to finish their homework in collaboration. ”

Participant 2 was of the view that:

“Learners learn differently and sometimes I will use videos for visual learners, and sometimes I use speakers with audio so they listen to someone else teaching. ”

Participant 1 continued to say:

“They can also learn by using visual aids that they can see. Certain things, when projected to learners rather than just talking, I believe learners will begin to learn more rather than simply explained, explained. ”

The survey revealed that most educators support the effectiveness of peer teaching as a teaching approach. They believe it enhances understanding among fellow learners by providing alternative explanations. Some learners exhibit comprehension when Educators provide explanations, while others show enhanced understanding when classmates engage in discourse. The survey also highlighted the importance of peer learning in facilitating teaching and learning.

The above findings are in line with the following comments from some of the participants.

“Okay, after introducing the topic I sometimes allow learners to do what is called peer teaching. I realised that some learners don ’t understand when the teacher teaches but when



fellow learners teach, they understand better. They use common language and they can understand each other to explained at their level they understand better (Participant 4.)”

The study identified the scaffolding method as an effective teaching approach that enhances learners' comprehension of concepts. Participants found that this method allows Educators to present questions orally before Learners respond, facilitating comprehension and improving academic achievement. This method is particularly beneficial for educators working with children with language acquisition difficulties.

The above findings are supported by the following excerpts made by one of the participants.

“Scaffolding as technique is one method which I use with auditory learners when they write informal assessments. I try to read questions and transactions to them, because some learners cannot read to understand (Participant 3).”

DISCUSSION OF THEMES AND ANSWERING THE RESEARCH QUESTIONS

To strengthen the paper, I will integrate the literature review and theoretical framework with the findings. Here's a possible approach:

This study emphasizes the importance of dynamic and skilled teaching approaches in Mathematics and Accounting education, aligning with literature highlighting the need for innovative methodologies to enhance learner engagement and understanding. Strategies such as group discussions, simplification of complex topics, technology integration, peer learning, and scaffolding methods are supported by research. Challenges faced by learners, such as limited motivation and language barriers, are also addressed. Practical applications and real-world examples are crucial for promoting understanding and engagement. The theoretical framework of behaviourism and social learning theory supports the role of reinforcement, motivation, and observation in shaping learner behaviours and attitudes. The study's findings are grounded in existing research and theoretical perspectives, strengthening the validity and relevance of the findings.

The summaries on the thematic analysis can be linked with answers to the research questions. This inquiry explores the strategies employed by educators in teaching Mathematics and Accounting. Participants believe that dynamic, highly skilled teaching approaches are most effective. Several teaching methodologies are employed, including group discussions, simplification of complex topics, technology integration, peer learning, and scaffolding methods. Research shows a correlation between good teaching and effective learning, which involves actively engaging learners in their own learning and personal growth.

Teaching methods include various effective strategies that promote knowledge acquisition in Mathematics and Accounting. Technology has enhanced accessibility of knowledge for learners. Group discussion can provide significant advantages by enabling active classroom participation and discourse. To improve Learners' learning abilities, educators should offer practical applications, helping them understand how to apply what they learn to various aspects



of their lives, particularly their future employment.

Educators can use examples from real-world events in class activities, such as calculating restaurant bills or drafting personal budgets. However, learners still do not prioritize Accounting as their preferred subject due to factors such as choosing the subject, struggling with English reading skills, and teaching due to school timetables. Despite being informed about teaching strategies, there are still challenges to motivate learners to prioritize the subject and continue studying it at the university level. Even technology can motivate learners to enthusiastically pursue the subject, but motivating learners at home remains a difficult task.

CONCLUSION

This study on teaching strategies for Mathematics and Accounting in South African secondary schools found that dynamic approaches like group discussions, simplification of complex topics, technology integration, peer learning, and scaffolding methods enhance learner understanding and motivation. Effective teaching strategies can improve learning outcomes, despite challenges in motivating learners to prioritize these subjects. Educators should incorporate diverse methodologies to cater to different learning styles, leveraging technology and peer learning to foster collaboration and engagement. By doing so, they can improve learner outcomes and academic achievement in Mathematics and Accounting.

RECOMMENDATIONS

1. Educators should incorporate different teaching methods to cater to different learning styles.
2. Technology integration should be enhanced for accessibility and motivation of Learners.
3. Peer learning and group discussions should be encouraged to foster collaboration and understanding of the subject Mathematics and Accounting.
4. Scaffolding methods should be used to support learners with language acquisition difficulties.

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FOSTERING MATHEMATICAL THINKING: THE IMPACT OF GRADE 9 TEACHER QUESTIONING STRATEGIES

¹Chipo Mangwiro and ²France Masilo Machaba

University of South Africa

This qualitative case study explored the impact of a structured intervention focused on enhancing teacher questioning strategies to foster learners' mathematical thinking during classroom discourse. Motivated by the persistent underperformance in mathematics among South African learners, the study was conducted with a purposively selected Grade 9 mathematics teacher and their class. The intervention involved professional support on how to use cognitively guided questioning aligned with socio-cultural learning principles. Data were gathered through pre- and post-intervention lesson observations and field notes, then thematically analysed using Boaler and Brodie's questioning framework. The findings revealed a significant shift from low-order to high-order questioning strategies post-intervention, leading to increased learner participation, critical thinking, and collaborative problem-solving. The study underscores the importance of targeted professional development in questioning techniques to enhance meaningful learner engagement in mathematics. These results have implications for teacher training and classroom practice beyond the study context, suggesting scalable strategies for improving mathematics instruction and learner outcomes.

INTRODUCTION AND BACKGROUND

Mathematics plays a critical role in developing learners' logical, critical, and abstract thinking skills. In South Africa, the Department of Basic Education (DBE), through its Curriculum and Assessment Policy Statement (CAPS), emphasizes the importance of cultivating such mental processes to enhance learners' performance (DBE, 2011). Despite these intentions, learners continue to perform poorly in mathematics, as reflected in national assessments such as the Annual National Assessment (ANA) and international evaluations like the Trends in International Mathematics and Science Study (TIMSS) (Haarhoff, 2020). Contributing factors include negative learner attitudes towards mathematics (Evendi & Verawati, 2021), limited opportunities to develop mathematical reasoning (Hansen & Hadjerrouit, 2022), and traditional teacher-centred practices that prioritize rote learning over conceptual understanding.

Research indicates that teachers' questioning strategies significantly influence the quality of classroom discourse and learners' cognitive development. Yet, many teachers tend to use low-order, recall-based questions that do not encourage meaningful mathematical engagement. Addressing this gap requires professional development interventions that support teachers in employing higher-order questioning aligned with constructivist and socio-cultural learning theories.

This qualitative case study investigated the impact of a professional development intervention on a Grade 9 mathematics teacher's questioning practices. The intervention introduced the



teacher to cognitively guided questioning strategies informed by Boaler and Brodie's (2004) framework, which classifies questions based on their potential to stimulate different levels of learner thinking. These strategies were applied in a real classroom context and evaluated through systematic pre- and post-intervention lesson observations. The study was theoretically grounded in Cognitively Guided Instruction (CGI) and Vygotsky's socio-cultural theory, both of which advocate for learner-centred approaches, scaffolding, and interactive learning within the Zone of Proximal Development (ZPD). In line with these frameworks, the study was guided by the following aim and research questions:

Aim:

To explore the impact of an intervention designed to improve a mathematics teacher's questioning strategies on fostering learners' mathematical thinking in a Grade 9 classroom.

Research Questions:

- What types of questions does the teacher pose during classroom discourse before and after the intervention?
- How does the intervention influence the teacher's use of questioning strategies?
- In what ways do the changed questioning strategies affect learners' mathematical thinking and classroom interaction?

LITERATURE REVIEW

Teacher Questioning in Mathematics Education

Teacher questioning plays a critical role in shaping learners' engagement and cognitive development in mathematics classrooms. It is widely acknowledged as a key instructional practice, with teachers often spending a significant portion of their instructional time posing questions (Aziza, 2021; Roberts, 2021). Effective questioning is associated with promoting learner understanding, critical thinking, and problem-solving (Conner et al., 2022; Glasson, 2021 & Nisrina, 2023). However, research suggests that many teachers rely heavily on low-order, recall-based questions that fail to stimulate meaningful learner engagement or deep thinking (Mangwiro et al., 2022; Chirinda, 2022).

The relevance of this study lies in addressing that gap by exploring how professional support in questioning strategies can enhance the quality of teacher questioning and thereby foster learners' mathematical thinking.

The Role and Impact of Questioning

Purposeful teacher questions are instrumental in developing higher-order thinking skills and learner autonomy. They serve various functions: nurturing interest, stimulating curiosity, scaffolding learning, and guiding learners toward conceptual understanding (Shanmugavelu et al., 2020; Fatmawati et al., 2020). The nature and intent of questions—whether to gather



information, explore relationships, or extend thinking—shape classroom discourse and determine the depth of learner participation.

Types and Levels of Teacher Questions

Boaler and Brodie's (2004) questioning framework categorizes teacher questions into nine types based on their pedagogical intent. These range from gathering information to probing, extending thinking, and establishing context. The taxonomy highlights the importance of using a variety of question types to move beyond rote learning and promote reasoning. Similarly, Bloom's taxonomy emphasizes cognitive levels from knowledge recall to evaluation, suggesting that strategic questioning can support learners at various levels of intellectual engagement (Bloom, 1956).

THEORETICAL FRAMEWORKS

This study is grounded in Cognitively Guided Instruction (CGI) and Vygotsky's socio-cultural theory, both of which highlight the pivotal role of teacher–learner interaction in supporting mathematical understanding. CGI positions learners' thinking at the center of instructional decision-making, encouraging teachers to use purposeful questioning to elicit, interpret, and build upon learners' mathematical ideas (Stuart McQueen, 2022). Complementing this, Vygotsky's socio-cultural theory underscores the significance of social interaction, scaffolding, and mediation within the Zone of Proximal Development (ZPD), where learning is most effectively supported through guided participation and dialogue (Vygotsky & Cole, 1978). Together, these theoretical perspectives offer a robust framework for examining how teacher questioning can promote learner development by fostering collaborative meaning-making and deeper conceptual engagement.

METHODOLOGY

Research Design and Paradigm

This study adopted a qualitative case study design, which enabled an in-depth exploration of a single teacher's classroom practices within a real-life school setting. A case study is particularly suited to investigating contemporary phenomena in context, especially when the boundaries between the phenomenon and its setting are not clearly defined (Yin, 2018). The research was framed within the interpretivist paradigm, which posits that reality is socially constructed and that knowledge emerges through interaction between researcher and participants (Hennink et al., 2020). This paradigm aligns with the study's aim to understand how a teacher's questioning strategies shape learner thinking and classroom discourse.

Participants and Sampling

The study focused on one Grade 9 mathematics teacher and their class of 42 learners in a public secondary school in South Africa. The school was conveniently selected from a local education



circuit, while the teacher was purposively sampled based on their availability, willingness to participate, and openness to engaging in professional development. The learners in the class were included by virtue of their participation in regular classroom instruction, which formed the central context for the study.

Research Context and Intervention

The research took place in a typical South African public-school classroom equipped with standard teaching resources. The intervention was designed and facilitated by the researcher and consisted of a structured professional development session aimed at enhancing the teacher's use of cognitively guided questioning strategies. The session drew on Boaler et al.'s (2004) framework, emphasizing question types that promote mathematical reasoning—such as probing, exploring relationships, and extending thinking. Following the session, the teacher implemented the new strategies in their teaching over the course of three observed lessons: one pre-intervention and two post-interventions.

Data Collection Methods

Data were collected through a three-phase cycle: a pre-intervention observation, the intervention session, and two post-intervention lesson observations. In total, three lessons were observed, and video recorded to capture both verbal interactions and classroom context. The primary instruments used were structured classroom observation schedules based on Boaler et al.'s (2004) nine-question typology, as well as detailed field notes. Video recordings were used to allow for repeated review and to ensure accurate transcription and analysis.

Data Analysis

The recorded lessons were transcribed verbatim and analysed using thematic analysis. A deductive coding approach was employed, guided by the nine questioning categories in Boaler et al.'s (2004) framework. Frequencies and types of questions were compared between the pre- and post-intervention lessons. Additionally, qualitative excerpts were analysed to understand shifts in learner responses and interaction patterns. To enhance reliability, a second coder independently coded 30% of the data. Inter-rater agreement was calculated and found to be 87%, indicating acceptable reliability. Discrepancies were resolved through discussion.

Trustworthiness and Credibility

Credibility was ensured through triangulation of data sources (observations, field notes, and video recordings) and peer debriefing during the analysis phase to validate interpretations. A thick description of the classroom context and instructional episodes was provided to enhance transferability. Dependability was supported through systematic documentation of the coding process, and confirmability was achieved through transparent reporting of methodological decisions.



Ethical Considerations

Ethical clearance was granted by the university's research ethics committee prior to data collection. Informed consent was obtained from the participating teacher and the school principal. Learners' participation was passive, and their contributions were anonymised in all transcripts and reports. All data were securely stored and used exclusively for research purposes.

Findings and Discussion

The findings are organized around the research questions and show meaningful changes in the teacher's questioning practices and their influence on learner engagement and mathematical reasoning. The discussion draws on the theoretical framework—Cognitively Guided Instruction (CGI) and Vygotsky's socio-cultural theory—as well as relevant literature to interpret these changes.

Changes in Teacher Questioning Practices

Analysis of three observed lessons—one pre-intervention and two post-interventions—revealed a significant shift in the teacher's questioning patterns. Before the intervention, 48% of the teacher's questions were lower-order, factual recall questions aimed at eliciting brief or singular responses. These questions limited opportunities for learners to reason or engage conceptually, aligning with Boaler et al.'s (2004) category of “gathering information,” which is typically associated with surface-level engagement.

After the intervention, there was a marked increase in higher-order questioning. For instance, the use of probing questions rose from 13% to 16%, questions aimed at exploring mathematical meaning increased from 10% to 19%, and those involving linking and applying concepts jumped from 3% to 15%. Most notably, questions designed to extend thinking—which were entirely absent before the intervention—comprised 14% of the post-intervention questions. This shift indicates a growing alignment with CGI principles, which emphasize the use of learner thinking as a foundation for instructional decisions (Stuart McQueen, 2022). Table 4 summarizes these changes in question types.

Table 4: Summary of Changes in Teacher Question Types

Question Type	Pre-Intervention	Post-Intervention
Gathering information	Most dominant	Significantly reduced
Inserting terminology	Moderate	Reduced
Exploring meanings	Limited	Increased
Probing	Limited	Increased
Generating discussion	Weak	Slightly increased
Linking and applying	Weak	Greatly increased
Extending thinking	Absent	Greatly increased



Question Type	Pre-Intervention	Post-Intervention
Orienting and focusing	Absent	Increased
Establishing context	Absent	Emerged but limited

These findings are consistent with Boaler et al.'s (2004) argument that effective mathematics instruction involves not only asking more questions but asking different kinds of questions—those that promote connections, reasoning, and student autonomy.

Influence of the Intervention on Instructional Practice

The observed shifts suggest that the professional development intervention was effective in helping the teacher adopt cognitively demanding questioning strategies. Post-intervention, the teacher more frequently posed open-ended questions that prompted learners to explain their reasoning (e.g., “Can you explain your idea?”), encouraged multiple contributions, and required learners to apply concepts in unfamiliar contexts. These types of questions foster the kind of instructional scaffolding Vygotsky (1978) describes as essential within the Zone of Proximal Development (ZPD), where learners are supported to move beyond their current level of independent functioning.

The teacher's improved questioning practice also reflects the use of dialogic discourse, where meaning is co-constructed through conversation. This aligns with the socio-cultural view that learning is mediated through social interaction and confirms the role of questioning in facilitating such interaction.

Effects on Learner Engagement and Mathematical Thinking

Post-intervention classroom observations revealed that learners responded with greater depth, using full sentences and justifications rather than brief, factual answers. Learners also engaged more collaboratively—discussing ideas in pairs or groups—which suggests that the shift in questioning created more space for exploratory dialogue. One classroom excerpt illustrates this transformation:

Teacher: Based on this information, what can you say about the distances?
Learner: The distance of the original is equal to the distance of the image.
Teacher: We can conclude that the distance of the point and its image from the mirror line are equal.

This exchange demonstrates not just factual understanding, but reasoning and co-construction of knowledge—core features of Vygotsky's ZPD and CGI's emphasis on eliciting learners' thinking. Such interactions suggest that the teacher's new questioning strategies supported the development of conceptual understanding and mathematical reasoning.



Connecting Findings to Theory

The observed increase in open-ended, cognitively demanding questions supports the theoretical premise that teacher questioning can mediate learning. According to CGI, when teachers listen to and build on learners' ideas, they foster deeper mathematical understanding (Darmayanti et al. 2023; Stuart McQueen, 2022). Simultaneously, Vygotsky's theory underscores the importance of scaffolding learners just beyond their current level, which was evident in the teacher's post-intervention practice.

The teacher's movement from a transmission-based model of questioning (focused on correct answers) to an interactional model (focused on reasoning and meaning-making) marks a pedagogical shift toward socio-constructivist teaching. This supports Boaler et al.'s (2004) conclusion that improving the quality—not just the quantity—of classroom questions can lead to more meaningful mathematics learning.

CONCLUSION

This study demonstrated that teacher questioning strategies play a critical role in fostering learners' mathematical thinking. The intervention—designed to improve the quality and variety of classroom questions—resulted in a clear shift from predominantly lower-order questions to those that promote higher-order thinking, conceptual understanding, and active learner engagement. Post-intervention observations confirmed that learners were more involved in reasoning, discussing, and explaining their ideas, indicating a stronger presence of mathematical thinking during classroom discourse.

The findings affirm that targeted professional development in questioning techniques can significantly enhance teaching practices and classroom interactions. Equipping teachers with the ability to pose purposeful and cognitively demanding questions can transform mathematics classrooms into spaces of inquiry, reflection, and collaborative learning.

Based on the findings, several recommendations are proposed to enhance teachers' questioning practices in mathematics. First, regular in-service training and professional workshops should be conducted, with a specific focus on developing effective questioning strategies. Second, questioning pedagogy should be embedded within pre-service teacher education programs to ensure that new teachers enter the profession equipped with these essential skills. Finally, teachers should receive ongoing support and be encouraged to engage in reflective practice, enabling them to continuously refine and improve their questioning approaches to foster deeper mathematical understanding among learners.



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INTEGRATING SPECIALIZED TEACHING METHODS FROM SPECIAL SCHOOLS INTO MAINSTREAM MATHEMATICS EDUCATION AT THE FOUNDATION PHASE

Dineo Molise and Kakoma Luneta

University of the Free State University of Johannesburg

The study aimed to establish an effective teaching method for mainstream foundation phase teachers by comparing the mathematics teaching approaches used in special schools. Learners in the foundation phase enter the school system without assessing their prior knowledge, which should indicate their readiness for mathematics learning. As a result, these learners are forced to adapt to a teaching method that fails to meet their learning needs. Teachers often rely on a single teaching method that may not address learners' gaps in understanding before instruction begins. This situation leads to challenges in learning and overall learner performance. This study utilized a qualitative research approach in which teachers were observed teaching mathematics in the foundation phase, specifically from grade 1 to grade 3. Six teachers from both mainstream and special schools were interviewed. The study's findings address the research question concerning effective methods for teaching mathematics in the foundation phase. They indicate that using concrete and practical approaches to teach mathematical concepts is essential. Furthermore, adapting the curriculum to incorporate repetition and practice of mathematical concepts is crucial, as it helps to foster number sense through counting, hands-on activities, real-world illustrations, visual aids, and manipulatives while encouraging learners to think flexibly about numbers. Finally, incorporating play into mathematics education is a vital strategy. This research investigates further how mainstream teachers can be fully trained, like the training provided by special schoolteachers, to acquire diverse knowledge and skills in various teaching methods for teaching mathematics to learners in the foundation phase.

INTRODUCTION

Foundation phase learners in grades R-3, aged 5 to 9 years, begin their education with varying learning needs. Some may have stumbling blocks in their readiness for mathematics and possess limited prior knowledge (Naude & Meier, 2019). Unfortunately, these learners are often met by a single teacher who uses a narrow range of teaching methods, restricting their exposure to different ways of learning mathematical knowledge. According to White Paper 6 (2001) on building inclusive education, mainstream education aims to integrate learners into an existing system by providing additional support to specific individuals within regular classroom settings. In contrast, special education focuses on recognizing and respecting all learners' differences while identifying similarities. This support is designed to assist the learners, teachers, and the educational system in addressing the full range of learning needs. Moreover, special education seeks to overcome systemic barriers that prevent meeting these diverse learning requirements (Education, 2001). The primary goal is to adapt and enhance the support available within the classroom. The policy was established in 2001, making it 24 years



old, and has not been updated since then. There has been no revision of its goals to ensure they still meet the current demands of education and the evolving requirements. This situation highlights the lack of support for both learners and teachers in mainstream schools.

Learners' differences are often overlooked, with support provided only to specific learners expected to conform to the existing system. Meanwhile, teachers lack the necessary support to overcome systemic barriers that hinder their ability to meet the diverse needs of their learners (Barnard & Henn, 2023). A study has shown that some learners in South Africa struggle to benefit from the standard instruction offered in mainstream schools (Hove, 2014). This difficulty arises from various factors that predate their schooling (Prinsloo, 2008). Consequently, these barriers hinder their ability to learn effectively (Vlok, 2016). When these needs and barriers are not addressed, learners may experience difficulties, which could result in their exclusion from mainstream schools and placement in specialized schools for intensive support and targeted intervention (Education, 2001). Identifying learners with special learning needs before admission to mainstream schools is a significant challenge that must be addressed before teaching and learning commence. Kauffman, Hallahan, & Pullen (2024) have revealed that mainstream schools wait for a child to fail rather than focus on early intervention to prevent failure.

The failure of students is often seen as a crucial indicator by mainstream teachers; however, this perspective tends to neglect the various contributing factors behind that failure. Instead of analysing the teaching methods, curriculum, students' prior knowledge, and readiness to learn mathematics, the emphasis remains solely on failure. The guiding research question of this study was: What teaching methods do mainstream and special school teachers use when teaching mathematics in the foundation phase? The research question aims to identify the most effective teaching method for mathematics that addresses the diverse needs of learners in the foundation phase. This study proposes that mainstream schoolteachers in this phase should integrate their mathematics teaching methods used in special schools to accommodate the diverse needs of learners.

White paper policy six clearly states that teachers in special schools use approaches that focus on problem-solving and the development of learners' strengths and competencies rather than solely on their shortcomings (Education, 2001). Furthermore, a study suggests that teaching approaches in the foundation phase should address the diverse learning needs and varying knowledge levels of children, ensuring that all learners receive the education they require (Kauffman, Hallahan, & Pullen, 2024).

The context of special schools in the foundation phase.

In South Africa, a special school is designed for learners with specific learning needs, which can be categorized into various factors. These factors include physical, mental, sensory, neurological, and developmental impairments, psychosocial challenges, varying levels of intellectual ability, unique life experiences, and socio-economic disadvantages (Education, 2001). Therefore, this study examines the two points highlighted in the white paper concerning



learners' unique life experiences and socio-economic disadvantages. In mainstream schools, learners in the foundation phase often have unique life experiences. Some of these learners did not attend preschool because of socioeconomic factors, which can affect their mathematical readiness and prior knowledge. During the foundation phase, each grade builds upon the mathematical knowledge and skills acquired in previous grades. When the same mathematical concepts are presented in successive grades, they are advanced to higher levels. This progression requires that learners master the concepts and skills outlined for each grade to ensure a complete understanding of mathematics (Naude & Meier, 2019).

The primary challenge arises from the differing training backgrounds of teachers in special schools, who possess the necessary skills and knowledge to address the distinctive challenges encountered by their learners effectively (Hove, 2014). Compared to those in mainstream schools, who are often not adequately prepared to teach learners exhibiting these factors effectively. As a result, learners with limited prior knowledge and mathematical readiness may not receive the teaching methods that suit their learning needs. This indicates that the teaching methods used for mathematics differ between mainstream and special schools (Louw & Mofolo-Mbokane, 2019). The literature has revealed that various instructional methods can be used to teach mathematics in the foundation phase. In the foundation phase, effective mathematics instruction can employ multiple techniques, including concrete manipulatives, visual aids, games, storytelling, and cooperative learning, all aimed at building a robust foundation of mathematical understanding.

Problem statement

Teachers' instructional approaches are key factors that influence how learners acquire mathematical knowledge. A study has revealed that educators in mainstream classrooms often rely on a one-size-fits-all teaching method when instructing mathematics in the foundation phase (Izevbigie, 2021). This approach does not accommodate all learners, as students have varied cognitive abilities and different learning needs that require tailored teaching strategies. The problem statement was developed based on various studies that identified issues with foundation phase teachers' methods in teaching mathematics (Findell, Swafford, & Kilpatrick, 2001; Gqoli, Okeke, & Ugwuanyi, 2023; Luneta, 2014). Research has shown that learners have specific learning needs beyond the intrinsic challenges associated with learning disabilities; some are not easily detectable (Kauffman et al., 2024). Extrinsic factors stemming from structural and systemic issues prevent these learners from acquiring essential educational skills. As a result, they may require specialized education to address these gaps. Therefore, this study seeks to integrate teaching methods from special schools to address the learning needs of learners in mainstream education, which may affect their learning.

Research Question

How can teaching methodologies utilized in special education be integrated into mainstream educational settings to effectively address the diverse learning needs of foundation phase learners in mathematics?



THEORETICAL FRAMEWORK

The theoretical framework for this study examines how the constructivist approach influences teachers' instructional methods in the foundation phase. This approach is a valuable tool for teachers, encouraging them to choose effective teaching strategies that empower learners to construct their knowledge. Additionally, the constructivist approach equips teachers to address challenges that may arise during the teaching and learning process (Vintere, 2018). Despite the presence and knowledge of this approach, it has not significantly influenced the transformation of pedagogical methods in early childhood mathematics instruction, as many teachers in mainstream schools continue to use conventional techniques (Hove, 2014). This study explores foundation phase teachers' methods for teaching mathematics and focuses on understanding how teachers teach mathematics in the foundation phase.

LITERATURE REVIEW

The literature review explores essential aspects of early childhood mathematical learning. It focuses on several key areas, including the impact of parental involvement in helping children discover mathematics through social interactions. The review also examines preschool learning and learners' transition between different educational levels. Additionally, it discusses how social mathematical vocabulary is acquired and contrasts this with traditional views on how mathematical knowledge is transmitted from teachers to learners. Finally, the review evaluates which instructional approaches most effectively equip learners with mathematical knowledge. These aspects are crucial because they shape learners' experiences and prepare them for mathematics. If these factors are not addressed, learners may be at risk of having impaired prior mathematical knowledge, leading to gaps in their mathematical learning.

Teachers lack an understanding of mathematical concepts

The key strategy for effective mathematical teaching in the foundation phase is for teachers to have a strong understanding of mathematics to successfully capture the learning opportunities within the child's environment and provide various acceptable resources and demanding activities. By adequately applying their conceptions of mathematical knowledge during teaching and learning, teachers can provide scaffolding that expands the child's mathematical thinking while acknowledging the child's contribution (Gqoli et al., 2023). Luneta (2014) found that teachers often lack a fundamental understanding of geometry. This raises concerns about how the teachers' insufficient content knowledge may affect learners' mathematical learning. If a teacher does not grasp basic geometry concepts, how can they effectively teach the subject to their learners? Effective instruction is essential for helping learners progress through levels, which means that teachers must be knowledgeable and competent in their subject matter (Luneta, 2014).

Establishing a solid mathematical foundation for learners is essential for the future academic success of our country. Studies indicate that early proficiency in mathematics strongly predicts future achievements in this subject (Duncan et al., 2007; Gqoli et al., 2023; Sinay & Nahornick,



2016). By the time the foundation phase, learners reach higher grades. They should have a grounded understanding of basic geometry, including shapes and their properties (Luneta, 2014); ensuring a strong foundational understanding of mathematics is important. Numerous studies indicate that young learners' foundational mathematical knowledge may be compromised for various reasons. One significant factor is teachers' lack of effective instructional methods, hindering early childhood learners from acquiring mathematics knowledge. Numerous studies have explored various strategies for effectively teaching mathematics to young children through interactive activities. However, a significant issue remains: many teachers do not understand how learners cognitively construct and acquire mathematical knowledge. This understanding is crucial for teachers to understand how children develop mathematical knowledge and how they can support this process more effectively. Academically in the classroom and in their daily lives at home or during playtime (Findell et al., 2001). Few learners in the foundation phase are effectively acquiring the mathematical knowledge, skills, and confidence needed to use the mathematics they have learned (Adendorff, Mntunjani, & Siyepu, 2018).

The conventional view on teaching mathematics

Using concrete resources is an essential part of teaching methods and is crucial for learners' understanding of mathematics. Learners' thinking and reasoning skills are predominantly concrete in the foundation phase. According to Piaget, learners in this phase are in the concrete operational stage and require concrete objects to develop a solid understanding of mathematical concepts. Concrete resources are tangible objects that help learners connect with abstract ideas and enhance their comprehension of math concepts. According to Adendorff, Mntunjani, and Siyepu (2018), manipulatives refer to mathematical tools such as counters, bead strings, abacuses, base ten blocks, number lines, and charts. These concrete manipulatives are vital for incorporating explicit instruction and helping students grasp abstract mathematical concepts. Using these mathematical tools fosters hand-eye coordination, enhancing learners' visual and fine motor skills. Learners' interaction with concrete materials during lessons significantly enhances their understanding of math concepts (Adendorff et al., 2018).

Teachers should incorporate manipulatives into their teaching approach because they are adequate resources for teaching mathematics. Manipulatives allow children to understand key concepts through hands-on experiences, helping them later connect these ideas to more abstract concepts. This practice is essential for effective teaching and learning in all units of the mathematics curriculum, especially in the foundation phase (Sikhwari & Feza, 2024). Even though teachers recognize the benefits of using hands-on resources, a study has shown that they often rely on rote teaching and drill exercises to prepare students for assessments. As a result, these resources are typically employed when learners have difficulty grasping concepts and are primarily used for procedural understanding rather than for fostering conceptual knowledge and enhancing students' mathematical skills (Adendorff et al., 2018).

According to constructivist theory, instruction should enable learners to build their understanding and knowledge. In mainstream classes, teachers provided learners with abundant



information, whereas teachers in special classes preferred to elicit information by presenting challenges using visual aids, manipulatives, and questions (Shaw, 2017). The significant differences in children's opportunities at home and in their early childhood environments contribute to the variations in their numeracy skills at school.

Teachers should recognize that some learners may not have been exposed to mathematical experiences in their homes, so it is important to adjust their teaching approach accordingly. The mathematics classroom should be a welcoming space where concepts can be explained, nurtured, discussed, and understood. Learners should feel that every mathematical idea, including unconventional thoughts, is accepted and valued in the classroom (Liljedahl, 2020; Nickerson, 1999). The teaching methods used in mainstream schools in the foundation phase of teaching mathematics are geared toward preparing learners for systematic evaluations. Teachers often relied on rote learning and drill exercises to help learners succeed in assessments (Adendorff et al., 2018). This teaching approach has led to learners lacking a fundamental understanding of mathematics. Many learners have a weak foundation in basic mathematical concepts, which hinders their ability to comprehend and excel in more advanced topics (Mabena, Mokgosi, & Ramapela, 2021). Teachers should prioritize the outcomes of mathematics activities and the understanding of their students. This focus will help identify the skills these activities aim to develop, as well as the mathematical concepts involved. Therefore, teaching methods should promote children's mathematical growth through interactive classroom activities while encouraging participation in various instructional approaches (Gqoli et al., 2023).

RESEARCH METHODOLOGY

The study employed a qualitative research approach, specifically using a case study centered on two schools in an urban area. The researchers interviewed three teachers from mainstream schools and three from special schools. These interviews occurred after the researchers observed the teachers during their classroom teaching. Each teacher was representative of grades 1, 2, and 3. Below is a table summarizing the participants. The special school was chosen because a recent study has shown that special schools effectively address the needs of children with learning disabilities, leading to improved academic outcomes compared to mainstream schools. Special schools provide better support for learners and improve their overall educational experience because teachers can focus on each child's needs and learning pace (Hove, 2014). The interviewees responded to the study's research question about the instructional approaches that special schoolteachers use to teach mathematics to foundation phase learners. This involved examining the similarities and differences in their methods. Second, this study aimed to develop a specialized teaching approach for mainstream teachers, transforming traditional teaching methods into classrooms where teachers can effectively and intensively teach mathematics using intervention strategies established in special schools that enhance the teaching and learning of mathematics in the foundation phase.



Table 1: Summary of participants

Mainstream Foundation Phase Teachers (MFPT)		Special foundation phase Teachers (SFPT)	
Grade 1	MFPT1	Grade 1	SFPT1
Grade 2	MFPT2	Grade 2	SFPT2
Grade 3	MFPT3	Grade 3	SFPT3

FINDINGS AND DISCUSSION

The study's findings address the research question regarding effective methods for teaching mathematics in the foundation phase. These findings are organized around four key themes derived from interview responses.

First, using concrete and practical teaching methods for mathematical concepts is essential. Second, adapting the curriculum through repetition and drilling of mathematical concepts is important, as it helps develop number sense through counting. Lastly, incorporating play into mathematics education is a crucial approach.

Counting

The study's findings suggest that when foundation phase learners enter school, they are expected to know how to count from one to ten. Their counting skills should improve as they progress from one grade to the next. However, a lack of counting ability may indicate that a learner requires intensive intervention and support. Counting is a foundational skill for teaching mathematics; according to the CAPS document, each mathematics lesson should begin with counting. The responses from mainstream foundation phase teachers (MFPT) indicate that learners often struggle with counting, and they have shared various strategies they use to teach this crucial skill.

MFPT-1: She teaches counting by guiding learners on where to start and stop. She explains, *"I show them how to begin counting and when to stop. As they count, I walk around to observe their progress. For those who struggle, I give them a red dot and later call them for group counting."* She also explains how she connects counting to addition by encouraging learners to use their fingers. For instance, if the problem is $9 + 5$, she instructs the learners to think of the bigger number (9) in their heads and then count by using their fingers to add the smaller number (5).

However, this teaching approach differs from that of special schoolteachers, who utilize a more kinesthetic approach, then concrete objects for counting before transitioning to picture counting and number charts.

SFPT-1: The special school grade 1 teacher explained how they approach counting, emphasizing how Mathematics should be taught step by step without rushing learners. When teaching counting, it is important to start with kinesthetic activities that involve fine motor exercises using their hands and fingers. For example, learners can engage in finger-snapping,



play rock-paper-scissors with friends, or sing counting songs. One relevant counting song is *“Five Little Monkeys Jumping on the Bed.”*

Once learners are comfortable with the kinesthetics activities, the next step is to introduce concrete materials, such as counters. During one-on-one intervention, learners can start counting one counter at a time, gradually adding more counters as they progress. Individual attention is preferred by many learners, making one-on-one instruction valuable.

Following these steps in order and skipping any of them can make it challenging for learners to grasp counting. The kinesthetics approach provides physical training that develops fine motor skills and enhances eye-hand coordination. These skills lay the foundation for manipulating objects like counters effectively.

However, the grade 3 mainstream teacher argued that **MFPT-3**: *“Some learners reach grade 3 able to recite songs and rhymes but lack number sense. They struggle to identify which numbers are more or less, and determine which number comes before, after, or between others.”* She pointed out that this issue may arise from earlier grades, where counting was taught primarily through rhymes, with less emphasis on practical application.

Adaptation of curriculum, repetition, and drilling

According to the White Paper 6 policy (2001), special schools aim to address barriers within the education system that hinder their ability to meet the diverse learning needs of learners. The emphasis is on adapting classroom support systems, particularly in modifying the curriculum.

The special schoolteacher emphasizes the importance of adapting the prescribed curriculum to accommodate the diverse needs of learners. **SFPT-3** stated, *“It is easier for the child when we adapt the curriculum because we are guided by what the child is ready to learn. In our adaptation process, we refer to the curriculum and the Annual Teaching Plan (ATP) to determine what we must teach. Then, we modify it to align with learner readiness by incorporating more visual aids and individual assessments. We also provide reassessments for learners.”*

The grade 2 teacher from the special school supported her colleague, **SFPT-2**, by stating, *“Learners with diverse needs cannot follow and keep up with the standard curriculum, as it places too much expectation on them when they are not ready to learn what is required. We stick to one basic method for addition and subtraction because using multiple advanced methods can frustrate learners. We only move to advanced methods once the basic method has been mastered. The teaching methods need to be linked to concrete and practical applications.”*

Teachers at mainstream schools do not have the privilege of adapting the curriculum. Instead, they must ensure that much of the required content is covered in their teaching and learning. A mainstream teacher mentioned **MFPT-2**: *“There is no time to cover all the required written*



work according to the curriculum. The learners' pace is challenging and does not meet curriculum expectations. The number of Department of Basic Education (DBE) books that learners need to complete, along with the evidence of other materials that the Department requires, makes it difficult to engage in practical work, as there is a focus on gathering evidence."

Play-based approaches

Children in the foundation phase learn through play. Therefore, teachers need to integrate play into their mathematical teaching approaches.

SFPT-2 mentioned that *"most of the time, the knowledge acquired through play cannot be easily identified in books. For example, when we sing songs for math, learners acquire mathematical knowledge. This play experience is valuable because it cannot be captured in books. Through these activities, learners truly develop their understanding of math."*

The mainstream teacher shared her experience of teaching mathematics in the United Kingdom. She explained that classes were structured around specific concepts. For example, when the children were learning about the number six, a designated area would be set up to explore that concept. Each class would take turns engaging with the mathematics topic through play, primarily outside the classroom. This approach allowed learners to jump around and move rather than just sitting at their desks and writing in their books.

RECOMMENDATIONS

Recommendations for adapting the mainstream curriculum should emphasize the importance of repetition and practice of mathematical concepts. This approach fosters number sense and encourages more practical learning over theoretical, paper-and-pen activities. It should include hands-on activities, real-world illustrations, visual aids, and manipulatives to help learners think flexibly about numbers. Incorporating play into mathematics education is also a vital strategy. Teaching and learning should be organized around play-based learning, allowing learners to explore mathematical concepts engagingly and interactively. Furthermore, mainstream teachers should receive specialized training rather than just basic education. They should be prepared the same way as special education teachers, acquiring diverse knowledge and skills in various teaching methods tailored to teach mathematics to learners in the foundation phase, especially those with diverse learning needs.

CONCLUSION

This study found that one-on-one interventions are the most effective teaching approach to bridge the gap between learners' inadequate prior knowledge and the required grade-level content. These interventions should focus on repetition and revisiting previously learned concepts that the learner struggled to grasp. Additionally, the findings highlighted that a lack of counting skills indicates insufficient readiness to learn mathematics. Counting is a fundamental building block for mathematics, as many number operation concepts are based on



it. Another important aspect is employing a teaching strategy that allows learners to explore mathematical concepts through hands-on activities and self-discovery. This supports the theory of constructivism, which posits that learners gain and construct knowledge through play and exploration.

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THE ROLE OF ARTIFICIAL INTELLIGENCE IN ENHANCING THE TEACHING OF MATHEMATICS CONCEPTS

Benadette Aineamani

Maskew Miller Learning

This paper explores the transformative role of Artificial Intelligence (AI) in mathematics education, focusing on how AI technologies can support conceptual understanding, personalised learning, and formative assessment. Grounded in constructivist, sociocultural, posthumanist, and sociotechnical theoretical frameworks, the study critically reviews current literature to examine the pedagogical and ethical implications of AI integration in school-level mathematics. The review synthesises findings from over a decade of empirical research and high-quality reports, highlighting how intelligent tutoring systems and adaptive platforms can offer real-time feedback, differentiated instruction, and cultural mediation of mathematical ideas. At the same time, the paper interrogates issues of algorithmic bias, data privacy, and unequal access, particularly in under-resourced contexts. Through a multidisciplinary lens that includes Human-Computer Interaction and critical digital pedagogy, the paper argues that while AI presents opportunities to deepen learning and extend teacher capacity, it must be implemented with ethical awareness and contextual sensitivity. Ultimately, the study calls for inclusive, equitable AI designs that serve as pedagogical partners rather than replacements, and offers recommendations for educators, developers, and policymakers to ensure AI strengthens the human core of mathematics education.

INTRODUCTION

Artificial Intelligence (AI) is no longer a distant concept. It is quietly reshaping how mathematics is taught and understood in classrooms around the world. With intelligent tutoring systems and adaptive learning platforms, AI offers more than automation, it enables instruction that listens, responds, and evolves with each learner. These tools personalise learning, provide immediate, targeted feedback, and help teachers uncover gaps that might otherwise go unnoticed (Holmes et al., 2021).

This paper explores the growing role of AI in enhancing the teaching of mathematical concepts. It investigates how AI can support deeper conceptual understanding, empower teachers with actionable insights, and create more responsive, learner-centred classrooms. At the same time, it critically examines the risks, data privacy, algorithmic bias, and unequal access, that may shape who benefits and who is left behind. Grounded in current research, the paper calls for a deliberate and inclusive approach to AI, one that strengthens mathematics education without compromising its human core.

Motivation for the Significance of the Theme

Artificial Intelligence is transforming education, and mathematics stands at the centre of this change. Once viewed as a subject defined by rules and procedures, mathematics is now being



reimagined through AI systems that solve problems, explain solutions, and adapt to individual learners. This shift challenges us to rethink what it means to teach and understand mathematics today.

If AI can carry out many of the tasks once taught as core skills, what becomes the role of the teacher? What should learners focus on when machines can calculate easily? These questions are critical because mathematics shapes access to further education and career opportunities. It is not just a subject; it is a gateway.

This transformation also brings a deeper question: Is understanding about getting the right answer, or is it about reasoning, making connections, and thinking critically? AI offers the potential to support this richer view of learning. With personalised feedback, real-time scaffolding, and multilingual support, AI can help address learning gaps and expand access, especially in under-resourced schools.

But this potential must be handled with care. The integration of AI into mathematics teaching and learning has to be thoughtful, equitable, and focused on deep, meaningful learning. This paper explores why that challenge is not only urgent, but essential.

THEORETICAL AND PHILOSOPHICAL FRAMEWORK

This paper adopts a multidisciplinary theoretical framework to critically examine the integration of AI in mathematics teaching and learning. Drawing from constructivist learning theory, sociocultural perspectives, posthumanist and sociotechnical thought, Human-Computer Interaction (HCI), learning analytics, and critical digital pedagogy, this approach provides a rich foundation for understanding the pedagogical, cognitive, and ethical implications of AI in this field.

Rooted in the work of Piaget and Vygotsky, constructivist learning theory positions learners as active agents who build understanding through engagement, problem-solving, and reflection (Piaget, 1972; Vygotsky, 1978). In this view, effective mathematics learning requires environments that support inquiry and conceptual reasoning, rather than passive absorption of information. AI systems, such as intelligent tutoring tools and adaptive learning platforms, can potentially support constructivist approaches by offering real-time feedback and differentiated pathways. However, there is a risk that these systems emphasise procedural accuracy over deep conceptual engagement, thereby narrowing the learner's experience (Holmes et al., 2021).

Expanding on this, sociocultural theory, also developed by Vygotsky, highlights the role of social interaction and cultural tools in cognitive development. Learning is seen as a mediated activity, shaped by dialogue and tools that reflect the norms and values of a community (Vygotsky, 1978). AI systems function as cultural tools that mediate learners' engagement with mathematical content. Their role as mediating artifacts raises important questions about how they influence communication, collaboration, and the social nature of learning in both individual and group settings (Blikstein, 2018).



To move beyond human-centred views, this paper also engages with posthumanist and sociotechnical perspectives. These frameworks challenge the notion of humans as the sole producers of knowledge and explore how learning is co-produced through relationships between people, machines, and systems (Bayne, 2018; Fenwick & Edwards, 2010). AI in mathematics education is not merely a neutral tool but an active agent in shaping pedagogical outcomes, redefining authority, and determining what counts as valid knowledge. This raises critical epistemological and ethical questions: Who decides what a correct answer is? What are the consequences of offloading mathematical reasoning to intelligent systems?

In parallel, the field of Human-Computer Interaction (HCI) offers valuable insights into how learners engage with AI tools. HCI focuses on designing systems that support usability, reduce cognitive load, and promote meaningful user experiences (Norman, 2013). In mathematics education, where abstract thinking can be challenging, the design of AI interfaces must balance clarity, responsiveness, and cognitive scaffolding. Poorly designed systems can alienate learners, misinterpret their needs, or unintentionally hinder rather than support understanding (Luckin et al., 2016).

Alongside HCI, the rise of learning analytics introduces new forms of data-driven decision-making. AI systems often rely on learner data to adapt instruction and predict future performance. While this can enhance personalisation, it may also lead to reductive interpretations of learning by focusing on quantifiable behaviours rather than rich, qualitative understanding (Williamson, 2017). Moreover, algorithmic predictions can reinforce pre-existing inequalities, particularly if the training data reflects biased assumptions about learners' capabilities or backgrounds (Selwyn & Jandrić, 2020).

To critically interrogate these dynamics, this paper also draws on critical digital pedagogy, which foregrounds issues of equity, agency, and power in technology-enhanced learning. This perspective challenges the narrative of AI as inherently progressive, urging educators to consider who benefits from AI tools, whose knowledge is prioritised, and how algorithmic systems may reproduce existing patterns of exclusion, particularly in mathematics, a subject already experienced as high-stakes and exclusionary by many learners (Watters, 2021).

Together, these theoretical and philosophical perspectives form a comprehensive framework for analysing AI in mathematics education. They invite critical reflection not only on the design and function of AI tools but also on their broader implications for knowledge, learning, and justice.

Methodology and Critical Review of Literature

This paper uses a traditional literature review approach, guided by a critical perspective, to explore how Artificial Intelligence is enhancing the teaching of mathematical concepts. The aim is not just to summarise existing research but to interrogate the depth and direction of current thinking. What is known about AI's role in mathematics education, what remains unclear, and how can this knowledge be used to support all learners, not just a select few



A structured search was conducted using academic databases including Google Scholar, ERIC, JSTOR, and SpringerLink. The search focused on peer-reviewed journal articles, systematic reviews, and high-quality research reports published between 2015 and 2024. Search terms included combinations such as Artificial Intelligence in Education, AI and mathematics teaching, AI and conceptual understanding, adaptive learning in mathematics, AI and formative assessment, equity and AI in education, and critical perspectives on AI in schools. Only English-language publications were considered.

Studies were selected if they focused on AI in school-level mathematics education and examined one or more of the following themes: conceptual understanding, formative assessment, or personalised and adaptive learning. In addition, included studies needed to engage with issues such as learner diversity, equity, or the evolving role of teachers in AI-supported classrooms. Studies were excluded if they focused on general STEM education without a clear mathematics focus, lacked empirical or conceptual substance, or were limited to higher education without relevance to school settings.

The selected literature reveals a dynamic and evolving field. While many studies highlight the potential of AI to personalise learning, provide targeted feedback, and support deeper conceptual understanding, others raise important concerns about access, bias, and the ethical dimensions of AI use in schools. This review provides the foundation for the discussions that follow and helps to map both the progress and the pressing questions that still need attention in AI-driven mathematics teaching and learning.

Critical Synthesis of Research

The body of literature reviewed in this study reveals a rapidly evolving field shaped by interdisciplinary contributions from education, computer science, cognitive psychology, and the learning sciences. While the promise of Artificial Intelligence in mathematics education has captured global attention, the research remains concentrated around a few core themes that highlight both its pedagogical potential and its limitations.

This synthesis draws together key insights from recent empirical and theoretical studies to examine how AI is currently being used to enhance mathematics teaching and learning. It highlights not only what AI can do, but how it is reshaping classroom roles, instructional practices, and the meaning of mathematical understanding itself.

Across the literature, three dominant areas of inquiry emerge. First, a strong focus on conceptual understanding explores how AI systems help learners move beyond procedural fluency toward deeper reasoning and flexible problem-solving. Second, there is growing interest in personalised and adaptive learning, examining how AI technologies respond to individual learner profiles, pace, and needs. Third, formative assessment and feedback has emerged as a major theme, investigating how AI enables real-time diagnosis and targeted support during the learning process.

Together, these strands form a critical foundation for understanding how AI is changing the



landscape of mathematics education. However, they also surface important tensions, around equity, teacher agency, algorithmic transparency, and the cultural assumptions embedded in AI design. The following sections unpack these three themes in detail, synthesising what current research reveals and identifying where further inquiry is urgently needed.

Supporting Conceptual Understanding

At the heart of mathematical learning lies conceptual understanding, which is defined as the ability to grasp not just how to solve a problem, but why the solution works. It involves seeing the connections between ideas, reasoning flexibly, and applying knowledge in new contexts (Kilpatrick, Swafford, & Findell, 2001).

Artificial Intelligence is opening new doors to this kind of learning. Moving beyond rote procedures, AI-powered platforms draw on constructivist and sociocultural principles (Piaget, 1972; Vygotsky, 1978) to create interactive spaces where learners explore, test, and manipulate mathematical ideas. Intelligent tutoring systems and dynamic visual tools allow learners to see relationships unfold in real time, shifting the emphasis from memorisation to meaning making (Holmes et al., 2021; Kiss & Póczos, 2020).

Evidence from large-scale studies reinforces this potential. For example, Pane et al. (2014) found that students using Carnegie Learning's Cognitive Tutor showed marked improvements in algebraic reasoning. The system's strength lay in its ability to personalise support, visualise multiple solution paths, and respond to learners' input with precision.

AI tools also function as cultural and cognitive mediators, guiding learners through scaffolded dialogue, prompts, and peer-like interaction (Blikstein, 2018). They can extend learners' zones of proximal development and support collaborative thinking, if designed with care.

Yet challenges remain. Many AI systems still struggle with cultural responsiveness, accessibility, and clarity of design (Norman, 2013). More critically, AI is not neutral. Algorithms shape what counts as valid reasoning and can reinforce dominant norms (Bayne, 2018; Fenwick & Edwards, 2010). Without thoughtful design and ethical awareness, these systems risk narrowing rather than enriching mathematical understanding.

Supporting conceptual understanding through AI, then, is more than a technical task. It is a pedagogical and social responsibility, one that demands inclusive design, critical reflection, and a commitment to equity (Watters, 2021).

Enabling Personalised and Differentiated Learning

AI is redefining what it means to meet learners where they are. In mathematics classrooms, where learners bring different starting points and progress at different paces, AI-powered adaptive systems offer a dynamic solution. These tools monitor learner input in real time, detect misunderstandings, and adjust tasks on the spot, creating a personalised learning journey that evolves with each learner (Zawacki-Richter et al., 2020; Pane et al., 2014).



Rooted in constructivist thinking (Piaget, 1972; Vygotsky, 1978), this personalised scaffolding allows learners to actively build understanding with just-right challenges. In overcrowded or under-resourced classrooms, AI can serve as an extra set of hands, helping to keep every learner engaged, supported, and stretched.

A powerful example is the ASSISTments platform, which provides immediate feedback, flags misconceptions, and helps teachers tailor instruction. In a large study across 43 schools, students using ASSISTments outperformed their peers on mathematics tasks, highlighting how AI can do more than grade; it can guide (Roschelle et al., 2016).

But personalisation isn't always neutral. Scholars like Williamson (2017) and Watters (2021) warn that algorithm-driven pathways can limit learner agency, nudging students toward narrow outcomes based on built-in assumptions. If AI chooses what's next without transparency or flexibility, it may close off inquiry rather than expand it.

That's why design matters. Systems must reflect learners' cultural, linguistic, and cognitive diversity, offering multilingual support, relevant examples, and responsive pacing. From a human-computer interaction perspective, clarity, usability, and trust are just as critical as accuracy (Norman, 2013).

Personalised learning through AI holds enormous promise, but only if built on ethical design, meaningful choice, and a deep understanding of learners as individuals. When guided by theory and care, systems like ASSISTments show how AI can power equity, not just efficiency, in mathematics education.

Enhancing Feedback and Formative Assessment

AI is transforming formative assessment from a reactive process into a real-time, personalised dialogue between learners and content. In mathematics, where understanding how a learner *thinks* is just as important as whether they get the answer right, AI-powered tools offer powerful new possibilities. These systems do not just mark answers, they trace problem-solving paths, detect misconceptions, and provide instant, tailored feedback. This kind of timely, diagnostic insight helps teachers intervene more effectively and supports learners in building deeper mathematical reasoning (Ifenthaler & Yau, 2020; Piaget, 1972; Vygotsky, 1978).

Platforms like ASSISTments show how adaptive feedback can boost confidence, correct misunderstandings, and encourage metacognition, all while supporting teachers with rich data on learner progress (Roschelle et al., 2016). From a sociocultural view, AI can act as a helpful classroom assistant, offering hints and scaffolds when the teacher cannot be everywhere at once. In remote or large classrooms, this can be the difference between stagnation and growth. But the promise comes with real caution. Feedback can be too vague, too rigid, or even misaligned with teaching goals (Roll & Wylie, 2016; Luckin et al., 2016). Over-reliance on AI risks turning learners into passive recipients of machine judgments rather than active participants in meaning-making. Moreover, many AI systems are designed without the contexts of under-resourced or multilingual learners in mind, limiting their reach and reinforcing



existing inequities (Selwyn & Jandrić, 2020; Watters, 2021). To be transformative, AI-powered assessment must go beyond efficiency. It must be inclusive, transparent, and grounded in sound pedagogy. When thoughtfully designed, AI can illuminate the learner's thinking, not just track their performance, and help build a mathematics classroom where every learner has the chance to grow.

Ethical Considerations and Limitations

While AI holds real promise for transforming mathematics education, its use demands careful ethical reflection. One of the most pressing concerns is data bias. AI systems rely on learner data to adapt instruction, but when this data lacks cultural or contextual diversity, the results can be exclusionary. Misinterpreted responses or mismatched feedback can disadvantage learners, especially those from underrepresented backgrounds. Access is another critical issue. Many schools, particularly in rural or low-income areas, still lack the infrastructure, devices, or internet connectivity needed to use AI effectively. This digital divide risks deepening existing inequalities, giving well-resourced learners yet another advantage (Williamson & Eynon, 2020). There are also growing concerns about the shifting role of the teacher. When AI systems begin to drive instruction, they can unintentionally sideline teacher expertise, judgement, and relational insight. Education risks becoming less human, less creative, and less responsive to learners' lived realities. For AI to truly enhance mathematics education, it must be designed to support, not supplant, the teacher, and to empower all learners, not just those already positioned to succeed.

Implications for Teaching, Learning, and Research in Mathematics

The rise of AI in mathematics education calls for a shift in how teachers teach and learners learn. Teachers will need to build confidence in using AI tools not just for practice drills, but to deepen mathematical understanding. Professional development should also address the ethical and philosophical questions AI introduces into the classroom. Learners, too, must move beyond seeing AI as an answer machine. They should be guided to explore how AI tools work, why certain feedback is given, and how to think critically about the suggestions they receive. AI should be a tool for reasoning and reflection, not just speed and accuracy. Future research must explore how AI shapes mathematical thinking across diverse cultural and linguistic contexts. It should engage both teachers and learners in the co-design of tools that respond to real classroom challenges. Understanding how AI influences cognition, judgement, and equity in mathematics remains a critical area of inquiry.

CONCLUSION

AI offers valuable opportunities to enhance the teaching of mathematical concepts by deepening learners' conceptual understanding, enabling personalised and adaptive instruction, and improving formative assessment through real-time feedback. Systems like Cognitive Tutor and ASSISTments illustrate how AI can help learners engage more meaningfully with mathematical ideas and support teachers in identifying misconceptions and tailoring instruction.



At the same time, these tools present important challenges. Algorithmic bias, lack of cultural responsiveness, and opaque decision-making processes can undermine equity, limit learner agency, and distort the goals of mathematics education. Moreover, most research on AI in education remains concentrated in high-income contexts, with limited attention to diverse learning environments or the voices of learners themselves.

To fulfil its promise, AI must be integrated into mathematics education with pedagogical care, ethical awareness, and a commitment to inclusion. Rather than replacing teachers, AI should serve as a supportive resource, enhancing conceptual learning and expanding access to high-quality mathematics instruction in ways that are equitable, context-aware, and learner-centered.

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EXPLORING THE DEDUCTIVE NATURE OF MATHEMATICS ON SOME TRIGONOMETRY RESULTS

Vasuthavan Gopaul Govender

Nelson Mandela University

Trigonometry is introduced to South African learners for the first time in term one of grade 10. There are a number of topics, mostly from grades 8 and 9, that contribute to this important content area of mathematics. This study discusses these topics' contributions to some key areas of Trigonometry. Teachers need to carefully plan their lessons for this key content area and their preparation should reflect the prior knowledge that learners should have from these grades. This approach is likely to lessen the cognitive load on learners, as they view Trigonometry as a continuation of the work learnt in other familiar sections of Mathematics, thereby, emphasising the deductive and integrated nature of Mathematics, as a school subject.

INTRODUCTION

Trigonometry is an important section of Mathematics in South Africa. It is one of the ten content areas for Mathematics in the Further Education and Training band. In terms of assessment, trigonometry forms part of Mathematics Study 2 (DBE, 2011b). Table 1 shows the composition of trigonometry in grades 10 -12.

Table 1: Trigonometry marks in Grades 10 - 12

Grade	10	11	12
Trigonometry marks	50	60	50
Total marks for the study	100	150	150
Percentage of trigonometry marks	50%	40%	33,33%

According to the table 1, Trigonometry forms a significant part of the assessment of Mathematics Study 2. Thus, learner performance in Trigonometry is likely have an impact in their performance in Mathematics Study 2, as well as their overall performance in Mathematics.

Trigonometry has numerous applications in other branches of mathematics as well as statistics, economics, surveying, architecture and engineering (Weber, 2005). However, learners tend to have difficulty with trigonometry, especially when traditional teacher-centred approaches are used in its teaching (Demir, Sutton-Brown and Czermiak, 2012).

Problem Statement

According to the annual teaching plan (ATP) for Mathematics, as located in CAPS document (DBE, 2011b), Trigonometry is introduced to grade 10 learners in weeks 9 and 10 of term 1. It



starts with the trigonometric ratios of $\sin \theta$; $\cos \theta$ and $\tan \theta$ in right-angled triangles. These definitions are then extended to $0^\circ \leq \theta \leq 360^\circ$.

While this introduction seems to be done fairly well by teachers, there may be a reluctance on the part of teachers to integrate some of the mathematics learners have done in grades 8 and 9, into this new section. This becomes crucial when concepts such as the sine, cosine and area rules are introduced in grade 11 and the compound angle identities in grade 12.

Table 2 shows the composition of the area/sine/cosine rules and compound angle identities in the grade 12 Mathematics studies for 2022 to 2024 (DBE, 2022; DBE, 2023; DBE 2024).

Table 2: Grade 12 Compound angles and Area/Sine/Cosine Rule

Years	Compound angle identities	Area/Sine/Cosine rule	Total
2024	Questions 5.1.2; 5.1.3; 5.2; 6.1.1: 18 marks	Question 8: 9 Marks	27 marks
2023	Questions 5.3 and 5.4; 14 marks	Question 7: 7 marks	21 marks
2022	Questions 5.3; 5.4 and 5.5: 18 marks	Question: 10 marks	28 marks

These sections form a significant total of the marks for trigonometry in each of these years.

Table 3 shows the composition of the area, sine and cosine rules in the grade 11 Mathematics studies for 2016 to 2018. There were no recent grade 11 studies on the DBE website (DBE, 2016; DBE, 2017; DBE, 2018).

Table 3: Grade 11 Area/Sine/Cosine Rule

Years	Compound angles	Area/Sine/Cosine rule
2016	None	Question 7: 13 marks
2017	None	Question 7: 11 marks
2017	None	Question 7: 16 marks

The area/sine/cosine rule is a crucial part of both grade 11 and grade 12 assessment while compound angle identities is a key part of grade 12 assessment. However, much will depend on how these topics are taught.

Research question

In the light of the discussion of these trigonometry topics in the previous section, the following research question was posed for this study:

What background knowledge and skills do mathematics learners need to answer questions involving area/sine/cosine rules in grades 11 and 12, and compound angle identities in grade 12?

The following sub-questions were formulated in line with the research question.



- What type of mathematical knowledge and skills are required to understand and solve questions in certain trigonometry sections in grades 11 and 12?
- When are learners likely to be taught or encounter, for the first time, these mathematical knowledge and skills?

LITERATURE REVIEW

This short literature review features a discussion on the different types of prior knowledge in mathematics and the need for both conceptual and procedural knowledge to feature in its teaching and learning. This is, especially, important in Trigonometry where such prior knowledge plays a crucial role in understanding firstly basic, and then more advanced trigonometric concepts.

Alreshidi (2023) speaks about two types of prior knowledge in mathematics, conceptual knowledge and procedural knowledge. To develop conceptual knowledge, students should first learn the concepts and then learn the relationships between the concepts. If they lack the necessary prior concepts, then learning connections between the concepts is likely to be unsuccessful. Similarly, procedural knowledge is visualised as the ability to follow steps in a sequence to reach a mathematical goal or solve mathematical problems. Thus, learning new procedures that have relationships with other procedures that learners may not know may lead to a memory overload. Furthermore, differences in students' prior knowledge strongly affect their learning, including the gaining of new procedural knowledge.

In a related study on procedural and conceptual knowledge, Hurrell (2021) contends that mathematics teaching should include both procedural knowledge and conceptual knowledge as both are equally important and help strengthen each other. Conceptual knowledge usually precedes procedural knowledge. In this regard, there is more chance of learners developing conceptual understanding if they start with conceptual knowledge and then move to procedural knowledge.

The preceding two paragraphs on procedural and conceptual knowledge are applicable to all mathematics topics. However, this becomes very important when Trigonometry is introduced to grade 10 high school learners in South Africa.

Prasad (2023) outlines the reasons why Trigonometry is an integral part of the Mathematics curriculum for high schools. He gives an overview of the various applications of Trigonometry from an historical perspective where examples in its use in astronomy and geometry are given, as well as a 21st century perspective, where applications of trigonometry feature in architecture, engineering, marine biology, oceanography, criminology and other fields.

Hulya (2009), in a study of trigonometry learning in Turkey, characterised some misconceptions that learners have in this important section. These were misused data, misinterpreted language, logically invalid reference, distorted definition and technical or mechanical errors. Learners made errors in the “usage of simple formulae,” “calculation of trigonometric values of angles,” and “to populate the values into a formula”. Some learners chose the correct ratios but were not able to manipulate these ratios. Learners also had



difficulties with trigonometric notation. The errors made by learners were likely to have a negative impact when learners take their university entrance examinations.

A related South African study by Ngcobo, Madonsela and Brijlall (2019) on the teaching and learning of the trigonometry topic of solution of triangles to grade 12 learners, which was underpinned by APOS (Action; Process; Objects; Schema) as the theoretical framework, found that the majority of the learners were operating at the action stage of APOS. While the action stage is necessary for the development of other stages such as process and object, 67% of the participants had not developed the object conception. The authors conclude that, despite learners moving on to the next grade, the lack of the development of the prerequisite concepts resulted in their conceptualisation of mathematical concepts being only at the action stage. This was likely to compromise their understanding of these and other trigonometry topics.

THEORETICAL FRAMEWORK

The theoretical framework for this study is guided by two key principles, learners' prior knowledge and connecting this prior knowledge to the "new knowledge". One of the theories which links prior knowledge to new knowledge is Cognitive Load Theory (CLT) which was introduced by John Sweller in the late 1980s. This theory provides valuable insights for teachers, enabling them to optimise teaching practices by considering the brain's cognitive capacity. At the heart of CLT, is the understanding that the human brain has a limited working memory which is easily overwhelmed when processing large amounts of new or complex information, leading to learner difficulty and poor retention. By designing lessons that takes these limitations into account, teachers can enhance learning experiences and prevent cognitive overload (Benjamin & Main (2022).

Ningsih and Retnowati (2019) state that prior knowledge forms part of a learner's long-term memory. This becomes important when teachers plan their lessons as this long-term memory, which gives an indication of learners' prior knowledge, is used in constructing further knowledge. Thus, teachers should establish what learners already know so that they are able to improve their own pedagogy. This becomes important in a subject like mathematics where much of learners' newly acquired knowledge is dependent on what they had already known.

In this study, the focus is on key trigonometry topics, the area, sine, and cosine rule in grade 11, and the compound angle identities in grade 12. These trigonometric topics depend on learners' prior mathematical knowledge in grades 8 to 11.

Thus, an appropriate framework in which to locate this study is

"Bridging the gap between prior knowledge and new knowledge, within a trigonometric context, to mitigate against cognitive overload in learners".

RESEARCH DESIGN AND METHODOLOGY

As mentioned earlier, Trigonometry is an important section of Mathematics in South Africa and other countries. It occupies a significant part of the final examinations for Mathematics



Study 2 in grades 10, 11 and 12. According to some research studies on Trigonometry, it would appear that learners are able to function at a superficial level in Trigonometry, possibly due to the way it is taught at schools in South Africa and elsewhere.

In this study, the writer traces the background or prior knowledge that is needed for learners to understand some of the intricacies involved in the studying some of its key topics in grade 11 and 12. This is done in a structured manner, using the CAPS documents for Senior Phase and FET as a guide (DBE, 2011a; 2011b).

It is hoped that this would change the way these topics are taught, with a focus on the deductive nature of mathematics, where new content is deduced from content that is already known or has been proven (Morsanyi, McCormack & O'Mahoney, 2017).

Since this study is about how the teacher is able to facilitate the acquisition of knowledge of these trigonometry topics to learners by making use of learners' prior knowledge and using mostly a deductive approach, this study is located within the constructivist paradigm. Constructive learning strategies are developed using principles of social constructivism to improve learner achievement and develop high-order thinking, social and emotional skills of learners. In this process, the teacher presents a topic to the learners and guides them to explore the topic through using their prior knowledge. Learners may ask questions with the teacher responding by showing how the prior knowledge helps learners understand the new knowledge, through scaffolding (Dagar and Yadav, 2016)

Furthermore, this study could also be classified as descriptive research (Manjunatha, 2019) as it gives a detailed description with the appropriate diagrams on how these grade 11 and 11 topics may be taught.

Data

The writer uses the CAPS documents for Senior Phase (DBE, 2011a) and FET (DBE, 2011 b) to examine the background knowledge that is required for the teaching and learning of certain trigonometry topics.

The Cartesian Plane

Teachers should make learners aware of the Cartesian plane, also known as the number plane or x-y plane. The Cartesian plane, is divided into four quadrants by two perpendicular lines called the **x**-axis, which is horizontal line, and the **y**-axis, which is vertical. These axes intersect at a point called the origin. Once a unit distance has been chosen, the position of any point in the plane can be uniquely represented by an ordered pair of numbers (x,y) (AMSI, 2025).

The Cartesian Plane features for the first time in the fourth term for grade 8, under the content area, Patterns, Function and Algebra, and the topic, Graphs (DBE, 2011a: 114). Here learners would have to “*use tables of ordered pairs to plot points and draw graphs on the Cartesian plane*”. This is repeated in grade 9, in the same content area, under the topic Functions and Relationships. Here learners have to “*determine, interpret and justify equivalence of different*



descriptions of the same relationship or rule presented verbally, in flow diagrams, in tables, by formulae, by equations, by graphs on a Cartesian plane” (DBE, 2011a:129).

Thus, by the end of grade 9, learners should be able to plot points on the Cartesian Plane and then join points to draw the corresponding straight-line graphs.

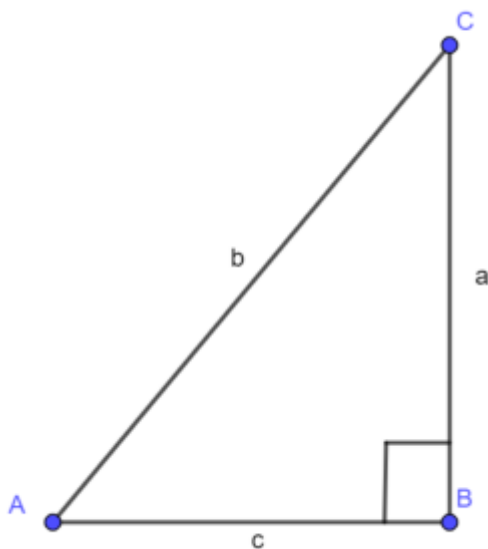
The theorem of Pythagoras

Another important result from grade 8 and 9 is the Theorem of Pythagoras. This is introduced in grade 8 in term 2. Learners are introduced to the properties of triangles; and then in a special case for a right-angled triangle, note that the square of the hypotenuse is equal to the sum of the squares of the other two sides (DBE, 2011a:96). This is usually achieved through guided discovery learning. Learners are able to solve for the missing lengths of a right-angled triangle using this theorem. The theorem of Pythagoras is repeated in grade 9 so that teachers able to further consolidation of its applications in geometry and measurement (DBE, 2011a:138). It is also specified on page 138 of the Senior Phase CAPs document (DBE, 2011a) that “*In the FET phase, the Theorem of Pythagoras is crucial to the further study of Geometry and Trigonometry*”.

Ratio and Rate

A crucial concept that forms part of the Senior Phase is Ratio and Rate. This, according to the CAPs document is done in grades 7 (DBE,2011a:24) where learners have to solve problems in contexts involving ratio and rate, where speed, distance and time are mentioned and in grade 9 (DBE,2011a:120) where the contexts are not restricted.

The ratio part of Ratio and Rate is, especially, significant when Trigonometry is introduced for the first time in the first term of grade 10. The definitions of the three basic trigonometric ratios, sine, cosine and tangent (and their reciprocals) are introduced using a right-angled triangle in the first quadrant and then extended to other quadrants (DBE, 2011b:23).





We draw a right-angled triangle, in the first quadrant, without showing the grid and axes lines. We may use A as the reference angle and from this we may define the three basic ratios, sine, cosine and tangent in terms of angle A. Now we use the letters a, b, and c as follows: BC = a (the side opposite angle A); AB = c (the side adjacent to angle A) and AC = b (the longest side of the triangle, called the hypotenuse)

Thus, we have the following definitions:

$$\sin A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{a}{b}$$

$$\cos A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{c}{b}$$

$$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{c}$$

We may turn the triangle around, so C becomes the reference angle

$$\cos C = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{b}$$

$$\sin C = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{c}{b}$$

$$\tan C = \frac{\text{opposite}}{\text{adjacent}} = \frac{c}{a}$$

These ratios make it easier for learners to find out the missing sides and angles in right-angled triangles without using the theorem of Pythagoras. Thereafter, the ratios of special angles such as 30° ; 45° 60° and 90° are determined using right-angled triangles which are then extended to the second, third and fourth quadrants. Learners also have to solve two-dimensional problems involving right-angled triangles and simple trigonometric equations for angles between 0° and 90° .

For example, we can prove that

$$\sin 30^\circ = \frac{1}{2}$$

Thus, the opposite side is 1 unit and the hypotenuse is 2 units. So the adjacent side is



$$\sqrt{(2)^2 - (1)^2} = \sqrt{3}$$

So we have:

$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

We now have:

$$\sin^2 30^\circ + \cos^2 30^\circ = \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1$$

This statement is then generalised as follows:

$$\sin^2 \theta + \cos^2 \theta = 1 \text{ for } \theta \in [-360^\circ; 360^\circ]$$

Learners are also introduced to the simplification of basic trigonometric ratios involving the following angles:

$$180^\circ \pm \theta; 360^\circ \pm \theta \text{ and } -\theta, \text{ where } \theta \in [0^\circ; 90^\circ]$$

The following results are also be derived:

$$\sin(90^\circ - \theta) = \cos \theta$$

$$\cos(90^\circ - \theta) = \sin \theta$$

Analytical Geometry

A key content area which is introduced to learners for the first time in the third term of grade 10 is Analytical Geometry (DBE, 2011b:26). Here, learners have to do the following:

- Represent geometric figures on a Cartesian co-ordinate system
- Derive and apply for any two points $(x_1; y_1)$ and $(x_2; y_2)$, the formula for calculating the distance between the two points; the gradient of the line segment connecting the two points (and identify parallel and perpendicular lines; and the coordinates of the midpoint of the line joining the two points.

In grades 8 and 9, learners had to draw straight lines through point-by-point plotting. They would soon realise, probably with the teacher's urging, that a minimum of two points are



needed to draw a straight line. Now they join points in an attempt to represent different geometry figures in the Cartesian plan.

The example given in the CAPs document (DBE, 2011b:26) is shown below, with cognitive level in brackets:

Consider the points P(2;5) and Q (-3;1) in the Cartesian Plan.

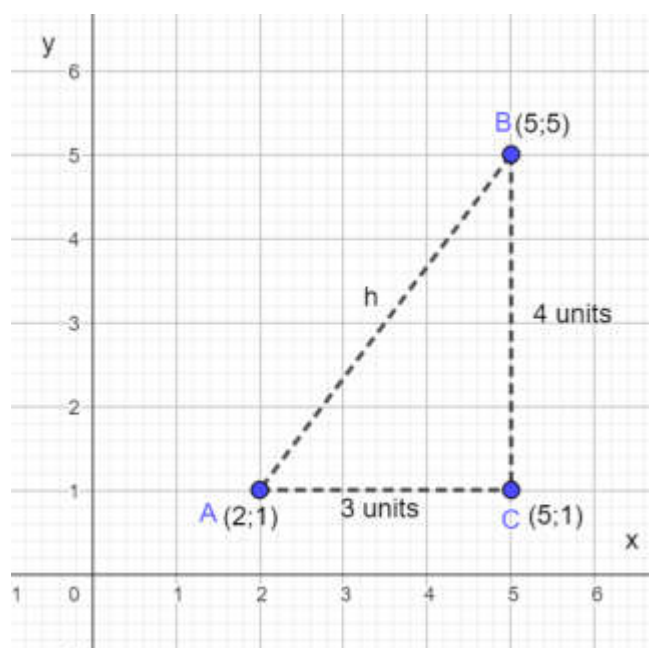
1.1	Calculate the distance PQ	(K)
1.2	Find the coordinates of R if M(-1;0) is the midpoint of PR	(R)
1.3	Determine the coordinates of S if PQRS is a parallelogram	(C)
1.4	Is PQRS a rectangle? Why or why not?	(R)

NB: K: Knowledge; R: Routine Procedures; C: Complex Procedures

These and other applications of Analytical Geometry are useful in other sections of Mathematics, including Euclidean Geometry and Trigonometry.

The Distance formula

Teachers need to consider what is the best way to introduce the distance formula. In this regard, we go back to the Cartesian Plane.



Now consider $\triangle ABC$ which has a right angle at C. From the background knowledge of points, we note that $\triangle ABC$ has the points A(2;1); B(5;5) and C (5;1). By counting horizontally, we get $AC = 3$ units and counting vertically we get $BC = 4$ units. Thus, from the theorem of Pythagoras from grades 8 and 9, we know that $AB^2 = AC^2 + BC^2 = 3^2 + 4^2 = 9 + 16 = 25$ So $AB = 5$ units.

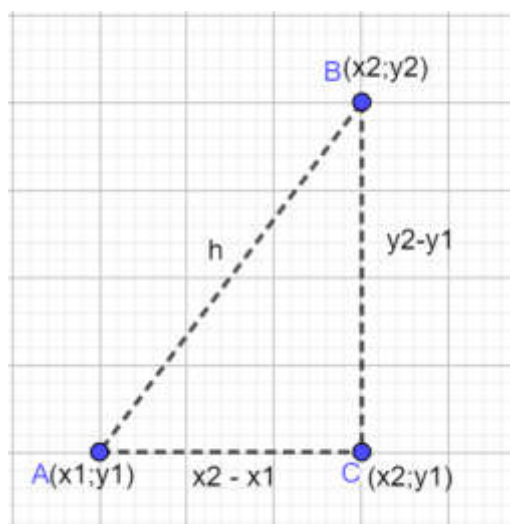


Is there another way of getting AB directly, without considering C? We note that the x-coordinates of A and B are 2 and 5 respectively, while the y-coordinates of A and B are 1 and 5 respectively.

Now we difference of x-coordinate is $5 - 2 = 3$; and the difference of the y-coordinate is $5 - 1 = 4$. So $AB^2 = (5 - 2)^2 + (5 - 1)^2 = 3^2 + 4^2 = 9 + 16 = 25$

Thus, $AB = 5$ units

We can now generalise the formula for the calculation of the distance between two points in the Cartesian plane:



Given that $A(x_1, y_1)$ and $B(x_2, y_2)$ then the distance AB is given by

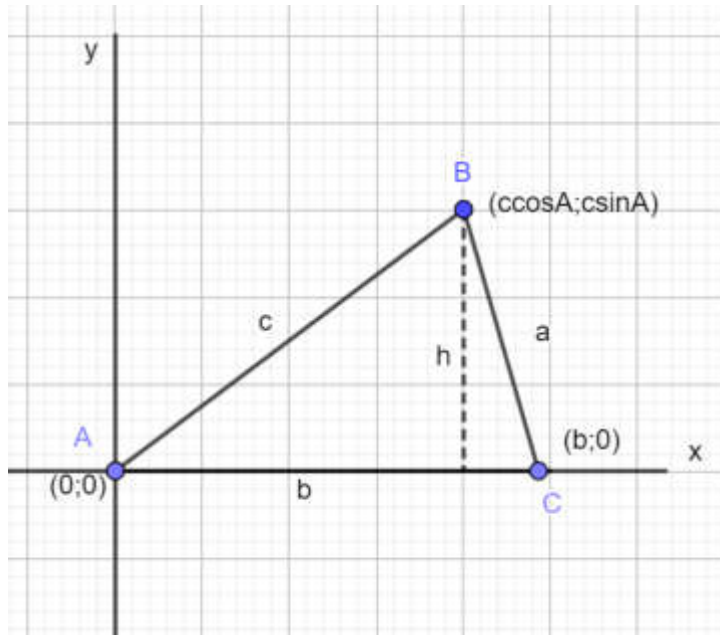
$$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\Rightarrow AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

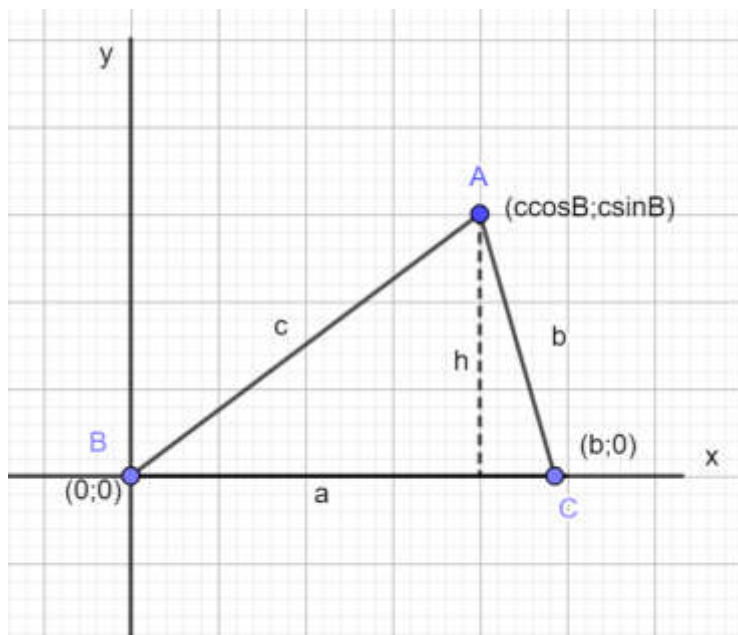
This formula can be used to calculate the distance between any two points in the Cartesian plane.

The Area, Sine and Cosine Rules

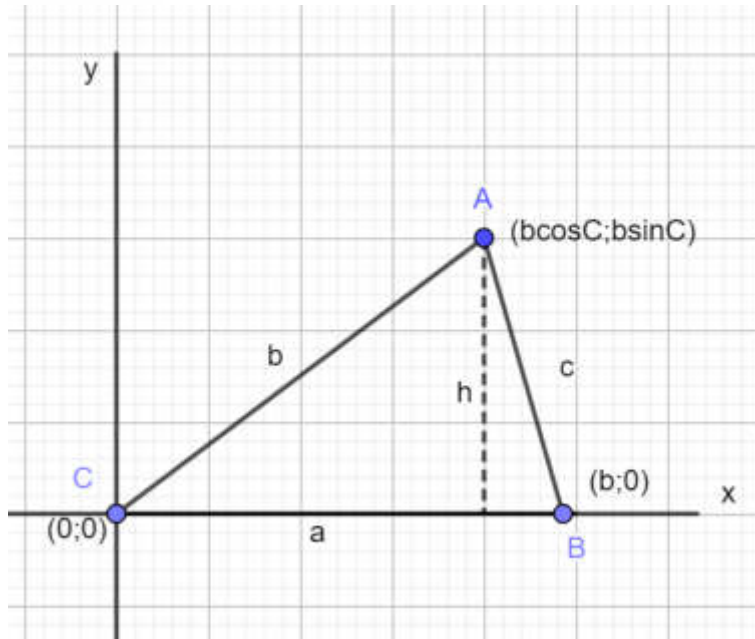
We draw $\triangle ABC$ is the first quadrant where A, B and C each have the opportunity of being placed at the origin (0;0). The key in each diagram is to get the coordinates of vertex using the sine and cosine ratios for each of the angles placed at the origin. The y-value of the vertex point represents the horizontal distance from the vertex point to the x-axis. This distance (h) is the height in each case.



In this diagram we note that $h = c \sin A$.
The area of $\triangle ABC = \frac{1}{2} AC \times h = \frac{1}{2} bc \sin A$



In this diagram we note that $h = c \sin B$.
The area of $\triangle ABC = \frac{1}{2} BC \times h = \frac{1}{2} ac \sin B$



In this diagram we note that $h = b \sin C$

The area of $\triangle ABC = \frac{1}{2} BC \times h = \frac{1}{2} ab \sin c$

Thus, there are three ways of writing the area of $\triangle ABC$ and all are equal so

$\frac{1}{2} bc \sin A = \frac{1}{2} ac \sin B = \frac{1}{2} ab \sin C$ (the area rule)

We multiply the above equation by $\frac{2}{abc}$ to get

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

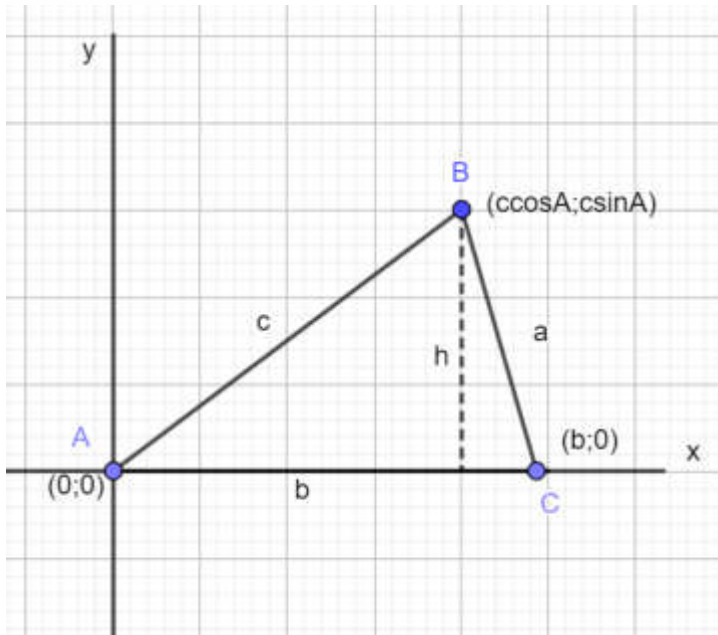
$$\Rightarrow \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \text{ by inverting the above equation)}$$

Thus, in $\triangle ABC$, the sine rule is given by:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The area and sine rule are taught in the third term of grade 11. Learners are given an opportunity to use the area rule to determine the area of any triangle; the sine rule is used to find missing sides or angles in triangles. However, teachers may ask learners what happens when you have two sides and the angle included by these two sides or when you are given three sides of a triangle? Are they able to find the missing angles or sides using the sine rule?

This, then, leads to the other important rule, introduced just after the sine rule in grade 11, which is the cosine rule.



Now $BC^2 = a^2$

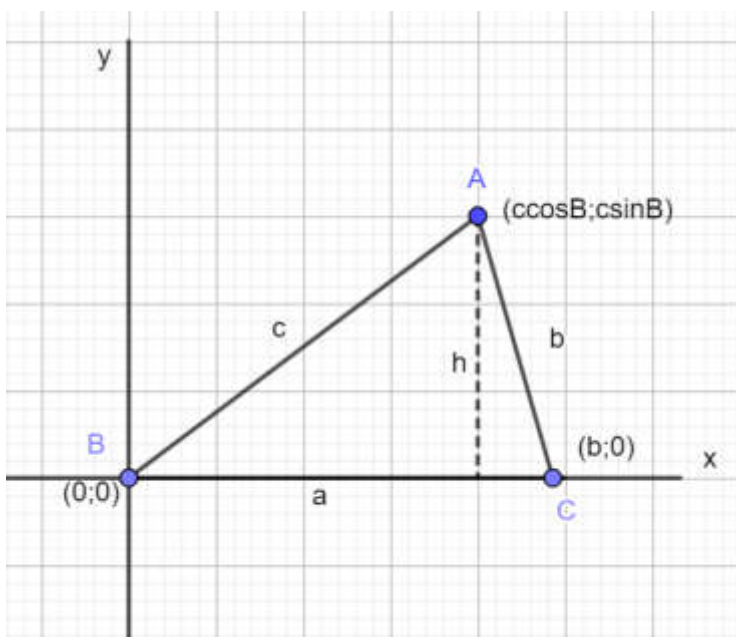
So using the distance formula to find BC we have

$$a^2 = (ccosA - b)^2 + (csinA - 0)^2$$

$$\Rightarrow a^2 = c^2 \cos^2 A - 2bccosA + b^2 + c^2 \sin^2 A$$

$$\Rightarrow a^2 = c^2 (\cos^2 A + \sin^2 A) - 2bccosA + b^2 \quad (\text{now } \cos^2 A + \sin^2 A = 1)$$

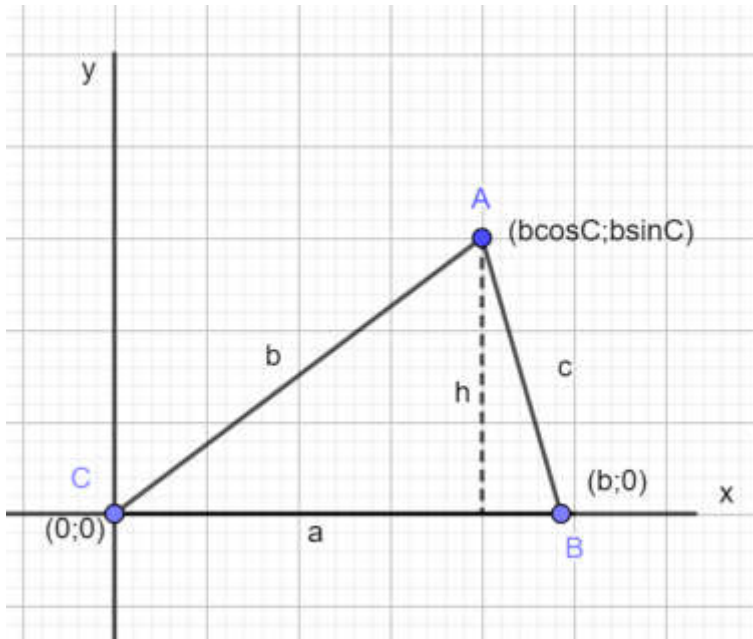
$$\Rightarrow a^2 = b^2 + c^2 - 2bccosA$$



With B at (0,0) we can use the distance formula to prove that:



$$b^2 = a^2 + c^2 - 2ac \cos B$$



With C at (0;0) we can use the distance formula to prove that

$$c^2 = a^2 + b^2 - 2ab \cos C$$

We can now write the cosine of a specific angle of a triangle in terms of the sides of that triangle.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Rightarrow \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$\Rightarrow \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\Rightarrow \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Thus, the cosine rule is useful to solve triangles when two sides and the included angle are given or when three sides are given.

Some points to consider:



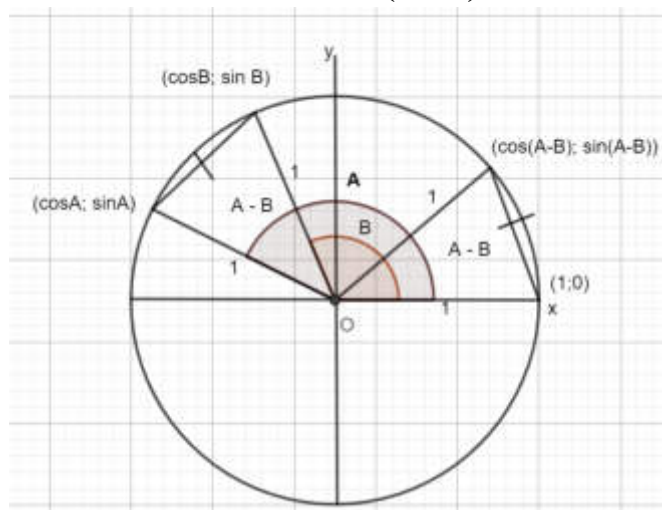
- When two sides and the included angle are given, we use the cosine rule to find the side opposite the given angle. Thereafter, the sine rule can be used to find one of the two remaining angles; the last angle is then found by using the angle sum of the triangle which is 180°
- When three sides are given, we use one of the cosine formulas, which are in terms of the angles, to find one angle. Thereafter, the sine rule could be used to find the second angle, and the angle sum is used for the third angle.

Compound angle identities

Compound angle identities are introduced in the first term of grade 12.

The identity below is first proved and then the other identities derived from this identity.

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$



We draw a unit circle (radius = 1 unit). We mark of angles A and B in the second quadrant with angle $A > \text{angle } B$. We are able to see angle $(A - B)$ in the diagram, which is in an isosceles triangle, in the second quadrant. We, then, this isosceles triangle in the first quadrant, with one side on the positive x-axis. We derive the identity for $\cos(A-B)$ as follows:

$$(\cos(A-B) - 1)^2 + (\sin(A-B) - 0)^2 = (\cos A - \cos B)^2 + (\sin A - \sin B)^2$$

$$\Rightarrow \cos^2(A-B) + 2\cos(A-B) + 1 + \sin^2(A-B)$$

$$= \cos^2 A - 2\cos A \cos B + \cos^2 B + \sin^2 A - 2\sin A \sin B + \sin^2 B$$

$$\Rightarrow 2\cos(A-B) + 1 + \cos^2(A-B) + \sin^2(A-B)$$

$$= -2\cos A \cos B - 2\sin A \sin B + \cos^2 A + \sin^2 A + \cos^2 B + \sin^2 B$$

$$\text{NB: } (\cos^2(A-B) + \sin^2(A-B) = 1; \cos^2 A + \sin^2 A = 1; \cos^2 B + \sin^2 B = 1)$$

$$\text{So we have: } -2\cos(A-B) + 1 + 1 = -2\cos A \cos B - 2\sin A \sin B + 1 + 1$$

$$\Rightarrow \cos(A-B) = \cos A \cos B + \sin A \sin B$$



Using this identity we are able to deduce the other identities.

From $\cos(A-B) = \cos A \cos B + \sin A \sin B$, we let $B = -B$

then we have:

$$\cos(A - (-B)) = \cos A \cos(-B) + \sin A \sin(-B)$$

$$\Rightarrow \cos(A+B) = \cos A \cos B - \sin A \sin B$$

From $\cos(A+B) = \cos A \cos B - \sin A \sin B$ we have

$$\sin(A - B) = \cos(90^\circ - (A-B)) = \cos((90^\circ - A) + B)$$

$$\cos((90^\circ - A) + B) = \cos(90^\circ - A) \cos(-B) - \sin(90^\circ - A) \sin(-B)$$

$$\Rightarrow \sin(A - B) = \sin A \cos B - \cos A \sin B$$

In $\sin(A - B) = \sin A \cos B - \cos A \sin B$, we let $B = -B$

So we have $\sin(A - (-B)) = \sin A \cos(-B) - \cos A \sin(-B)$

$$\Rightarrow \sin(A+B) = \sin A \cos B + \cos A \sin B$$

FINDINGS AND DISCUSSION

This study traces the mathematical knowledge in the curriculum that is required in the introduction of trigonometry to grade 10 learners and some key trigonometric content in grades 11 and 12. It is important for teachers to carefully plan their lessons in trigonometry and other content areas so that learners are able to relate their previous mathematical knowledge and skills, learnt in earlier grades to new mathematical knowledge and skills.

Thus, it is now opportune to respond to the research sub-questions, and consequently, the research question for this study. In this regard, the previous or prior mathematical knowledge and grades are stated here:

- The **plotting of points in the Cartesian plane** in the first, second, third and fourth quadrants; and joining points on a straight line. This is done in grades 8 and 9 when the Cartesian plane is introduced when learners join the points in their drawing of straight lines. In way, there are able to see for themselves that when they draw the straight lines, they see that there is some “**distance**” between the points.
- The **theorem of Pythagoras** which is first introduced in grade 8 when the properties of triangles are introduced. There is further consolidation of the theorem of Pythagoras in grade 9.
- When the **basic trigonometric ratios** are introduced early in grade 10, the right-angled triangle is used as the starting point. Here the three basic ratios and their reciprocals are introduced using the sides of the triangle. Learners should be familiar with the **ratio concept** as it was part of the grades 7 and 9 curriculum. Learners are then introduced to trigonometric ratios in the second, third and fourth quadrants, where they express the



angles in terms of the angle in first quadrant and simplify various trigonometric ratios, at a basic level in grade 10, and at a more advanced level in grades 11 and 12.

- In grade 10, learners are introduced to the **distance formula** in a new content area, Analytical Geometry. In Analytical Geometry, points are drawn to form figures such as triangles, squares, rectangles and parallelograms. Learners are able to calculate the distance between two points, find the midpoint of different lines, calculate missing points and make other discoveries. However, one of the key parts of Analytical Geometry that is applicable to Trigonometry is the distance formula which is used in the **derivation of area/sine/cosine** rules in grade 11 and **the compound angle identities** in grade 12.

CONCLUSION

Trigonometry is a key content area in the mathematics curricula in South Africa and other countries. This study traced the beginning of trigonometry and some of its key results to other topics which were done in the earlier grades, as well as early in grade 10 when Trigonometry is introduced for the first time. As indicated earlier in this study, learners tend to struggle with Trigonometry, possibly, due to its dependency on other branches of Mathematics. Thus, teachers have to take extra care when planning and preparing their lessons.

The recognition of the prior mathematical knowledge and skills shows the integrated and deductive nature of mathematics, where content from some sections of the work is used in the development of other sections of the work, as well as more advanced work, which is also the case in Mathematics related modules at higher education institutions.

If this is done carefully and in a structured manner, then learners' cognitive load is likely to be well-managed, and less stressful, possibly, leading to better conceptual understanding on their part, and improved performances.

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A COMPARATIVE STUDY OF TWO GRADE 6 TEXTBOOKS' PRESENTATIONS OF THE REPRESENTATION MODES AND SUB-CONSTRUCTS OF FRACTIONS

Duduzile Sibiya and Anthony A Essien

University of the Witwatersrand

This qualitative study compares the presentation of fraction exercises in two Grade 6 mathematics textbooks, Premier and Platinum. This study investigates the affordances for learning and their presentation of fraction exercise problems in Premier and Platinum, using sub-constructs and modes of representations. From the study, the findings reveal that Premier textbooks have more range in their coverage of sub-constructs and modes of representation and thus offer affordances for learning in developing learners' understanding of fractions. Platinum, however, offers learners the opportunity to explore reasoning and explanations on the topic of fractions.

INTRODUCTION

Fraction proficiency is a significant foundation for advanced mathematical topics (Bruce, Chang, & Flynn, 2013). Cady, Collins, and Hodges (2015) identified proportionality, algebra, and probability to be some of the few topics that demand fraction proficiency. A deep conceptual understanding of fractions is crucial for success in secondary school mathematics. In the first author's experience as a Grade 6 Mathematics teacher, anecdotal evidence reveals that learners face difficulties when they are faced with a task that requires them to formulate connections from fractional knowledge to transitioning between different modes of representations. Mathematics textbooks have been identified as exceedingly impactful classroom teaching and learning tools (Okeeffe, 2013; Mellor, Clark & Essien, 2018; Santoso, 2020; Tanto & Santoso, 2020; Rezat, Fan, & Pepin, 2021). Mathematics teachers rely on textbooks to a large degree in South Africa and according to Shield and Dole (2012), this is because textbooks are a very important tool for guiding classroom practices for teachers.

Research shows that the ability to move fluently between representations; manipulatives, pictures, symbols, verbal explanations, and real-life contexts is a key indicator of fraction understanding (Lesh, Doerr, Post, & Zawojewski, 2003; Siegler & Pyke, 2013). However, learners often struggle with this and with sub-constructs beyond part-whole, which receives the most focus in the curriculum. Kieran (1988) identified five essential sub-constructs: part-whole, measure, ratio, operator, and quotient, all of which are critical for building conceptual understanding (Tsai & Li, 2017).

The five sub-constructs of fractions are essential in developing learners' understanding of fractions. Therefore, the emphasis must be on all the various sub-constructs of fractions as they play an important role in the development of interconnectedness and not just one (Tsai & Li, 2017). The Curriculum and Assessment Policy Statement (2012) stipulates that one of the first



few mathematical skills learners should possess is the skill to fluently translate between representations, and “the Mathematical Education of Teachers identifies these multiple subconstructs of fractions as essential to learning for classroom educators” (Bruce et al. 2013, p. 10).

Valverde, Bianchi, Wolfe, Schmidt and Houang (2002) argue that textbooks are key to understanding learning opportunities. The purpose of this study is to analyse the affordances the selected two Grade 6 textbooks offer to learners’ development of fraction proficiency in terms of how modes of representations and sub-constructs of fractions are presented. This study is informed by the following research questions:

- How is the concept of fractions presented in Grade 6 *Premier Mathematics* and *Platinum* textbooks, taking the sub-construct of fractions into consideration?
- What affordances for learning about common fractions are offered in Grade 6 *Premier Mathematics* and *Platinum* textbooks?

LITERATURE REVIEW

Sub-constructs and Models in Fraction Understanding

To develop a conceptual understanding of rational numbers, learners need to have a strong understanding of fractions which includes visual representations, quantities represented by fractions, and their relationships (Way, 2011). Literature suggests that it is important to represent mathematical ideas in multifaceted modes of representation (including real contexts, physical models, pictures, verbalizations, and written symbols (Mainali, 2001; Lesh et al., 2003). Way (2011) suggests that children learning fractions need to be provided with the opportunity to explore fractions in different contexts, representations and engage with manipulatives such as cutting out paper strips, drawing shapes, and sharing items.

Research suggests that robust fraction knowledge requires learners to re-organise their numerical knowledge, as this is not gained through experience as it is with whole numbers (Charalambous & Pitta-Pantazi, 2005; Siegler, Carpenter, Fennell, Geary, Lewis, Okamoto, Thompson & Wray, 2010). Kieren (1976) introduced the concept of fraction sub-constructs and argued that to understand the general concept of fractions, it is essential to gain a deeper understanding of each and their convergence (Lazića, Abramovichb, Mrđaa & Romanoc, 2017). The development of the sub-constructs of fraction later suggested a fifth sub-construct: part-whole (Behr et al., 1983; & Kieren, 1995) which is connected to partitioning. Roche and Clarke (2011) propose that it is essential that teachers prioritise the various sub-constructs of fractions in instructions to develop learners’ fraction proficiency.

Naiser, Wright, & Capraro (2014), contend that teaching using manipulatives gives teachers an insightful understanding that learners learn differently, learners can get to the same answer using different ways and manipulatives to assist in mediating these various learning needs. Manipulatives in the classroom combined with real-life problems interest learners in solving fraction problems (Naiser et al., 2014).



The National Council of Teachers of Mathematics in 2000 published the Principles and Standards for School Mathematics' document that included representations as one of the essential elements in the mathematics curriculum that needed to be emphasized during teaching and learning of. The concept of representation can take internal and external forms (Mainali, 2001). However, for this study, the focus is on external modes of representations, defined by Lesh et al., (1987) as manipulatives, pictures, oral language, written symbols, and real-life situations. Deep understanding of the development of fraction proficiency is embedded in the transactional model as it enables learners to make connections.

Instructional Design and Representation in Fraction Teaching

Textbooks have been regarded as important instruments that facilitate and guide teachers (Johannson, 2005; Sunday, 2014, Haavold, Storaas, Johnsen, Strand & Heimstad, 2025) because of their influence on classroom practices (Mellor et al., 2018). As a result, textbooks warrant close examination as they impact learners' content knowledge and views of mathematics, and influence teachers' content and pedagogical approaches (Cady et al., 2015).

Research revealed that East Asian countries outperform the United States in the performance of mathematics in almost all mathematics topics including fractions (Zhou, Peverly, & Xin, 2006; Watanabe, 2007; Charalambous et al., 2010; Son, 2011). A comparative analysis study of Japanese textbooks has shown that there is a clear predominance of linear representations of fractions in the form of rulers and number lines to emphasize that fractions are numbers (Bruce et al., 2013). The five fraction sub-constructs are emphasized in East Asian textbooks (Watanabe, 2007). Korean textbooks have been documented to also prioritize sub-constructs, introducing the general concept earlier (Grade 3) and developing it throughout primary school (Son, 2011).

Taiwanese textbooks have the most effective and comprehensive fraction instruction program, the constantly highly ranked students in international mathematics assessments are Chinese students from Taiwan (Bruce et al., 2013). There is a board application of fractions representation in Taiwanese textbooks depending on a particular problem and/or subconstruct of fractions. Asian textbooks, unlike North American textbooks, emphasize all fraction sub-constructs (Son, 2011; Charalambous et al., 2010; Watanabe, 2007), using varied graphical representations tailored to problem contexts (Bruce et al., 2013), rather than relying heavily on part-whole models like the "pizza model".

THEORETICAL PERSPECTIVES

The transactional model is relevant to this study because of the emphasis it places on the importance of translating between different modes of representation and making meaningful connections. This is essential for the development of fraction proficiency (Cady et al., 2015). The transactional model suggests that multiple representations and transitions between them facilitate mathematical connections and understanding (Gagatsis & Shikalli, 2004). We applied this model to investigate various modes of representation in textbooks focusing on how they provide opportunities for transitioning between different modes.



The transactional model for this study is complimented by Kieren's (1988) notion of sub-constructs. Fraction proficiency requires understanding all five interrelated sub-constructs, which represent different ways fractions can be interpreted (Behr, 1983; Kieren, 1995). The transactional model and the theoretical model of sub-constructs of fractions will be used to investigate textbook opportunities for transitioning between different modes of representation and exploring the interrelations between fraction sub-constructs.

The transactional model highlights the relationships between different elements of mathematical representation. The model highlights the importance of using and translating between modes of representation; manipulatives, pictures, written symbols, verbal symbols, and real-life contexts. The model shows that deep mathematical understanding develops when learners can move flexibly between these representations, strengthening their conceptual knowledge and problem-solving abilities. In Figure 1A, the blue double-headed arrows represent the possible transitions between representations, illustrating how learners can move between different modes. This visual emphasis reinforces the model's strength in promoting flexible and interconnected mathematical understanding.

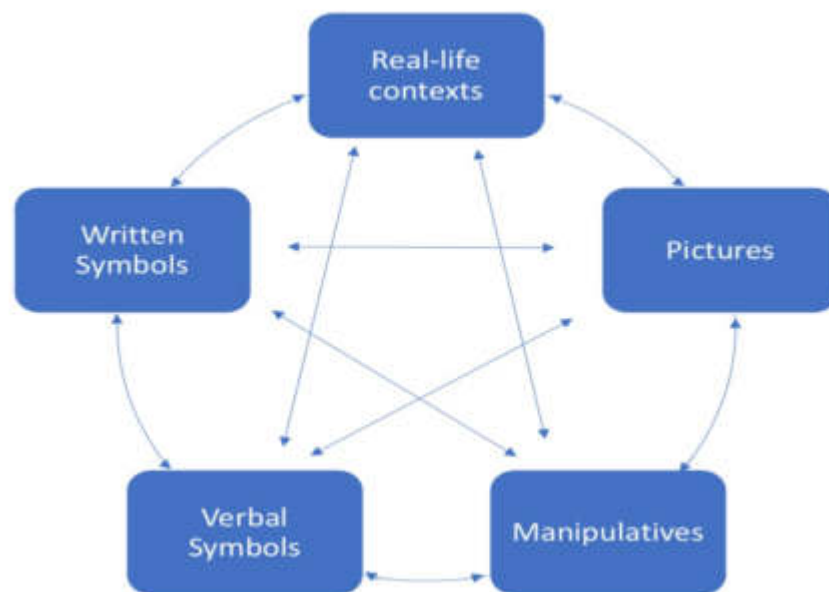


Figure 1A: Lesh et al., (2003) Transactional Model

Kieren's (1988) notion of sub-constructs of fractions suggests that learners might obtain insight into the essence of fractions. Fraction proficiency requires comprehension of all five sub-constructs of fractions (Behr, 1983; Kieren, 1995). Part-whole lays the foundation for comparing, ordering, and equivalence. Measure is crucial for additive operations and understanding fractions on a number line. Ratio is fundamental for proportionality and relationships between quantities. Additionally, the operator sub-construct involves fractions acting on a set to enlarge or shrink it, while the quotient sub-construct interprets fractions as division, such as sharing objects among groups.

This study applies the transactional model to examine representations and transitions specifically in the context of fractions. Kieren's (1988) five sub-constructs will be merged with



the transactional model to investigate how different interpretations are represented in various modes, facilitating meaningful connections for learners using textbooks.

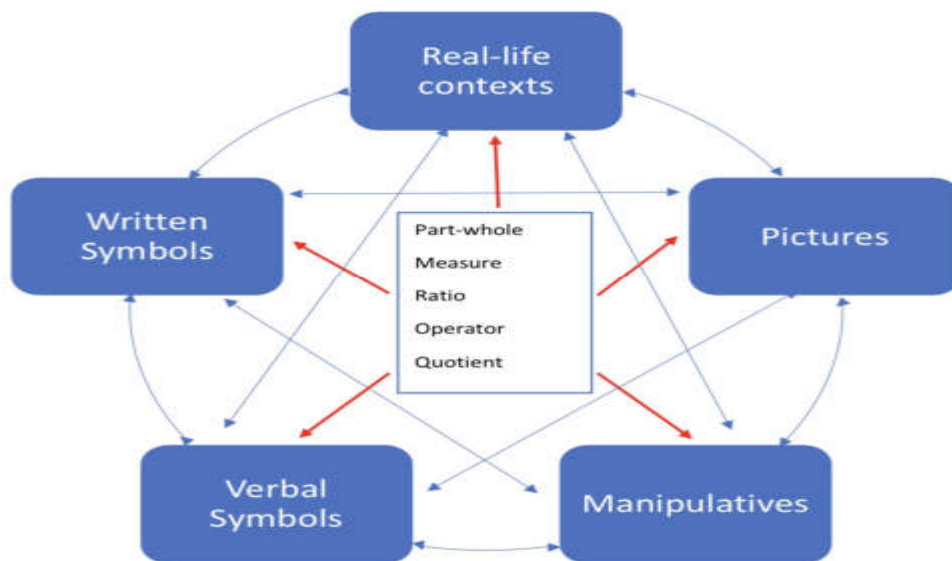


Figure 1B: Merged framework of the two theoretical perspectives.

Figure 1B shows a merged framework of the two theoretical perspectives. The framework features five modes of representation with blue double-headed arrows indicating potential transitions between them. The five sub-constructs are positioned at the centre, with red arrows extending to the modes of representations, indicating possible representation formats. The more various modes of representations a sub-construct can be presented in, indicates a deeper understanding of fractions. Successfully representing fraction sub-constructs requires a certain level of fractional knowledge. The merged framework for the study, along with Table 1, facilitates the investigation of the transitions between modes of representation and the representation of sub-constructs.

RESEARCH METHODOLOGY

Selection of Textbooks and Focus Topic

The two textbooks analysed in this study are *Premier Mathematics* and *Platinum Mathematics*, both approved by the Curriculum and Assessment Policy Statement (CAPS). They were purposively selected for their alignment with curriculum objectives, particularly problem-solving and engagement with multiple representations. Fractions were chosen as the focus due to their well-documented difficulty in primary mathematics. Research shows that learners often struggle with fractions because traditional curricula limit the use of varied representations and sub-constructs (Cady et al., 2015; Tsai & Li, 2017).

Analytical Framework and Procedure

A structured analytical framework in Table 1 was used to compare fraction content in both



textbooks, focusing on modes of representation and sub-constructs. Each question in the fractions section was examined and categorised by representation types such as symbolic, visual, verbal, or contextual and recorded to assess variety and frequency. Next, questions were coded based on the five sub-constructs of fractions: part-whole, ratio, operator, quotient, and measure (Kieran, 1988). A comparative analysis identified the presence or absence of each sub-construct in both textbooks. This approach allowed for a focused evaluation of the conceptual depth and learning opportunities offered for developing fraction proficiency.

ANALYTICAL FRAMEWORK

The transactional model and fraction sub-constructs were combined to create a framework in Table 1 to analyse how Grade 6 textbooks present fractions and what learning affordances they offer. The analysis focused on two areas: firstly, the use of different modes of representation to explore fraction concepts and secondly, the presence of the five key sub-constructs of fractions. The absence of any sub-construct may limit learners' understanding, while the appropriate use of multiple representations supports deeper comprehension.



Table 1: Definitions and rules of recognition of modes of representations and sub-constructs of fractions

Category	Rules of recognition	Example
Modes of representation		
1. Manipulatives Manipulatives come in different forms, they are physical objects that engage and encourage learners to hands-on learning.	A mode of representation is a manipulative in a textbook if: A question requires learners to make use of objects that can be grasped by hands and touched. e.g., fraction walls, number lines, etc.	Use the Fraction Chart to find the equivalent of $\frac{3}{4}$.
2. Pictures Pictures are a visual representation of a written or spoken mathematical concepts.	A mode of representation in a textbook is a picture if: A question requires a mathematical concept to be drawn in picture form a representation of a mathematical concept and not a decorative picture. e.g., hand sketches, tables, graphs, etc.	Draw a rectangle made up of 6 equal parts. Shade in $\frac{2}{6}$ of your rectangle.
3. Written Symbols Written symbols are structures that stand in the place of quantities and operational signs, instead of words, and mathematical notation (Hiebert, 1988).	A mode of representation in a textbook is a written symbol if: - A question requires to be expressed in a numerical symbol - A question requires the use and expression of operational signs and/or formula. e.g., $\frac{1}{2}$, $=$, 0,5, $+$, variables, etc.	Add and subtract the following fractions $\frac{8}{8} + \frac{2}{4} =$ $3\frac{3}{4} - \frac{1}{4} =$
4. Verbal Language Verbal language refers to words or phrases used to explain or define ideas in mathematics.	A mode of representation is verbal language in a textbook if: A question requires an explanation, discussion, or use of words or phrases are used to explain a mathematical concept or	With a pizza divided into 6 equal pieces, and you eat two pieces, how many thirds have you eaten? Explain.
5. Real-life context Refers to the use of objects and events that make use of real-life situations to make mathematical connections.	A mode of representation is a real-life context in a textbook if: A question uses real-life situations to make mathematical connections or requires learners to use imagination to create	In a class of 60 learners, $\frac{2}{5}$ were girls. How many were boys?



	life-like situations to solve mathematical questions.	
Five sub-constructs of fractions		
A. Part-whole Refers to the partitioning of a continuous quantity or a set of quantities into equal-sized pieces.	A sub-construct is recognised as part-whole in a textbook if: • A question is structured such that it requires learners to demonstrate the denominator as the total number of equal parts a whole or a set has been partitioned into and the numerator represents the number of parts that are shaded or spoken of the whole or a set. • A question requires learners to compare, arrange, add, or subtract fractions or find equivalent fractions because all these instructions are being applied to partitioned constituents.	In a circle divided into four equal parts, shade in $\frac{3}{4}$ of the circle.
B. Measure Refers to the measurement of an object's length, identifying a definite unit of measure, and then using the partitioned quantity repeated as well as dividing it into smaller parts.	A sub-construct is recognised as a measure in a textbook: • If a question requires the use of an object (a ruler or strip) or distance is subdivided into smaller equal partitions and is connected to the process of linear measurement.	Elmon's mother cut two pieces of ribbon to wrap a gift. One piece is $\frac{65}{100}$ of a meter and the other is $\frac{7}{10}$ of a meter. If mom wants the shortest ribbon, which one should she choose?
C. Ratio A numeric expression of a relationship between two quantities.	A sub-construct is recognised as a ratio in a textbook. • If a question requires the use of an expression to show a numeric relationship between two quantities.	In a box of crayons, Nomsa took three red crayons and Thabang took four green apples. What is the ratio of the red crayons to the green crayons?
D. Operator Is described as a fraction that can enlarge or reduce a quantity.	A sub-construct is recognised as an operator in a textbook if: • A question requires the use of a fraction to operate as a function on a quantity by either multiplying it or dividing it.	Find $\frac{1}{3}$ of 60. Or If Precious saved $\frac{2}{5}$ of her salary, which is R1800.



	It is usually in the form of $\frac{a}{b}$ of x, where $\frac{a}{b}$ is fraction and x is a quantity.	How much did she save?
E. Quotient Refers to the answer you get when dividing two numbers.	A sub-construct is recognised as a quotient in a textbook if: If a question requires two integers to be divided, a quotient is the result of a division of two numbers.	If 20 pens are shared amongst 10 learners, $\frac{20 \text{ pens}}{10 \text{ learners}} = 2 \text{ pens per learner.}$



Data Analysis

The comparison of the two Grade 6 textbooks, Premier and Platinum, reveals that while both textbooks share a similar structural format—organizing questions into exercises based on specific fraction sub-concepts—they differ significantly in the quantity and variety of content offered. A key distinction lies in the use of modes of representation: Premier includes 48 instances, while Platinum nearly doubles that with 93. This suggests that Platinum may offer richer learning opportunities through a wider variety of visual and conceptual tools, potentially supporting deeper understanding and engagement. However, it is important to note that a higher number of questions or representations does not necessarily equate to higher quality; the effectiveness of these materials will depend on how they facilitate the development of conceptual understanding through appropriate scaffolding, progression, and cognitive challenge. These elements will be further examined in the second level of analysis, which aims to determine the actual affordances each textbook provides for learners' understanding of fractions.

Levels of analysis

Level 1 of analysis: Sub-construct questions

The first analysis examines the sub-constructs in each textbook's questions to determine the learning affordances offered by each.

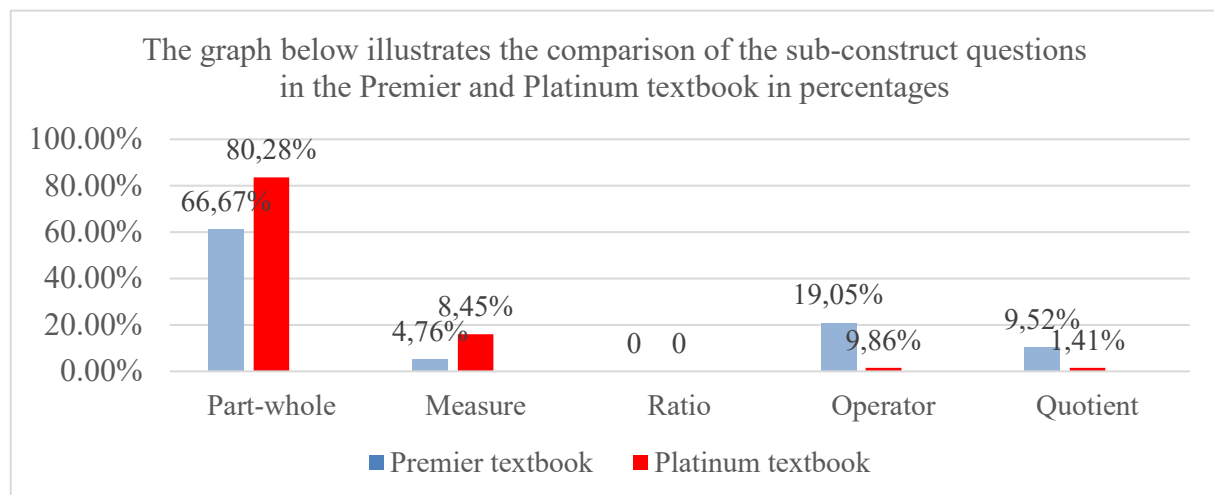


Figure 2A: The graph above illustrates the comparison of the sub-construct questions in the Premier and Platinum textbook in percentages.

According to the sub-construct data from the two textbooks illustrated in Figure 2A, there is no coverage of the ratio sub-construct. According to Charalambous et al., (2010), the coverage of all sub-constructs is important for the comprehension of advanced mathematics. Failure to cover all these sub-constructs, including ratio has been done in both textbooks and learners will have difficulties pursuing other concepts in mathematics (Bruce et al., 2013). There is a slight distribution of the sub-constructs of fraction in the Premier textbook compared to the Platinum textbook, even though in both textbooks there is an over-emphasis of the part-whole sub-



construct, and again Charalambous et al., (2010) suggest that this leads to difficulties when learners are supposed to understand fractions bigger than one. The Premier textbook covered measure, operator, and quotient at a slightly higher degree than Platinum, and as a result, learners are given the opportunity to explore the sub-construct to understand fractions at different interpretations, Platinum limits learners' accessibility to the interpretations of fractions and research (Bruce et al., 2013) indicates that learners need to be engaged in multiple sub-constructs to reinforce fraction understanding.

Level 2 of analysis: Modes of representation

This is the second part of the investigation, and it focuses on the modes of representation required to be used in the two textbooks.

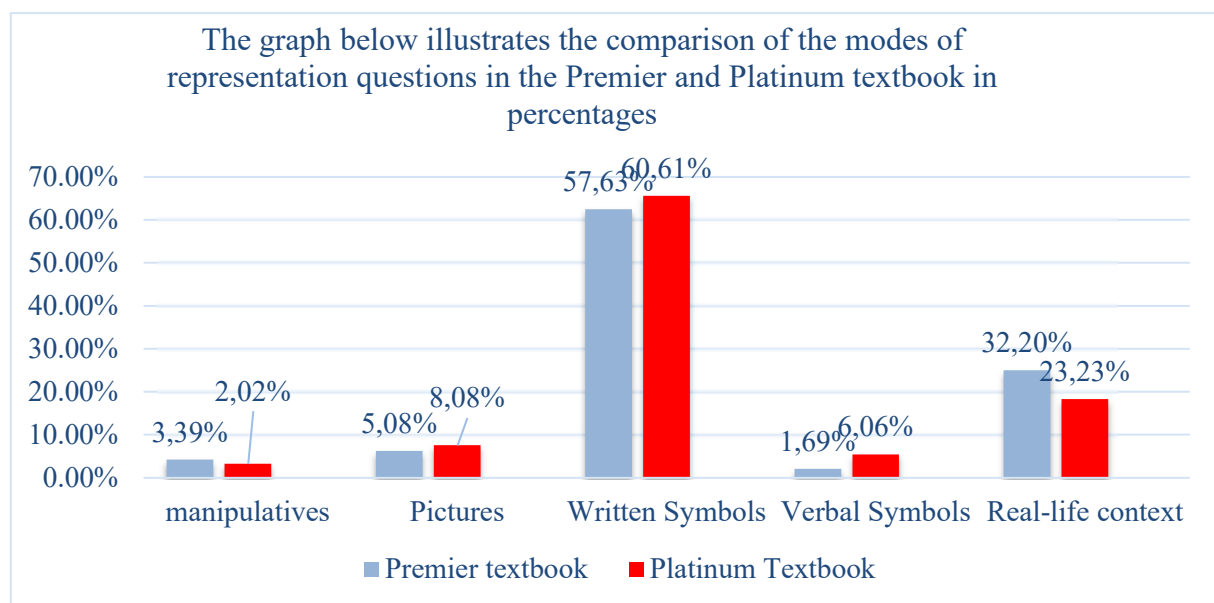


Figure 2B: The graph above illustrates the comparison of the modes of representation questions in the Premier and Platinum textbook in percentages.

As illustrated in Table 2B, the most used mode of representation in the Premier textbook is the written symbols, (57,63%) followed by real-life context (32,20%), pictures (5,08%), manipulatives (3,39%) and the least used mode of representation in Premier are verbal symbols (1,69%). In the Platinum textbook, the written symbols (60,61%) are the most used in this textbook as well, also followed by real-life context (23,23%), pictures (8,08%), verbal symbols (6,06%), and the least used mode of representation in the Platinum textbook's questions are manipulatives (2,02%).

Cross-Case Analysis

The analysis suggests that the Premier textbook presents exercise questions in a manner that provides learners with the opportunity to explore common fractions through different interpretations (sub-constructs) compared to Platinum. This suggests that learners using Premier have greater affordances to learn common fractions in-depth, as it enables learners to explore sub-constructs across a slightly broader range than Platinum. Even though both



textbooks did not use ratio, Premier still showed a greater range than Platinum, which focused a lot on part-whole questions. Research (Roche & Clarke, 2011) has suggested that emphasizing on different sub-constructs improves understanding. Premier has greater affordances than Platinum because it has put emphasis across sub-constructs (excluding ratio). Lastly, Crammer et al., (2010) have suggested that modes of representation and the ability to translate between them is vital in understanding mathematical ideas for learners in primary school. When learners understand a concept, transitions between representations are possible. Premier offers a slightly broader range of representation modes than Platinum, suggesting it better supports learners' development of fraction representation skills and overall mathematical learning.

DISCUSSION AND CONCLUSION

This comparative analysis revealed that while both textbooks are designed for Grade 6 fraction instructions, they employ a distinct presentation of the fraction content. The Premier textbook approach is marked by a broader scope of sub-construct coverage and more variation was used for modes of representation. Premier's textbook approach has the potential to foster a deeper and more multifaceted understanding of fractions. The deliberate integration of real-life contexts in Premier further bridges the gap between abstract mathematical concepts and practical applications, potentially enhancing students' ability to transfer knowledge. Conversely, Platinum's emphasis on the part-whole sub-construct, coupled with a more restricted range of representations, may limit students' exposure to the full spectrum of fraction understanding. Critically, the omission of the ratio sub-construct in both textbooks raises concerns. Ratio is pivotal for developing proportional reasoning, a crucial precursor to advanced mathematical competence.

Research (Tsai & Lai, 2017) has revealed the importance of representing mathematical concepts in different modes of representation to deepen learners' understanding and this is done by translating from one mode to another. The more the opportunities of translations being afforded to learners will result in deeper understanding. It is no surprise that looking at the graph above, written symbols are the most used mode of representation in both textbooks, considering that mathematics uses a lot of these. Both textbooks covered the use of the real-life context in questions, sufficient use of real-life content enables learners to see the application of mathematics in real-life situations. The use of pictures according to the graph is illustrated as present but at a low percentage, research (Kasmaienezhadford, Pourrajab, & Rabbani, 2014) indicates the significance of pictures textbooks as contributors to learners' motivation and creativity, making them creative thinkers. Manipulatives are the least represented mode of instruction in textbooks, yet they play a foundational role in helping learners move from concrete experiences to understanding abstract mathematical concepts (Carbonneau, Marley, & Selig, 2013).

Therefore, teachers must be acutely aware of these disparities when selecting or implementing these textbooks. To ensure a comprehensive and balanced exploration of fraction concepts, it is strongly recommended that teachers supplement their instruction with materials that address



the identified limitations. Specifically, incorporating activities that explore ratio and proportional reasoning, as well as providing diverse representational experiences, will be essential. Future research should delve into the long-term impact of these differing pedagogical approaches on student learning outcomes, focusing on how sub-construct coverage and representational diversity influence the development of robust fraction proficiency. This research will be vital for informing evidence-based textbook selection and instructional practices.

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INVESTIGATING PRESERVICE TEACHERS' KNOWLEDGE OF UNIVERSAL DESIGN FOR LEARNING IN 21ST CENTURY MATHEMATICS CLASSROOMS

**Felix O. Egara¹, Moeketsi Mosia¹, Mirriam M. Moleko², Puleng D.
Motseki³, Themba M. Mthethwa⁴, Mathomo M. Moila² and
Nomathamsanqa A. Wongo²**

¹University of the Free State (UFS), ²College of Education (CEDU), ³University of South Africa (UNISA), ⁴Mangosuthu University of Technology (MUT)

This study investigates preservice mathematics teachers' knowledge of Universal Design for Learning (UDL) at the University of Nigeria, Nsukka (UNN). A descriptive survey design was employed to assess participants' understanding, perceptions, and challenges regarding UDL principles. The study population was comprised of all third- and fourth-year preservice mathematics teachers (PMT) in the Department of Science Education, Faculty of Education. Since the total number of students in these levels was relatively small (n=16), a census sampling technique was adopted, ensuring that all eligible participants were included in the study. Data were collected using a structured questionnaire, the Preservice Teachers' Knowledge of Universal Design for Learning in Mathematics Classrooms (PTK-UDL), developed based on the UDL framework and adapted from prior studies. The instrument underwent expert validation and achieved a reliability coefficient of 0.84 using Cronbach's alpha. Descriptive statistics, including mean and standard deviation, were used for data analysis. Findings revealed that preservice mathematics teachers exhibit low knowledge of UDL, with a limited understanding of its core principles, differentiation, and application in diverse classrooms. Their extent of UDL application in teaching was also low, though they demonstrated relatively better use of multiple representations. Key challenges hindering UDL integration included a lack of awareness and training, limited resources, and large class sizes. These findings underscore the need for targeted interventions, such as UDL-focused coursework and professional development programmes, to enhance inclusive teaching practices.

Keywords: Universal design for learning (UDL), inclusive education, preservice mathematics teachers, mathematics instruction, teacher training

INTRODUCTION

Mathematics is a fundamental discipline that fosters critical thinking, problem-solving, and logical reasoning skills essential for success in various fields (Okeke et al., 2022, 2023, 2025). However, many students struggle with mathematical concepts due to differences in cognitive processing, learning preferences, and prior knowledge (Mosia & Egara, 2024; Mosimege et al., 2024; Nzeadibe et al., 2019, 2020, 2023). Addressing these challenges requires instructional strategies that ensure inclusivity and accessibility for all learners. The 21st-century classroom is characterised by diverse learner needs, necessitating teaching approaches that accommodate this variability (Meyer et al., 2014). The Universal Design for Learning (UDL) offers a



framework for inclusive education. UDL is an educational framework aimed at making learning accessible to all students, including those with diverse abilities, learning styles, and backgrounds. It is based on three core principles: providing multiple means of engagement (to motivate learners), multiple means of representation (to present content in different ways), and multiple means of action and expression (to allow students to show what they know in different forms). By embedding these principles into curriculum design, teachers can create more inclusive and flexible learning environments that accommodate individual learner needs (CAST, 2018; Griful-Freixenet et al., 2021; Lambert et al., 2023).

THEORETICAL FRAMEWORK

This study is underpinned by Inclusive Education Theory, as articulated in the Salamanca Statement by UNESCO (1994), which advocates for education systems that accommodate the diverse needs of all learners. The theory emphasizes equitable access, participation, and achievement in mainstream education settings, particularly for students who have traditionally been marginalized. The UDL, as a flexible and proactive instructional framework, aligns with these principles by promoting accessibility and inclusivity through multiple means of engagement, representation, and expression. Investigating preservice mathematics teachers' knowledge of UDL is thus situated within this broader theoretical commitment to inclusive education, as it provides insight into how future educators are being prepared to support diverse learners in 21st-century mathematics classrooms. This theoretical lens informs the formulation of research questions, guides the interpretation of findings, and provides the conceptual foundation for understanding how future mathematics educators are prepared to support diverse learners in 21st-century classrooms.

LITERATURE REVIEW

Globally, there has been a growing emphasis on equipping preservice teachers with the knowledge and skills to implement UDL in their teaching practices (Lowrey et al., 2019; Mavrovic-Glaser, 2017). However, research on UDL implementation in African contexts remains scarce, particularly concerning preservice mathematics teachers (Adigun, 2021; Sholanke et al., 2018). This gap raises important questions about whether future educators in Nigeria are adequately prepared to foster inclusive learning environments in 21st-century mathematics classrooms.

Studies on preservice teachers' knowledge and UDL application highlight its potential benefits and implementation challenges. Almutairi and Alsuwayl (2023) found that while elementary school teachers in Saudi Arabia were aware of UDL principles, they struggled with practical application due to insufficient training. Similarly, Lowrey et al. (2019) emphasised the importance of structured professional development in enhancing teachers' ability to integrate UDL effectively. In a systematic review, Rusconi and Squillaci (2023) concluded that ongoing training is crucial for embedding UDL in teacher education programmes.

In mathematics education, UDL has been shown to improve differentiated instruction. Arslan and Yavuzarslan (2020) found that integrating UDL strategies, such as multiple representations



and interactive tools, enhanced student engagement and comprehension. Similarly, Yılmaz (2022) reported that preservice mathematics teachers exposed to inclusive teaching approaches, including UDL, were better equipped to address diverse learning needs.

Despite these advantages, research indicates that many preservice teachers lack confidence in implementing UDL. Griful-Freixenet et al. (2021) found that while teachers appreciated the inclusive potential of UDL, they often faced institutional barriers to its application. Lambert et al. (2023) reinforced this finding, noting that inadequate professional training hindered widespread adoption. Additionally, Vitelli (2015) identified inconsistencies in how UDL is incorporated into general education teacher training, further complicating its effective implementation.

Challenges in integrating UDL extend beyond teacher education programmes. Melhem and Al-Rashid (2023) found that teachers working with students with learning difficulties recognised UDL's benefits but faced obstacles such as limited resources and institutional support. Similarly, Surya and Vishnubhotla (2024) reported that Indian teachers in South Carolina identified professional development as a key factor in improving UDL adoption.

In Nigeria, research on UDL remains limited. Adigun (2021) highlighted the lack of explicit UDL training in African teacher education programmes, leading to gaps in inclusive instructional practices. Sholanke et al. (2018) further noted that many preservice teachers in Nigeria are unaware of inclusive teaching frameworks, making it difficult for them to adapt their instructional strategies to support diverse learners. Given these gaps, it is crucial to investigate the awareness and preparedness of preservice mathematics teachers at the University of Nigeria, Nsukka (UNN), in implementing UDL principles. Understanding their level of knowledge and challenges will provide valuable insights for improving teacher training programmes and fostering inclusive mathematics education in Nigeria.

This study aimed to assess the knowledge and preparedness of preservice mathematics teachers at UNN regarding UDL principles. Given the increasing emphasis on inclusive education, understanding how future mathematics educators perceive and integrate UDL is essential for enhancing mathematics instruction. The findings will contribute to discussions on improving teacher training programmes to ensure that preservice teachers are well-equipped to support diverse learners.

Research Questions

This study seeks to answer the following research questions:

1. What is the level of UDL knowledge among preservice mathematics teachers at UNN?
2. To what extent do preservice mathematics teachers understand the application of UDL principles in the 21st mathematics classroom?
3. What challenges do preservice mathematics teachers face in integrating UDL into their teaching practices?



METHODOLOGY

Research Design

This study adopted a quantitative descriptive survey design to investigate the level of knowledge, understanding and challenges of UDL among PMT at UNN. A descriptive survey design was chosen because it allows data collection on participants' knowledge, perceptions, and challenges regarding UDL without manipulating variables (Creswell & Creswell, 2018).

Population of the Study

The target population comprised all PMT enrolled in the Department of Science Education, Faculty of Education, UNN. This included 21 undergraduate students in their third and fourth years of study, as they have completed core education courses and are preparing for teaching practice.

Sample and Sampling Technique

The study adopted a census sampling technique, as the total population of PMT in the 300 and 400 levels at UNN was relatively small. The initial target population consisted of 21 preservice teachers (16 from the 300 and 5 from the 400 levels). However, due to non-responses from three 300-level and two 400-level students, the final sample comprised 16 preservice mathematics teachers, representing 76.2% of the total population. This approach ensured a broad representation of students' knowledge of UDL while acknowledging non-response limitations.

Instrumentation

The instrument utilised for data collection for this study is the *Preservice Teachers' Knowledge of Universal Design for Learning in Mathematics Classrooms (PTK-UDL)*. The questionnaire was developed based on the UDL framework (CAST, 2018) and adapted from previous studies on teacher knowledge of inclusive education (Courey et al., 2013; Rao et al., 2014). It comprised three sections designed to capture relevant information from the participants.

The first section, Section A, focused on demographic information, including participants' gender, academic level, and previous exposure to UDL concepts. Section B assessed PTK of UDL principles, specifically examining their understanding of the three core components: multiple means of engagement, representation, and action/expression. The final section, Section C, explored the challenges faced by preservice teachers in implementing UDL-based teaching strategies in mathematics classrooms.

The questionnaire employed a 5-point Likert scale ranging from Strongly Agree to Disagree Strongly, enabling the measurement of preservice teachers' level of awareness and understanding of UDL principles. This design facilitated the collection of quantitative data to analyse participants' knowledge and perceptions regarding UDL.



Validity and Reliability of the Instrument

The instrument was subjected to face validity by three experts in Mathematics Education and Measurement and Evaluation units at the Department of Science Education, UNN. Their feedback was incorporated to ensure that the items accurately measured preservice teachers' knowledge of UDL. A pilot study was conducted with 30 preservice teachers from another university with similar characteristics to establish reliability. The reliability coefficient was determined using Cronbach's alpha, yielding a reliability score of 0.84, which indicates high internal consistency.

Data Collection Procedure

The questionnaire was administered in person with the assistance of a research assistant who provided necessary clarifications to the participants. Participants were assured of confidentiality and anonymity, and their participation was voluntary. The data collection process spanned two weeks to allow all participants to respond.

Data Analysis

The collected data were analysed using descriptive statistics, specifically mean (M) and standard deviation (SD), to assess preservice teachers' knowledge of Universal Design for Learning (UDL). The results were presented in tables and charts for clarity, ensuring a clear interpretation of the findings. Data analysis was conducted using SPSS (Statistical Package for the Social Sciences) version 28. Different interpretation scales were used based on the nature of the variables. Knowledge was assessed using a 'knowledge scale' (Very Low to High Knowledge) to measure conceptual understanding. At the same time, applications and challenges were evaluated using an 'extent scale' (Very Low to High Extent) as they measure implementation frequency. For knowledge, mean scores of 1.00–1.99 indicated Very Low Knowledge, 2.00–2.99 Low Knowledge, 3.00–3.99 Moderate Knowledge, and 4.00–5.00 High Knowledge. For the extent of application and challenges, 1.00–1.99 signified Very Low Extent, 2.00–2.99 Low Extent, 3.00–3.99 Moderate Extent, and 4.00–5.00 High Extent. These cut-off points provided a structured approach to determining the extent of PMTs' knowledge, awareness and understanding, and challenges of UDL at the University of Nigeria, Nsukka.

Results

This section presents findings on preservice mathematics teachers' UDL knowledge, their understanding of its application, and challenges in implementation at UNN.



Level of UDL Knowledge among Preservice Mathematics Teachers

Table 1: Level of Knowledge of UDL Among Preservice Mathematics Teachers

Item	Item Statement	Mean	Std. Deviation	Decision
1	I clearly understand what Universal Design for Learning (UDL) is.	2.13	.81	Low knowledge
2	I am familiar with the three core principles of UDL (multiple means of engagement, representation, and expression).	2.44	.81	Low knowledge
3	I understand how UDL can support diverse learners in a mathematics classroom.	2.31	.48	Low knowledge
4	I can define and differentiate the three UDL principles.	2.13	.81	Low knowledge
5	I have read or studied research materials on UDL.	2.19	.66	Low knowledge
Grand Mean		2.24	0.71	Low knowledge

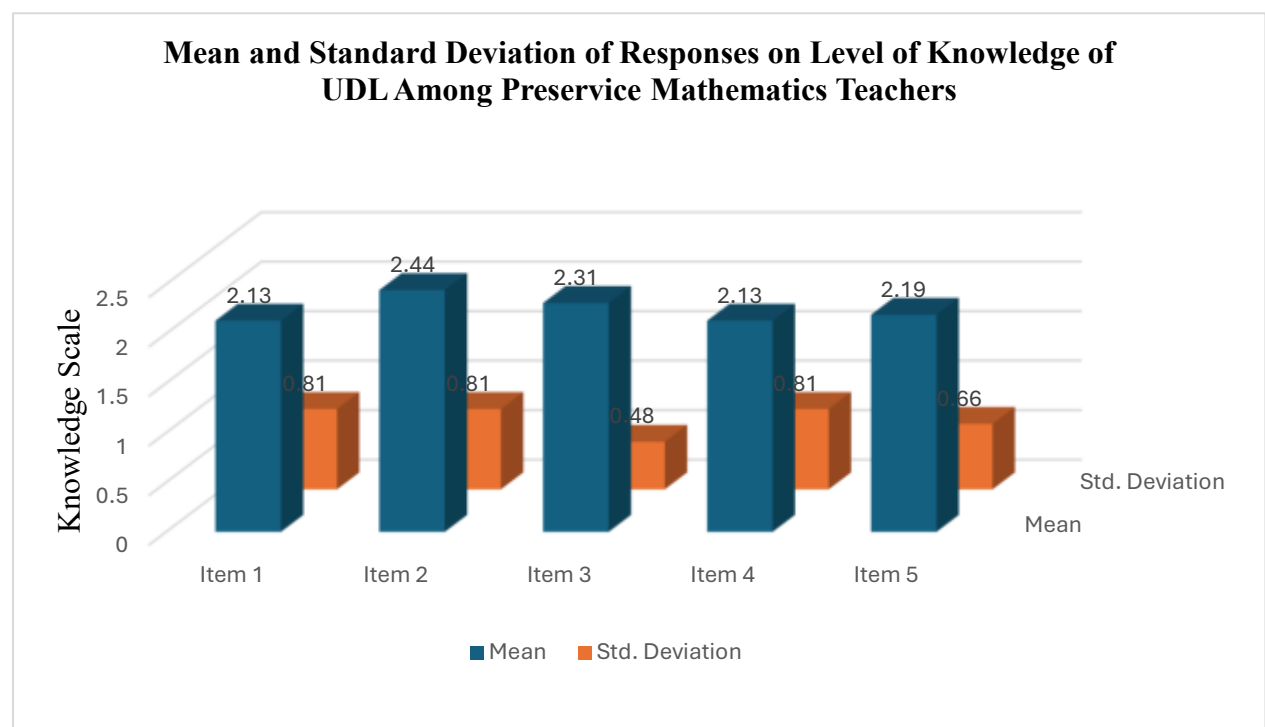


Figure 1: Preservice Mathematics Teachers' Level of Knowledge of UDL

The results presented in Table 1 and Figure 1 indicate that preservice mathematics teachers at the University of Nigeria, Nsukka, exhibit a low level of knowledge regarding UDL, as reflected in the grand mean of 2.24 ($SD = 0.71$). Across all five items assessing their understanding of UDL, the mean scores are below the benchmark of 3.00, suggesting limited familiarity with UDL concepts.

Specifically, respondents demonstrated insufficient comprehension of what UDL entails ($M = 2.13$, $SD = 0.81$) and its three core principles - multiple means of engagement, representation,



and expression ($M = 2.44$, $SD = 0.81$). Their understanding of how UDL supports diverse learners in a mathematics classroom was also low ($M = 2.31$, $SD = 0.48$), as was their ability to define and differentiate the three UDL principles ($M = 2.13$, $SD = 0.81$). Additionally, the mean score for reading or studying research materials on UDL ($M = 2.19$, $SD = 0.66$) suggests that many preservice teachers have not engaged deeply with scholarly resources on UDL.

As illustrated in Figure 1, the consistent pattern of low mean scores across all items highlights a significant gap in UDL knowledge among preservice mathematics teachers. This lack of foundational understanding may hinder their effective implementation of inclusive instructional strategies, underscoring the need for targeted interventions in teacher education programmes.

Understanding of UDL Application in 21st Century Mathematics Classrooms

Table 2: Preservice Mathematics Teachers' Understanding of the Application of UDL in Mathematics Classrooms

Item	Item Statement	Mean	Std. Deviation	Decision
6	I can effectively design a UDL-based lesson plan for a mathematics classroom.	1.94	.85	Very low Extent
7	I have incorporated multiple means of engagement (e.g., interactive activities) while teaching mathematics.	3.44	1.03	Moderate Extent
8	I have used multiple means of representation (e.g., diagrams, videos, models) in my mathematics lessons.	4.00	1.03	High Extent
9	I have applied multiple means of action and expression (e.g., allowing students to demonstrate learning in various ways).	3.38	.89	Moderate Extent
10	I can modify teaching methods to meet the diverse learning needs of students using UDL.	1.87	.96	Very low Extent
11	I am aware of how assistive technology can support UDL in mathematics education.	1.75	.78	Very low extent
Grand Mean		2.73	0.92	Low Extent

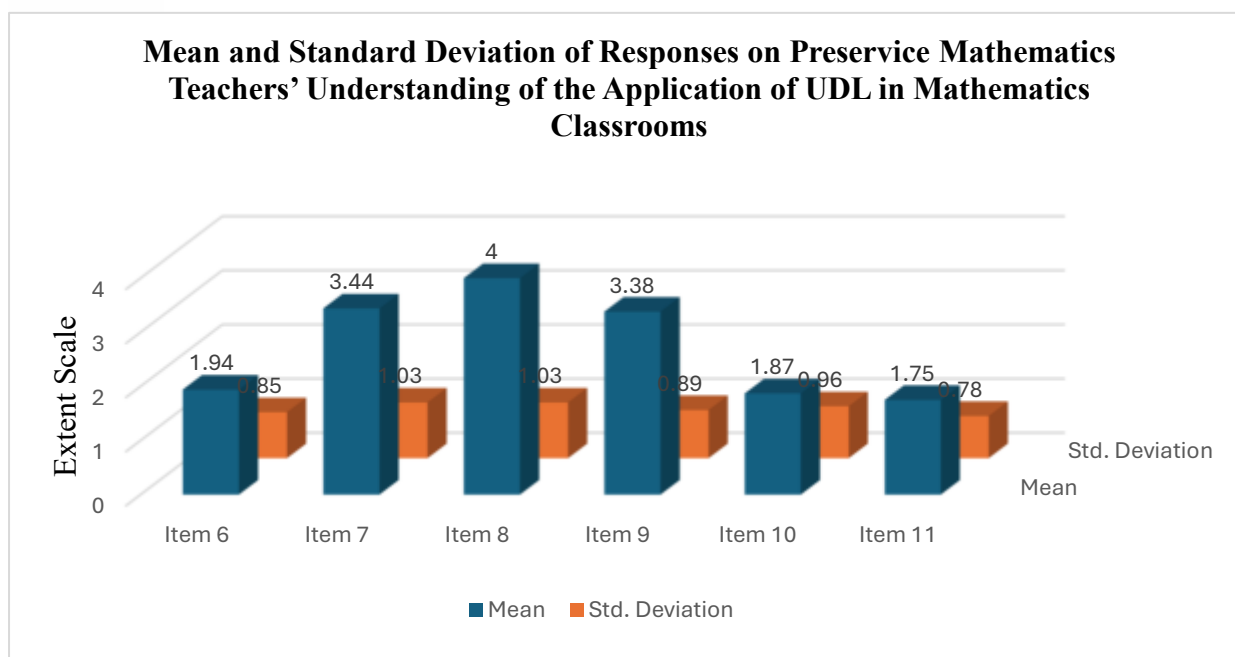


Figure 2: Preservice Mathematics Teachers' Understanding of the Application of UDL in Math Classrooms

Table 2 and Figure 2 present the extent to which preservice mathematics teachers at the University of Nigeria, Nsukka, understand and apply UDL principles in mathematics instruction. The grand mean of 2.73 ($SD = 0.92$) suggests a low extent of understanding and implementation of UDL strategies in their teaching practices.

A closer examination of individual items reveals areas of relative strength and weakness. The highest mean score was observed for using multiple means of representation ($M = 4.00$, $SD = 1.03$), indicating that preservice teachers are more comfortable incorporating diverse instructional materials such as diagrams, videos, and models in their teaching. Similarly, moderate levels of application were reported for multiple means of engagement ($M = 3.44$, $SD = 1.03$) and multiple means of action and expression ($M = 3.38$, $SD = 0.89$), suggesting that some interactive strategies and flexible assessment methods are being employed.

However, significant gaps exist in critical aspects of UDL implementation. The lowest mean scores were recorded for awareness of how assistive technology can support UDL ($M = 1.75$, $SD = 0.78$), modifying teaching methods to meet diverse learners' needs ($M = 1.87$, $SD = 0.96$), and designing a UDL-based lesson plan ($M = 1.94$, $SD = 0.85$). These findings indicate that while preservice teachers demonstrate some familiarity with UDL principles, particularly in using multiple representations, their ability to plan, modify, and integrate assistive technology into instruction remains limited. As illustrated in Figure 2, this uneven distribution of UDL applications underscores the need for targeted professional development programmes to enhance preservice mathematics teachers' effective implementation of inclusive teaching practices.



Challenges Faced in Integrating UDL into Teaching Practices

Table 3: Challenges Faced by Preservice Mathematics Teachers in Integrating UDL into Their Teaching Practices

Item	Item Statement	Mean	Std. Deviation	Decision
12	Lack of awareness and training on UDL makes it difficult to implement this in my teaching.	4.13	1.03	High Extent
13	Limited resources and instructional materials hinder the application of UDL in mathematics classrooms.	4.19	.54	High Extent
14	Large class sizes make it challenging to apply UDL principles effectively.	3.44	1.32	Moderate Extent
15	Time constraints prevent me from incorporating UDL strategies into my lessons.	3.19	1.11	Moderate Extent
16	I feel unprepared to integrate UDL principles due to inadequate training in my teacher education program.	2.06	1.12	Low knowledge
Grand Mean		3.40	1.02	Moderate Extent

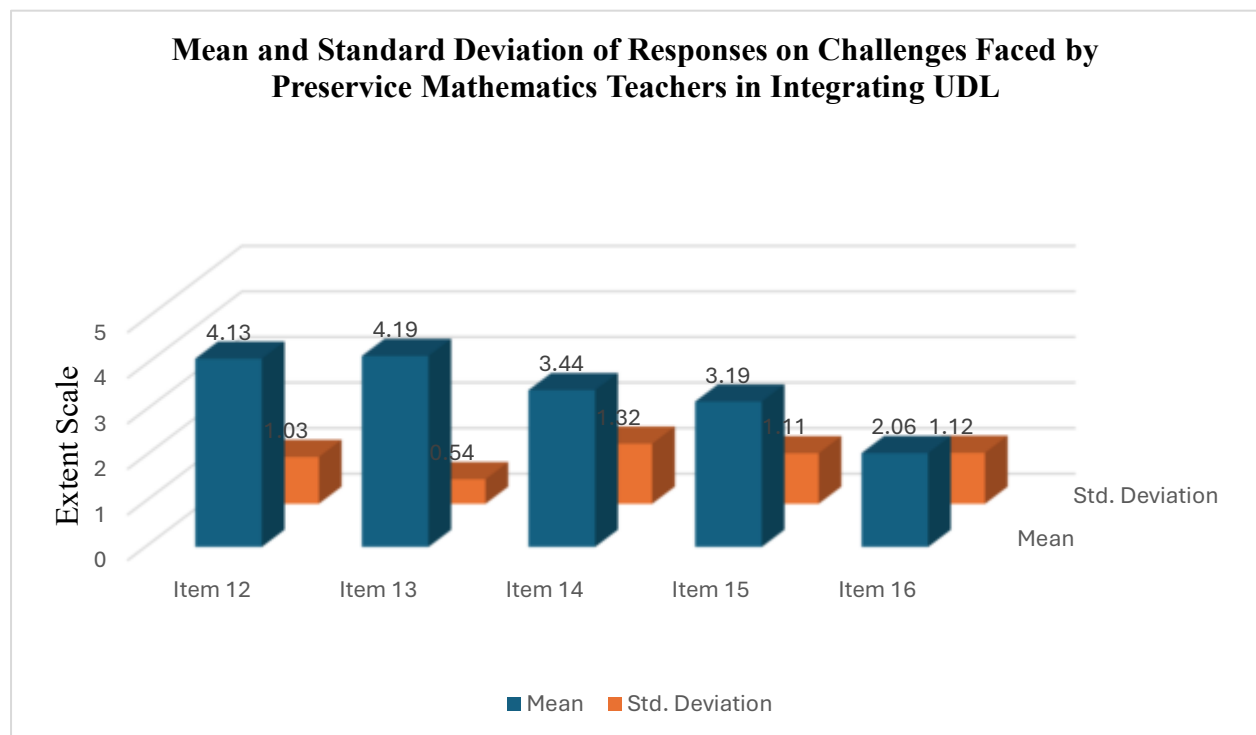


Figure 3: Challenges Faced by Preservice Mathematics Teachers in Integrating UDL

Table 3 and Figure 3 present the challenges preservice mathematics teachers encounter at the University of Nigeria, Nsukka, in integrating UDL principles into their instructional practices. The grand mean of 3.40 ($SD = 1.02$) suggests that preservice teachers face moderate challenges in implementing UDL strategies.



The most significant barriers identified were a lack of awareness and training on UDL ($M = 4.13$, $SD = 1.03$) and limited resources and instructional materials ($M = 4.19$, $SD = 0.54$), both rated at a high extent. These findings indicate that preservice teachers struggle primarily with insufficient knowledge and inadequate support systems for effective UDL implementation. Additionally, large class sizes ($M = 3.44$, $SD = 1.32$) and time constraints ($M = 3.19$, $SD = 1.11$) were rated moderately, suggesting that structural and logistical challenges within the classroom setting also hinder the application of UDL principles.

Conversely, the lowest mean score was recorded for unpreparedness due to inadequate training in their teacher education programme ($M = 2.06$, $SD = 1.12$). This result suggests that while preservice teachers acknowledge their training limitations, they may not perceive this as a primary barrier compared to other more immediate challenges, such as limited resources and time constraints. As illustrated in Figure 3, addressing these challenges, particularly through enhanced training, increased resource availability, and better classroom support structures, could significantly improve preservice teachers' ability to integrate UDL principles effectively.

DISCUSSION

The findings of this study indicate that preservice mathematics teachers at the University of Nigeria, Nsukka, exhibit a low level of knowledge regarding UDL. This aligns with previous studies highlighting inadequate exposure to UDL concepts among preservice teachers in various educational settings (Almutairi & Alsawayl, 2023; Griful-Freixenet et al., 2021; Vitelli, 2015). The limited understanding of UDL principles, particularly in differentiating between multiple means of engagement, representation, and expression, suggests that teacher education programmes may not provide sufficient instruction on inclusive pedagogical strategies. This deficiency in knowledge is concerning, as research has demonstrated that effective UDL implementation can significantly enhance student engagement and achievement, especially in mathematics education (Arslan & Yavuzarslan, 2020; Rusconi & Squillaci, 2023).

Despite this limited knowledge, the findings reveal that preservice teachers demonstrate some ability to apply elements of UDL in their teaching practices, particularly through multiple means of representation. Using diagrams, videos, and models suggests a recognition of diverse learning preferences. This aligns with the findings of Arslan and Yavuzarslan (2020), who noted that visual and multimedia resources are among the most commonly adopted UDL strategies in mathematics education. However, the study also highlights substantial gaps in the ability to design UDL-based lesson plans, modify instructional methods to accommodate diverse learners, and utilise assistive technology. These findings resonate with the work of Lowrey et al. (2019), who found that a lack of training and exposure to assistive technology limits the ability of preservice teachers to integrate UDL principles into their classrooms fully.

One of the key barriers to effective UDL implementation identified in this study is the lack of awareness and training among preservice mathematics teachers. This finding is consistent with previous research by Griful-Freixenet et al. (2021), which emphasised that teacher training programmes often do not incorporate sufficient coursework on inclusive instructional strategies. Limited access to resources and instructional materials further exacerbates this challenge, as effective UDL implementation requires access to diverse teaching tools (Rusconi



& Squillaci, 2023). Additionally, large class sizes and time constraints were identified as moderate challenges, which align with studies by Melhem and Al-Rashid (2023) that found that overcrowded classrooms hinder the application of differentiated instruction techniques, including UDL.

Interestingly, while many preservice teachers acknowledged their lack of training in UDL, they did not perceive this as the most significant barrier to its implementation. This may suggest a gap in self-awareness regarding the pedagogical demands of inclusive instruction or prioritisation of structural and resource-related challenges over professional development. Research by Lambert et al. (2023) supports this interpretation, noting that teachers often attribute instructional difficulties to external factors rather than gaps in their training.

Given these findings, teacher education programmes must integrate comprehensive training on UDL principles to enhance preservice teachers' preparedness for inclusive instruction. Empirical studies (e.g., Rusconi & Squillaci, 2023; Lowrey et al., 2019) suggest that embedding UDL training within coursework and practicum experiences can improve conceptual understanding and practical application. Furthermore, increased access to instructional resources, assistive technology, structured mentorship, and professional development opportunities can address the identified gaps in UDL implementation.

CONCLUSION

This study examined the level of knowledge, application, and challenges related to UDL among preservice mathematics teachers at the University of Nigeria, Nsukka. The findings indicate a generally low level of UDL awareness, with limited understanding of its core principles and application in lesson planning, differentiation, and the use of assistive technology. While some preservice teachers reported moderate use of engagement and representation strategies, their overall application of UDL remains inadequate. Major barriers include insufficient training, resources, large class sizes, and time constraints.

Educational Implications

The results of this study have significant implications for mathematics teacher education programmes. Given the increasing emphasis on inclusive education, the low knowledge of UDL among preservice teachers suggests a critical gap in teacher preparation curricula. If preservice teachers lack the foundational knowledge and skills to implement UDL, they may struggle to support diverse learners effectively in their future classrooms. Therefore, integrating UDL-focused coursework into teacher training programmes is essential to preparing mathematics teachers for inclusive instruction. Additionally, professional development workshops and hands-on experiences in UDL-based teaching strategies could enhance preservice teachers' competencies.

Limitations of the Study

While this study provides valuable insights into the knowledge and application of UDL among preservice mathematics teachers, certain limitations must be acknowledged. The study was conducted within a single university, which may limit the generalizability of the findings to



other institutions with different teacher preparation structures. Moreover, although a census sampling technique was adopted to include all eligible participants, five preservice teachers could not be reached, reducing the final sample size. This non-response could have affected the representativeness of the findings. Additionally, the reliance on self-reported data may have introduced response bias, as participants might have overestimated or underestimated their familiarity and use of UDL strategies. The absence of classroom observations also means that the study could not directly verify the practical implementation of UDL principles. Future research should address these limitations by including a broader sample from multiple institutions and incorporating observational and interview-based methodologies for a more comprehensive assessment.

RECOMMENDATIONS

To improve the integration of UDL in mathematics teacher preparation, several recommendations are proposed. Firstly, universities offering teacher education programmes should revise their curricula to include structured UDL training, ensuring that preservice teachers gain both theoretical knowledge and practical experience with UDL. Additionally, continuous professional development workshops and training programmes on UDL should be provided for both preservice and in-service teachers to enhance their capacity for inclusive teaching. Educational institutions are also encouraged to invest in resources such as assistive technologies and multimedia tools that support effective UDL implementation. Finally, policymakers should mandate UDL training as a core component of teacher education programmes to guarantee that all future educators are adequately prepared to meet the diverse needs of learners.

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ENHANCING MATHEMATICS PERFORMANCE THROUGH BOOTCAMP INTERVENTIONS FOR AT-RISK STUDENTS

Frikkie George¹ and Nwabisa Mbeki²

Cape Peninsula University of Technology (CPUT)

This study explores the effectiveness of Bootcamp interventions in promoting academic success among first-year mathematics students deemed at risk at a South African university. These students often face a range of challenges, including inadequate secondary school preparation, limited access to academic resources, language barriers, and socioeconomic constraints. Such factors contribute to low performance and high attrition rates in university mathematics courses. In response to these challenges, a targeted four-day mathematics Bootcamp was implemented during the September recess. The primary aim was to reinforce foundational mathematical concepts, improve exam preparedness, and support the holistic development of student academic identities. The study was guided by the central research question: To what extent does a short-term mathematics Bootcamp intervention improve the academic performance, confidence, and engagement of first-year at-risk students in a South African university context? The intervention was framed through a pluralistic theoretical lens, incorporating Student Identity Theory, Ubuntu (an ethics of care and community), and the Dialogical Argumentation and Assessment for Learning Instructional Model (DAAFLIM). This integrated framework provided a basis for addressing not only students' academic needs but also their emotional, social, and identity-related dimensions of learning. A mixed-methods research design was employed. Quantitative findings revealed a statistically significant increase in students' mathematics test scores after the Bootcamp, with a large effect size (Cohen's $d = 0.88$), indicating meaningful academic gains. Qualitative data collected from focus group interviews further highlighted students' increased confidence, improved conceptual understanding, and heightened levels of classroom engagement. Students also reported feeling more motivated and better equipped to face the challenges of university-level mathematics. The findings carry important implications for educators, academic support units, and policymakers in higher education. Importantly, the success of this intervention suggests that similar Bootcamp models could be adapted for high school learners, particularly those in under-resourced schools. By introducing such interventions at the secondary level, educators can begin to close conceptual gaps early, improve learners' confidence in mathematics, and better prepare them for the transition to tertiary education. This proactive approach could play a significant role in addressing systemic inequalities in mathematics education and improving overall learner performance in national assessments.

Keywords: Bootcamp intervention, at-risk students, mathematics education, academic support, exam readiness



INTRODUCTION

Underperformance in first-year mathematics remains a critical issue in higher education, particularly in South Africa, where many students enter university underprepared due to systemic socioeconomic and educational disparities. These at-risk students often encounter persistent academic and non-academic challenges, including poor foundational knowledge, limited access to academic support, and psychosocial barriers such as low self-efficacy. In response, most South African universities offer semester-long tutorial programs, which typically include weekly group sessions led by senior students or academic staff, peer-assisted learning (PAL), and voluntary consultation hours. While these initiatives provide ongoing support, they are often limited in their reach and effectiveness due to inconsistent attendance, large student-to-tutor ratios, limited time for individualised attention, and a lack of alignment with assessment demands. Moreover, these programs frequently adopt a remedial rather than a developmental approach, focusing on procedural problem-solving rather than deep conceptual understanding. They are also constrained by the academic calendar, leaving little room for intensive, focused intervention just before high-stakes assessments. As a result, there remains a notable absence of targeted, high-impact initiatives that concentrate specifically on exam preparation and conceptual reinforcement in a condensed timeframe.

This study assesses the impact of a four-day mathematics Bootcamp conducted during the September recess, aimed at supporting academically struggling first-year students in a more intensive and structured format. Unlike traditional tutorial models, the Bootcamp was designed to foster both academic and affective growth, incorporating active learning strategies, peer collaboration, and focused revision of key mathematical concepts aligned with upcoming assessments. The intervention not only seeks to improve immediate academic performance but also informs strategies for earlier interventions at the secondary school level, with the goal of building stronger mathematical foundations before students transition to university.

As STEM education evolves, Universities of Technology (UoTs) are increasingly tasked with fostering applied learning and innovation. Bootcamps, as intensive, immersive educational experiences, can bridge the gap between theoretical knowledge and practical application (Campbell, 2015). This paper was guided by the central research question: *To what extent does a short-term mathematics Bootcamp intervention improve the academic performance, confidence, and engagement of first-year at-risk students in a South African university context?*

Background and Context

High dropout rates in the first year of study at South African universities are often linked to poor outcomes in foundational subjects such as mathematics (Acher, 2000). While support systems like tutorials exist, they frequently fall short in equipping students for examinations and are often not sufficiently personalised. Bootcamps offer a solution through focused, short-term interventions aimed at consolidating essential knowledge and fostering academic confidence (Munoo, 2018). The Bootcamp presented in this study provided an interactive and supportive environment for engaging with core mathematical concepts. Led by experienced tutors, it featured diagnostic assessments, structured tutorials, and group-based problem-



solving activities, all tailored to foster comprehensive learning. The main objective of the Bootcamp was to assist students who were identified by lecturers to be at risk of failing.

THEORETICAL FRAMEWORK

This study is underpinned by a multifaceted theoretical framework which is underscored by the constructivist constructs of Vygotsky (1978) consisting of:

Student Identity Theory: Recognises that students' self-perception influences their learning behaviour. Many of the first-year students who attended the Bootcamp experienced challenges during the transition from high school to university mathematics to become accustomed to the university environment. Bootcamps can contribute to reshaping these identities toward academic confidence and capability.

Ubuntu (Ethics of Care): Emphasises a communal, empathetic approach to education. Facilitators act not just as teachers but as mentors who nurture a supportive learning space (Freire, 1970). Besides the academic support provided at the Bootcamp, it is also important that students psychosocial needs should be catered for (Yosso, 2012).

DAAFLIM (Dialogical Argumentation and Assessment for Learning Instructional Model): Promotes deep learning through dialogic engagement, formative assessment, and collaborative problem-solving (George, 2021). The DAAFLIM approach allows the facilitators to create appropriate learning spaces – intra-argumentation, inter-argumentation, trans-argumentation, and meta-argumentation – for students to enhance their conception (Figure 1).

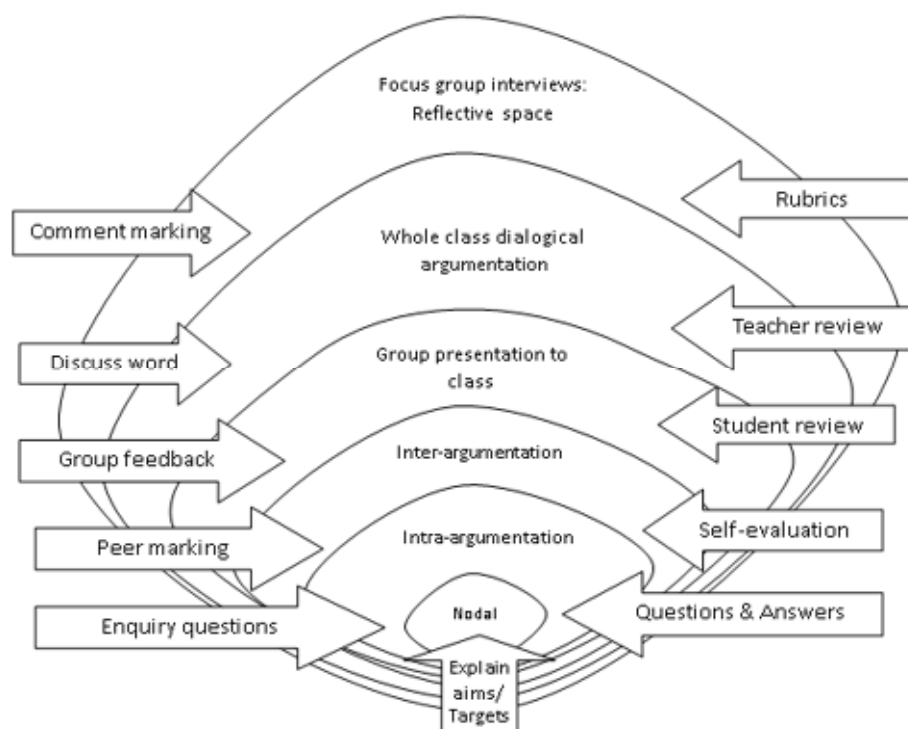


Figure 1: The DAAFLIM pedagogical approach (George, 2021)



Justification of the pluralistic theoretical approach

The integration of Student Identity Theory, Ubuntu, and DAAFLIM provides a holistic and complementary theoretical framework that addresses the cognitive, emotional, and social dimensions of student learning, particularly for at-risk first-year mathematics students transitioning into higher education.

Student Identity Theory highlights the critical role of self-perception in learning. Many first-year students arrive at university with fragile academic identities, often probably being shaped by prior negative experiences in mathematics or systemic barriers in their educational journey. These students may view themselves as “not good at math,” which limits their engagement and persistence (George & Rzyankina, 2024). The Bootcamp intervention, grounded in this theory, aims to help reshape students’ academic identities by creating opportunities for success, reinforcing a sense of competence, and normalising struggle as part of the learning process. As students experience growth through collaborative learning and problem-solving, their confidence and belief in their mathematical ability increase—paving the way for sustained engagement and improvement.

Ubuntu, as an African philosophy of communalism and care, complements Student Identity Theory by promoting a supportive, empathetic learning environment where each individual is valued and nurtured (Ramose, 1999). In the Bootcamp, facilitators adopt a mentoring role rather than a purely instructional one, building relationships based on mutual respect, encouragement, and shared responsibility. This approach is particularly important in the South African context, where many students face psychosocial challenges linked to socioeconomic hardship, marginalisation, or being first-generation university attendees. By affirming students’ worth and fostering a sense of belonging, Ubuntu-based pedagogy ensures that academic growth is accompanied by emotional and social support—essential for holistic development and resilience.

DAAFLIM further enhances this framework by operationalising dialogical and formative learning practices. Through structured spaces of intra-, inter-, trans-, and meta-argumentation, students are actively engaged in meaning-making, explanation, and critical reflection illustrated in Figure 1.

Intra-argumentation: refers to the internal cognitive process a learner engages in when constructing, organising, and evaluating their own understanding or argument before interacting with others. It is the first stage in the DAAFLIM framework and plays a foundational role in promoting metacognitive awareness and conceptual clarity. The key characteristics of Intra-argumentation:

- Individual reasoning: The student reflects on a mathematical problem or concept privately, trying to make sense of it using their prior knowledge, intuition, or logical reasoning.



- **Formulation of initial ideas:** The learner begins to build a personal argument or hypothesis to explain or solve a problem, even if it's incomplete or incorrect.
- **Preparation for dialogue:** It sets the stage for deeper engagement in the next phases of dialogical learning (inter-, trans-, and meta-argumentation), as students become aware of what they understand, what they don't, and what they need to clarify.

Inter-argumentation: is the second stage of the DAAFLIM framework, following intra-argumentation. It refers to the interactive exchange of ideas, reasoning, and arguments between learners. During this phase, students articulate their individual understandings and engage in dialogue with peers to compare, challenge, justify, and refine their thinking. The key characteristics of Inter-argumentation:

- *Collaborative dialogue:* Learners share their initial reasoning (developed during intra-argumentation) and respond to others' interpretations or solutions.
- *Argument exchange:* Students are encouraged to listen, question, agree, disagree, and provide evidence or reasoning to support their viewpoints.
- *Exposure to multiple perspectives:* The dialogue promotes cognitive conflict, where differing opinions help students reassess and strengthen their own understanding.
- *Co-construction of knowledge:* Through peer interaction, students collaboratively negotiate meaning and deepen conceptual clarity.

Meta-argumentation: involves students reflecting on the entire argumentative process—learners' own reasoning, the contributions of peers, the strategies used, and how understanding evolved through the discussion. It is during the meta-cognitive phase, where learners step back to think about how they think, learn, and engage with mathematics. The key characteristics of meta-argumentation:

- *Reflection on reasoning:* Students evaluate the quality of their arguments and the effectiveness of different approaches.
- *Awareness of learning processes:* Learners consider how their thinking has changed and what influenced that change (e.g., peer feedback, errors, facilitator input).
- *Articulation of strategy:* They explain why certain methods worked better, what misconceptions were corrected, or how they resolved confusion.
- *Development of self-regulation:* Students begin to monitor and direct their own learning, recognising areas for improvement and planning how to approach similar problems in the future.

Rather than passively receiving information, learners engage in discussions, construct arguments, challenge assumptions, and receive continuous feedback. This framework strengthens conceptual understanding, develops mathematical reasoning, and reinforces metacognitive skills. Moreover, the collaborative nature of DAAFLIM aligns seamlessly with Ubuntu, fostering shared inquiry and collective problem-solving within a nurturing space. Collectively, these three theories work synergistically to transform the learning experience.



Student Identity Theory motivates internal change by nurturing self-belief, Ubuntu creates an emotionally safe and relationally rich learning environment, and DAAFLIM activates intellectual engagement through structured, participatory pedagogical strategies.

Together, they form a cohesive framework that enables students not only to perform better academically but also to *feel* and *be* more capable, connected, and confident learners. This integrated approach ensures that the Bootcamp does not merely serve as an academic intervention but as a transformative educational experience that supports both academic success and personal empowerment.

METHODOLOGY

This study adopted a mixed-methods research design to explore the effectiveness of a four-day mathematics Bootcamp in supporting the academic success of at-risk first-year students at a South African university. The rationale for employing this approach lies in its ability to provide a comprehensive, nuanced understanding of both the measurable outcomes of the intervention and the subjective experiences of participants. Quantitative methods, particularly pre- and post-test, were essential for evaluating changes in student performance, including statistical significance and effect size. These metrics allowed the researcher to objectively determine whether the Bootcamp contributed to academic improvement. However, performance data alone could not capture the complex psychosocial, identity-related, and motivational factors that underpin learning—particularly in students facing systemic socioeconomic and educational challenges.

To address this limitation, qualitative methods were incorporated through focus group interviews, which explored participants' perceptions, emotional responses, confidence levels, and reflections on their learning experiences. This interpretive lens was particularly important given the study's grounding in Student Identity Theory, Ubuntu, and DAAFLIM, which emphasise affective, social, and dialogical dimensions of learning. The integration of both data types enabled the researcher to:

- Validate findings through triangulation, enhancing the credibility and robustness of the results.
- Link learning outcomes to student experiences, thereby offering a richer explanation of why and how the Bootcamp worked.
- Generate actionable insights for practice and policy, as the findings address both performance outcomes and the conditions necessary for sustainable learning support.

In addressing the research question, “*How do short-term Bootcamp interventions influence the academic performance, engagement, and identity development of at-risk first-year mathematics students?*”, the mixed-methods approach was particularly well-suited. It allowed for the investigation of what changed (quantitative) and how and why those changes occurred (qualitative), leading to a more holistic understanding of the Bootcamp intervention's impact.



Participants

Fifty at-risk first-year mathematics students participated, with 47 completing both the pre- and post-intervention assessments.

Intervention Design

Spanning four days during the September recess, the Bootcamp featured topic reviews, problem-solving sessions, mock assessments, and collaborative discussions. Facilitators emphasized deep conceptual understanding.

Data Collection:

Data was collected from diagnostic tests, observation schedules, and focus group interviews.

Pre- and Post-Diagnostic Tests: Assessed knowledge gains before and after the Bootcamp. The students completed a pre- and post-diagnostics test to be also used to determine the effectiveness of the Bootcamp. The diagnostic test included questions from previous written tests of mathematics assessments. Students only had to answer questions relevant to their discipline e.g., electrical engineering, marine engineering, civil engineering, and education.

Focus Group Interviews: Conducted with both students and facilitators to capture perceptions and experiences. The focus group interviews were conducted with students in each of the different discipline fields throughout the duration of the Bootcamp.

Data Analysis:

Thematic analysis and SPSS software were used to analyse the data collected. The different theme that emerged from the data were:

- *Increased Motivation and Confidence* – how students responded during the Bootcamp sessions and in the focus group interviews.
- *Enhanced conceptual understanding of mathematic topics* – the performance in the diagnostic tests were used for this theme.
- *Ethic of care (ubuntu)* – the effect of psychosocial support on the performance of the students.
- *Supportive Facilitation* – how students responded to the pedagogical process.

Ethical consideration:

The participants gave informed consent and voluntary took part in the study. The university approved the study's ethical clearance application.



RESULTS

Quantitative Findings

The quantitative data in Table 1 revealed a statistically significant and educationally meaningful improvement in student performance following participation in the mathematics Bootcamp. A comparison of pre- and post-intervention diagnostic test scores demonstrated a substantial gain in conceptual understanding and problem-solving ability over the short duration of the intervention.

Table 1: Scores of the pre-post-diagnostic test

Diagnostic tests	N	Mean score (%)	SD	t-value
Pre-test	50	9.98 (49.9%)	3.19	-3.74*
Post-test	47	12.47 (62.4%)	3.08	

*Note: Cohen's $d = 0.88$; $p < 0.05$; $t_{critical} = 2.02$; * = statistical significant difference*

The mean post-test score of 62.4% reflects a 12.5% improvement from the pre-test average of 49.9%. A paired sample t-test yielded a t-value of -3.74, which exceeds the critical threshold ($t_{critical} = 2.02$) and confirms that the increase in scores was statistically significant at the 5% level ($p < 0.05$). Furthermore, the effect size, as measured by Cohen's $d = 0.88$, indicates a large effect, suggesting that the Bootcamp had a strong and meaningful impact on student learning. It is worth noting that the pre-test results showed that students had some initial understanding of the material (mean score = 49.9%), indicating that they were not entirely unfamiliar with the foundational concepts. However, the marked improvement in the post-test scores points to the Bootcamp's effectiveness in deepening conceptual clarity, enhancing retention, and boosting students' ability to apply mathematical techniques more confidently and accurately.

These results provide compelling evidence that even a short, intensive intervention—when strategically designed—can yield significant academic gains for at-risk students. The structured focus on core concepts, combined with targeted instruction and dialogic engagement, appears to have played a crucial role in reinforcing mathematical understanding in a compressed timeframe.

Qualitative Findings

The qualitative data collected through focus group interviews with students and facilitators revealed a consistently positive perception of the Bootcamp experience, with several key themes emerging. These findings not only complement the quantitative results but also provide deeper insight into the affective, cognitive, and social impact of the intervention.

Enhanced Conceptual Understanding

A dominant theme across student responses was a marked improvement in conceptual clarity, particularly in foundational topics such as functions, differentiation, and calculus. Students



attributed their improved understanding to the structured, scaffolded approach adopted in the Bootcamp, which emphasised step-by-step guidance and peer discussion.

As one participant explained, *“I understand the implicit differentiation technique now much better than how my lecturer explained it.”* — Student 12

This quote reflects a common sentiment among the group, illustrating how the Bootcamp’s pedagogical design, grounded in DAAFLIM, helped students develop a more coherent understanding of complex topics. Initially, students engaged with problems independently (intra-argumentation), followed by collaborative peer discussions (inter-argumentation), allowing them to confront misconceptions, compare reasoning strategies, and consolidate accurate methods.

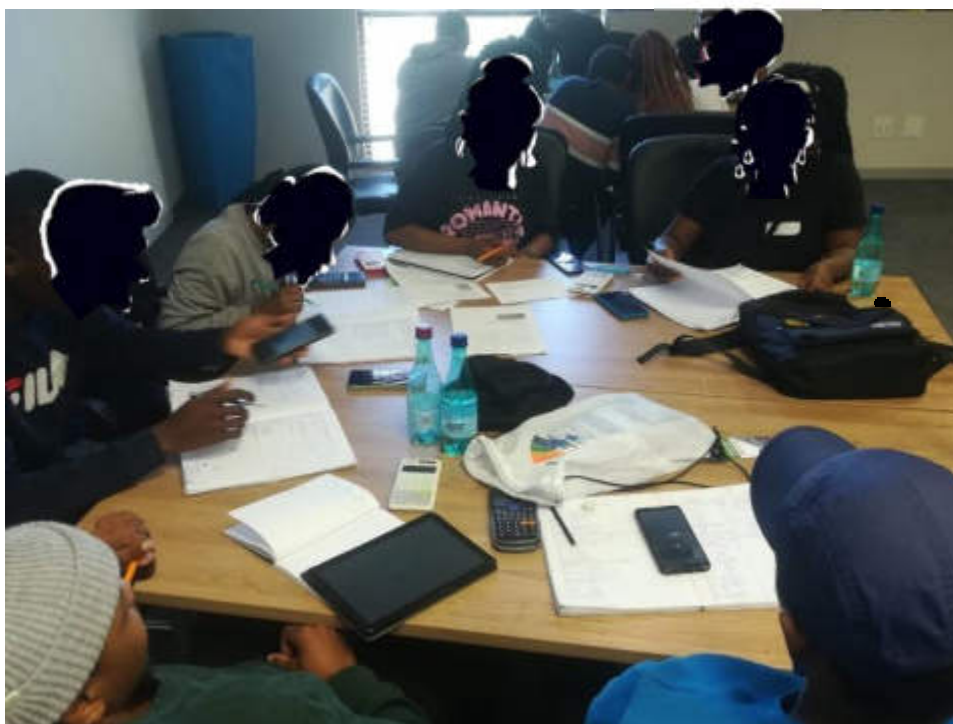


Figure 2: Student during the intra-argumentation space (Photo: Author)

Facilitators also noted that students were more willing to ask questions, justify their reasoning, and actively participate in mathematical discourse compared to their usual semester-long tutorials (Figure 2). This indicates that the Bootcamp environment fostered psychological safety, which is a critical enabler of meaningful learning—particularly for students who may normally feel marginalised or underconfident in traditional academic settings.

Increased confidence and academic identity development:

Students repeatedly mentioned feeling more confident in their mathematical abilities after the Bootcamp. For many, the intervention helped reshape their self-perceptions from "struggling" to "capable" learners. This aligns closely with Student Identity Theory, which posits that students' beliefs about themselves directly influence their motivation and engagement. Several students shared that the Bootcamp was the first time they felt “smart in maths” or believed they



could pass upcoming assessments, suggesting that the intervention not only impacted performance but also began to reconstruct academic identities in empowering ways.

The relaxed and collaborative environment made learning less intimidating and more enjoyable. Students 23 expressed the following view, “During the lectures I do not ask questions even if I do not understand, but at this Bootcamp I feel free to ask questions to the facilitators and my group members”.



Figure 3: *Students confidently discussing during inter-argumentation (Photo: Author)*

During the group work sessions students demonstrated and increased confidence levels as the Bootcamp progressed. In Figure 3 the students explained concepts to each other in a safe and brave space (inter-argumentation).

Supportive Facilitation

The Bootcamp also created a strong sense of community and care, reflecting the principles of Ubuntu, a communal and empathetic philosophy that underscores the value of mutual support and human dignity. Students appreciated the facilitators’ approachability, encouragement, and non-judgemental attitudes, which contributed to a warm and inclusive learning atmosphere.

“It felt like the tutors really cared about us. They explained things like they wanted us to succeed, not just tick boxes.” — Student 21

This ethic of care, combined with collaborative learning, appeared to motivate students to persist, help each other, and engage more deeply with the material. Such emotional and social support mechanisms are particularly vital for students navigating complex transitions from high school to university under conditions of socioeconomic disadvantage. Tutors’ availability and empathetic approach created a caring atmosphere aligned with the Ubuntu philosophy. Student 35 respond the following in the focus group interviews, “I really like how the facilitators helped me with the questions that I do not understand and the are very friendly”.



Figure 4: Facilitators engaging students (Photo: Author)

The facilitators interacted with the different groups and intervened when they realise students need assistance (Figure 4). They also make sure that students have appropriate conceptions at the end of the Bootcamp sessions during the trans- and meta-argumentation spaces.

Value of the Learning Structure and Materials: Students praised the quality and design of the worksheets, which served as both instructional guides and self-assessment tools. The gradual progression of difficulty, combined with dialogical engagement in problem-solving, allowed students to reflect on their understanding, gain feedback, and learn from others' strategies. Facilitators leveraged the DAAFLIM model to create opportunities for trans-argumentation and meta-argumentation, where students generalized concepts and reflected on their learning processes.

Summary of Impact

Overall, the qualitative findings suggest that the Bootcamp achieved more than academic improvement. It fostered confidence, community, and conceptual growth. By integrating structured pedagogy (DAAFLIM), psychosocial support (Ubuntu), and a focus on academic identity, the intervention provided a holistic response to the multifaceted challenges faced by at-risk students in higher education.

DISCUSSION

The Bootcamp's success is reflected in both measurable academic gains and improved student attitudes. It validated the potential of intensive, short-term interventions in helping students reframe their academic identities and engage more confidently with mathematics. The theoretical frameworks applied played a crucial role in this outcome. Student Identity Theory helped shift learners' self-conceptions. Most of the students portrayed coyness at the start of the Bootcamp and developed more confidence towards the end. Ubuntu ethics fostered a nurturing academic space and the facilitators' attitude, stationery and catering provided at the Bootcamp contributed to the students' participation and concur with the study of Nkoane (2012). DAAFLIM's interactive approach promoted critical engagement and deeper learning. These results align with international literature on effective STEM education practices and



underscore the unique value of Bootcamps in bridging academic and emotional support. The different learning spaces provided by the DAAFLIM approach afforded the students with authentic learning opportunities.

During intra-argumentation, a student may be asked to think independently about a mathematical question, such as solving a system of equations or interpreting a graph. Instead of jumping straight into group discussion, the learner first writes down their thought process, highlights their assumptions, or even drafts a mini explanation. This phase encourages self-questioning like:

- *“Why does this formula work?”*
- *“What is the underlying principle here?”*
- *“What do I already know that relates to this?”*

Why Intra-argumentation Matters:

- It develops critical thinking and reflection, helping students to internalize concepts rather than rely solely on peer or teacher input.
- It empowers students to identify their own misconceptions and gaps in understanding before social comparison.
- It supports academic identity development, particularly for students who may lack confidence, by giving them a private space to engage without fear of judgement.

During the Bootcamp, intra-argumentation was valuable because it allowed students to engage deeply and individually with mathematics problems before moving into inter-argumentation (peer discussion), ensuring that all voices—especially those of shy or underconfident students—had the chance to form ideas and participate meaningfully.

After students individually attempt a problem (intra-argumentation), they are placed in small groups (maximum five learners) to discuss their reasoning. For example, if the task involves solving a quadratic equation, one student might argue that completing the square is best, while another prefers using the quadratic formula. As they debate the efficiency or clarity of each method, they begin to unpack underlying mathematical concepts, correct misconceptions, and reinforce accurate procedures.

During inter-argumentation, students might say things like:

- *“I used substitution—did you get the same answer using elimination?”*
- *“I think your method works, but isn’t there a mistake in your second step?”*
- *“Why did you divide both sides by 2 here?”*



Why Inter-argumentation Matters:

- It promotes dialogic learning, where knowledge is socially constructed rather than passively received.
- It strengthens communication skills and mathematical language use.
- It helps students identify errors and gaps in reasoning through peer feedback.
- It builds confidence and engagement, especially in environments like Bootcamps that foster safe, collaborative learning spaces.

By integrating inter-argumentation, the Bootcamp moved beyond traditional teaching methods and fostered a participatory culture where students learned from each other—mirroring real-world mathematical reasoning and problem-solving processes.

After discussing and debating multiple solution methods (inter-argumentation), students might begin to notice patterns, compare strategies, or suggest a general rule that can be applied to a wider class of problems. For example, in a discussion about functions, students may integrate ideas about linearity, domain restrictions, and real-world applications to create a more nuanced understanding of function behaviour.

In a trans-argumentative moment, students might say:

- “It looks like this method works best when the coefficient is even. Can we generalize that?”
- “What if we apply this strategy to exponential equations? Would it still work?”
- “I think the graphical approach gives us a clearer picture—maybe we can explain the algebra using visuals.”

This phase is often marked by "aha" moments, where learners shift from surface-level understanding to deep insight and flexible thinking. Why Trans-argumentation Matters:

It helps students internalize and extend their learning.

- It encourages intellectual independence, empowering students to think critically and creatively.
- It promotes knowledge transfer, essential for success in assessments and real-world problem-solving.
- It develops adaptive expertise, allowing students to navigate unfamiliar problems with confidence.

In the Bootcamp context, trans-argumentation enabled students to move from simply solving mathematics problems to thinking like mathematicians—making connections, generating strategies, and seeing mathematics as a dynamic, interconnected discipline. This stage is especially powerful for transforming how students perceive themselves as learners and thinkers, aligning with the goals of both Student Identity Theory and Ubuntu.



After solving and discussing problems, the facilitator might prompt meta-argumentation by asking students:

- *“What strategy worked best for you and why?”*
- *“How did your thinking change during the discussion?”*
- *“What would you do differently next time you see a similar problem?”*
- *“What helped you understand the concept more clearly?”*

A student might reflect:

- *“At first, I thought substitution was easiest, but after hearing the other group's approach, I realized elimination helped avoid fractions. Now I prefer that method.”*
- *“I misunderstood the graph at first, but after explaining my reasoning and getting feedback, I saw the mistake in how I interpreted the x-intercepts.”*

These reflections help consolidate learning and deepen understanding.

Why Meta-argumentation Matters:

- It fosters metacognitive growth, which is essential for independent learning and academic resilience.
- It encourages ownership of learning, where students recognize their progress and areas for development.
- It supports formative assessment, enabling both students and educators to gauge the effectiveness of the learning process.
- It strengthens the connection between mathematical reasoning and self-awareness, which is especially powerful for at-risk students who may doubt their academic potential.

In the context of the Bootcamp, meta-argumentation was instrumental in helping students become active agents in their own learning journeys. It gave them the tools to evaluate their thinking, learn from mistakes, and carry forward new strategies with confidence—an outcome that supports both Student Identity Theory and the principles of Ubuntu, while fully realizing the goals of DAAFLIM. The facilitator’s role is to guide but not dominate the discussion—encouraging students to articulate their thinking and listen actively to others.

IMPLICATIONS AND RECOMMENDATIONS

To support student retention and success in mathematics, the study suggests institutional adoption. Bootcamps should be formally incorporated into academic calendars, especially



during pre-exam periods. Resources should be allocation and funding are needed for facilitator training, materials, and participant incentives. Furthermore, facilitator training sessions for tutors must be equipped with pedagogical tools that are academic skilled, culturally responsive, and emotionally supportive. Further research is needed to evaluate and upscale for broader application of Bootcamps across faculties and institutions.

Implications for practice

The findings offer valuable insights for higher education practitioners, curriculum developers, and policymakers. Mathematics Bootcamps represent a promising, cost-effective strategy to support academically vulnerable students, particularly in gateway STEM disciplines. The integration of culturally responsive pedagogies, such as Ubuntu, and dialogical teaching methods that prioritize peer interaction and formative feedback, contributes to deeper learning and emotional support. These Bootcamps also have the potential to foster students' sense of academic identity, belonging, and resilience—critical factors for long-term success.

Importantly, the success of this intervention suggests that similar Bootcamp models could be adapted for high school learners, particularly those in under-resourced schools. By introducing such interventions at the secondary level, educators can begin to close conceptual gaps early, improve learners' confidence in mathematics, and better prepare them for the transition to tertiary education. This proactive approach could play a significant role in addressing systemic inequalities in mathematics education and improving overall learner performance in national assessments.

In conclusion, the study underscores the potential of short-term, theoretically grounded interventions to make a lasting impact on student outcomes. Bootcamps, when thoughtfully designed and responsive to learners' diverse needs, can enhance academic success, reduce dropout rates, and contribute to more equitable access to mathematics education at both secondary and tertiary levels.

This study explores the role of Bootcamp interventions in supporting the academic success of first-year mathematics students considered at risk at a South African university. Many of these students face significant socioeconomic and educational disadvantages that hinder their ability to succeed academically. A four-day Bootcamp, conducted during the September recess, aimed to reinforce core mathematical concepts and enhance exam preparedness. The intervention was framed through a pluralistic theoretical lens, incorporating Student Identity Theory, Ubuntu (an ethics of care), and the Dialogical Argumentation and Assessment for Learning Instructional Model (DAAFLIM). Using both quantitative and qualitative methods, the study found notable improvements in student performance, with a statistically significant increase in test scores and a large effect size (Cohen's $d = 0.88$). Focus group interviews highlighted students' increased confidence, conceptual understanding, and engagement. The findings suggest that Bootcamps can serve as an effective academic support strategy, reducing dropout rates and improving outcomes in mathematics and related subjects.



CONCLUSION

Bootcamps emerge as an effective intervention strategy that boosts both academic performance and student confidence. This study affirms the potential of combining academic rigor with relational pedagogy in helping at-risk students thrive. As universities strive to address dropout rates and improve students' performance in mathematics, Bootcamps represent a scalable and impactful solution (Tinto, 1993). When rooted in care, identity transformation, and dialogic learning, such interventions do more than improve test scores — they cultivate empowered learners ready to succeed in higher education and beyond.

Bootcamps that emphasize digital integration, real-world problem-solving, and accessibility can transform how students engage with STEM, preparing them to become future innovators and leaders in their fields.

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THE INFLUENCE OF BALANCED ASSESSMENT TASKS ON GRADE 7 LEARNER'S ENGAGEMENT IN MATHEMATICS

Nomthandazo M. Bhekiswayo and Matshidiso M. Moleko

University of the Free State University of South Africa

The importance of learner engagement in mathematics has long been underscored and cannot be disregarded due to its critical role in fostering a deeper understanding of mathematical concepts, as well as being essential for improving academic performance and outcomes. Balanced assessment tasks are often designed and intended to align with diverse learning objectives, offering learners varied opportunities to demonstrate their understanding through an array of methods, such as problem-solving, collaborative activities, and reflective practices. Drawing from this, therefore, the qualitative case study reported in this study aimed to explore the influence of balanced assessment tasks (encompassing formative and summative components) on learners' engagement levels in grade 7 mathematics classrooms. The study interrogates how balanced tasks promote active participation, sustain interest, and address different learning styles through the use of focus group discussions with learners. Data was collected through focus group discussions. Thematic analysis was used to analyse data. Social constructivist theory is used as a theoretical framework guiding the study. The findings of the study indicate that balanced assessment tasks not only improve engagement but also foster learners' positive attitudes toward mathematics, reduce anxiety, increase motivation to learn, improve feedback and progress tracking, and foster the metacognitive skills. The study highlights the critical role of well-structured assessments in creating an inclusive and stimulating learning environment. By integrating balanced assessment strategies, educators can effectively bridge the gap between instruction and evaluation, thus making mathematics more accessible and enjoyable for learners.

Keywords: *balanced assessment tasks; mathematics classroom; learner engagement*

INTRODUCTION AND BACKGROUND

In today's educational landscape wherein the use of technology increases at a rapid rate, fostering learner engagement has become an important goal, more especially in a subject, such as mathematics, where there are a myriad of challenges that learners encounter, often resulting in learners' lacking interest and motivation to learn. Engagement in mathematics is crucial, since it profoundly impacts learners' understanding, academic performance, and ability to apply mathematical concepts to real-world problems (Fung, et al. 2018). One of the major factors that has a substantial effect on student engagement is the development and execution of assessment activities (Li, et al. 2025). Traditional assessments mainly focus on rote memorisation and procedural fluency, which often does not meet and capture the diverse learning needs and potential of learners (Assencio, et al. 2015). Contrary to the traditional assessments are the balanced assessment tasks, which offer a more comprehensive and holistic approach that incorporates several structures and goals to support meaningful learning



experiences (Greenstein, 2012).

Balanced assessment tasks aim to examine learners' abilities beyond procedural knowledge and reinforce their conceptual understanding, critical thinking, and problem-solving skills (Bhekiswayo, 2023). These tasks (balanced assessment tasks), often include an amalgam of formative and summative assessments, collaborative activities, and real-world applications, thereby addressing and catering to the different aspects of learning (Canty, 2012). Such an approach to assessment shifts the focus from pushing learners and causing them become fixated and fussed about achieving correct answers, to making them engage with the processes and reasoning behind mathematical concepts (Vatterott, 2015). This paradigm shift in assessment is consistent with the goals of current educational practices, promoting imaginative problem-solving, collective effort, and analytical skills (Care, et al. 2018). It reflects the ideals of modern pedagogy and highlights resourcefulness, cooperative learning, and advanced reasoning.

The relationship between balanced assessment tasks and student engagement in mathematics is a growing area of interest among educators and researchers. Preliminary insights suggest that the balanced assessment tasks have the potential to transform mathematics classrooms into dynamic spaces where learners actively participate, explore, and connect with the subject matter (Abrahamson et al., 2020). Furthermore, the preliminary findings reveal the need for the focus to be placed on student engagement and balanced assessment tasks (Csikszentmihalyi et al., 2014). Based on this, therefore, this paper explores the influence of balanced assessment tasks on student engagement by examining how these tasks can create a supportive and stimulating environment for learners. The study aims to provide insights into improving assessments to enhance student interest, motivation, and overall success in mathematics education by analysing key strategies and practices. In line with all this therefore we ask the question: *What is the influence of balanced assessment tasks on learners' engagement in mathematics?*

LITERATURE REVIEW

Assessment is an integral part of the teaching and learning process and can either be baseline, diagnostic, formative or summative (Dreyer & Mawela, 2020). During the course of the year, assessments are either formative or summative with formal assessments comprising a number of school-based assessments (SBA) and end-of-year examinations. The SBA should take various forms, such as tests, projects, assignment, investigations and examinations (DBE, 2011). Teachers are expected to apply the Bloom's Revised Taxonomy when designing learner assessments (DBE, 2011). The taxonomy is divided into cognitive, affective and psychomotor domains (Bloom et al., 1956). The cognitive domain discusses the mental process of knowing and understanding. The affective domain describes the experience of feeling or emotion, and the psychomotor explains the relation between cognitive functions and physical movement (Bloom et al., 1956).

Bloom's Taxonomy of the Cognitive Domain encompasses knowledge, comprehension, application, analysis, synthesis and evaluation (Huitt, 2011). The cognitive levels are arranged from the lowest level, the simple recall of mathematical facts, to high-order, complex and



abstract reasoning. To address the cognitive domain, the CAPS (DBE, 2011) identified four cognitive levels for the design of assessment activities. These four levels include knowledge, routine procedures, complex procedures, and problem-solving, which should assist teachers in developing assessments in the content of mathematics, towards achieving the broader aims of the subject. Each cognitive level has a description of the skills to be demonstrated (DBE, 2011). Therefore, the mathematics teacher is expected to plan tasks, bearing in mind the cognitive levels to elicit information on current learning patterns (Vingsle, 2014). The task is given to learners during teaching and then marked, with both the teacher and learners receiving feedback on the performance. Abu Bakar and Ismail (2020) highlighted the importance of giving learners assessment tasks that are balanced. The balanced assessment task refers to an assessment activity that incorporates a variety of assessment methods, including both formative and summative elements, to provide a comprehensive picture of a student's learning progress and mastery of a subject. It offers multiple opportunities to demonstrate their understanding while also providing timely feedback for improvement throughout the learning process of CAPS (DBE, 2011). Subsequently, the balancing of assessment tasks influences the engagement of learners in Mathematics.

The influence of balanced assessment tasks on learner engagement in mathematics is a broad area of research that includes many aspects of learner interaction, motivation, and pedagogical approaches (Zhang, 2022). A balanced assessment approach, which includes both formative and summative assessments, has been found to greatly increase learner engagement by creating a supportive learning environment (Nazeef & Fareed, 2024). According to research (Reyes et al., 2012; Kytälä & Björn, 2021), the classroom's emotional environment has a significant impact on learner engagement. Kytälä and Björn (2021) emphasise that the learning environment, including the emotional climate and instructional quality, has a substantial influence on learners' mathematics experiences and subsequent engagement levels. This is consistent with the results of Reyes et al. (2012), which state that classrooms that prioritise learner views and foster autonomy are important predictors of long-term engagement.

The emotional and cognitive dimensions of engagement are intricately linked; for example, Purnomo et al. (2021) find a strong relationship between cognitive and emotional engagement, implying that learners who are emotionally invested in their learning are more likely to engage cognitively with the material. Furthermore, the importance of teachers in increasing engagement cannot be sufficiently emphasised (Purnomo et al., 2021). According to Attard's (2011) longitudinal study, strong pedagogical interactions between learners and teachers are critical for increasing mathematical engagement. This is supported by the findings of Watt et al. (2017), which show that teacher excitement and the learning environment have a substantial effect on learners' engagement profiles.

Implementing balanced assessment tasks can improve these connections by offering chances for feedback and reflection, both of which are important for building learners' intrinsic motivation and self-concept in mathematics (Brookhart, 2021; Shaikh et al., 2023). The use of technology in assessment processes is also important for increasing engagement (Attard & Holmes, 2020). Furthermore, technology can help learners engage by offering dynamic and



relevant learning experiences (Lazarides et al., 2020). Similarly, Bahati et al. (2019) discovered that learners were very satisfied with formative e-assessment procedures, which increased their engagement levels. This implies that designing engaging and relevant assessment problems can dramatically increase learners' cognitive and emotional participation in mathematics.

Furthermore, the benefit of balanced assessment tasks on engagement is seen in a variety of learning situations (Nazeef & Fareed, 2024). Lazarides et al. (2020) investigated how various motivational beliefs combine with classroom factors to promote learner engagement and future goals. This is especially important in blended learning contexts, where the use of several assessment techniques may be accommodated into diverse learning styles and preferences, thereby increasing overall engagement (Torrejos, 2024).

Finally, balanced assessment tasks help to increase learner engagement in mathematics by providing a positive emotional environment, thus improving teacher-learner connections, and utilising technology (Nazeef & Fareed, 2024). The interaction of cognitive and emotional involvement, as well as the impact of educational techniques, highlights the significance of a comprehensive approach to evaluation in mathematics education.

THEORETICAL FRAMEWORK

In this study social constructivist theory is adopted as a theoretical framework underpinning this study. Social constructivism is a learning theory propounded by Lev Vygotsky in 1968. This theory claims that learners construct and translate knowledge. Social constructivism highlights the role of social interaction in classroom learning (Vygotsky, 1978). Social constructivist theory states that knowledge is "developed in coordination with other human beings" (Amineh & Asl, 2015, p. 3). As a result, the learning process is coordinated on a social level (Amineh & Asl, 2015). Instead of learners passively obtaining knowledge from the instructor, the learning process happens through interaction between all learners and their teacher, involving learners' social and cognitive presence throughout classroom instruction (Lowyck & Pöysä, 2001). Thus, learners want environments that allow them to take control of their learning, rather than depending on traditional teaching, learning, and evaluation methods in mathematics classes.

Vygotsky's theory of learning emphasizes the importance of interaction and dialogue in learners' interpretation and meaning-making (Rieber & Carton, 1987). This is reflected in the social, cognitive, emotional, and behavioral dimensions of engagement and disengagement (Wang et al., 2017). Furthermore, engagement in a classroom is shaped by interactions between the learner, subject, and environment, which can be influenced by relational, personal, contextual, and sociocultural factors. Vygotsky proposed that learning should be placed at the top of a learner's zone of proximal development (ZPD), allowing for self-regulation and maximum learning. Thus, teachers should ensure that assessments in a mathematics classroom are relatable to learner's everyday lives and allows learners to take control. In the data society, digital tools are sometimes used to support learners, and mediation revolves around the notion that tools mediate reality to individuals' practices (Ritella, 2018). Teaching and assessment practices require teachers to orchestrate digital technologies that mediate



individual and collaborative learning processes. As digitalization spreads, teachers must learn to orchestrate multiple digital technology tools in complex learning situations to ensure smooth and effective support for individual needs and preferences.

METHODOLOGY

This study reported in this paper employed qualitative research methods to explore the influence of balanced assessment tasks on learner engagement in mathematics. Data were generated through focus group discussions, as this instrument provide in-depth insights into participants' experiences, perceptions, and attitudes. Focus group discussions facilitated collaborative dialogue among participants, encouraging the emergence of shared experiences and diverse perspectives (Reznitskaya et al., 2009). Five learners were purposefully selected to participate in the study. These learners represented diverse academic abilities, engagement levels, and socio-cultural backgrounds, thus facilitating diverse experiences. The selection process involved consulting with mathematics teachers to identify learners likely to provide rich and varied data. Criteria for selection included a balance of high- and low-performing learners and learners with differing levels of participation and interest in mathematics. This purposeful sampling approach aimed to capture a holistic view of how balanced assessment tasks influence engagement across diverse learner profiles.

The study generated nuanced data that illuminated the complex interplay between assessment design and learner engagement by employing focus group discussions. This data generation methods captured individual experiences and fostered a collective exploration of the impact of balanced assessment tasks, thereby contributing to a comprehensive understanding of their role in enhancing mathematics education.

Data were analysed through a thematic analysis technique. Braun and Clarke's (2006) six steps of thematic analysis were used to analyse data, which include becoming familiar with the data, generating initial codes, searching for themes, reviewing themes, defining themes and finally, using the findings for the write-up. Thematic analysis provides accessible and organised procedures for generating codes and themes from qualitative data (Maguire & Delahunt, 2017). The technique made it easy for us to identify the emerging themes and to organise them in line with the objectives of the study. Thus the sub-themes were highlighted.

FINDINGS AND DISCUSSION

This section provides discussions on the analysis and interpretation of the data collected during the focus group discussions with learners. The subsequent sections thus highlight the four themes that emerged in relation to the influence of balanced assessment tasks on a learner's engagement in mathematics.

Theme 1: Increased Motivation

In the focus group discussions wherein learners were asked about how they felt about working on higher-order tasks, learners increased motivation was highlighted as one of the influences



of learner engagement. To that effect, the following was said:

“I enjoy working on a task that is difficult because it allows me to work hard and reach my potential.....” (Learner A)

“challenging tasks motivates me.....” (Learner B)

“working on a challenging task is enjoyable to me because it develops resilience in me it also gives me an opportunity to learn from my mistakes.....” (Learner C)

“I enjoy working on these tasks as they motivate me to be better than others and compete with others.....” (Learner D)

From the remarks above, it was apparent that administering high order tasks motivates learners. As a result, most learners find the task enjoyable and meaningful. Learner A’s and Learner C’s statements indicate that higher-order tasks create meaningful learning; whereas learner B’s statement indicates that a challenging task requires intense focus and engagement. As a result, it requires learners to fully immerse themselves in the activity, so as to experience a sense of enjoyment and satisfaction. The findings above concur with those of the study by Reyes et al. (2012), where they discovered that when assessments are varied and allow learners to demonstrate their learning in different ways, they feel a great sense of ownership of a task, which leads to increased motivation.

Therefore, teachers should employ a variety of assessments (low order, middle order, and high order), as well as different assessment strategies to make the learning process enjoyable and meaningful. If teachers do not balance their tasks, learners will not engage in the assessment, as well as in the lesson. If learners do not engage in the lesson, it could affect their achievement in mathematics (Bakar & Ismail, 2020). Furthermore, social constructivism theory (Vygotsky, 1978) advocates learners’ engagement in an endeavour to improve learners’ performance (in this context, in mathematics).

Theme 2: Improved feedback and progress tracking

A second identified influence of the balanced assessment task was improved feedback and progress tracking. During focus group discussions with learners, they pointed out that feedback is essential to them as it provides them with knowledge of their weaknesses and strengths. Therefore, learners are able to see where they need to improve in their learning. These were evidenced by the transcript below:

“it is important that my teacher provide me with feedback so that I can see my strengths and weaknesses...” (Learner A)

“it is important for teachers to give us feedback because it helps to avoid misunderstandings ...” (Learner B)

“...feedback is important because it gives us clarity” (Learner C)



“...teachers should give us feedback because it helps us to see our progress” (Learner D)

“...feedback is important to me because I get more focused in learning and encourages me to work harder.... ” (Learner E)

From the transcript above, it was apparent that giving feedback is essential and improves a learner's engagement in an assessment task. Learner A stresses that feedback assists in identifying one's strengths and weaknesses. This indicates that the learner benefits from the feedback on assessment tasks, which might lead to improved achievement in mathematics. Learners B and C allude to the fact that feedback assists in the clarification of any misunderstandings that might have occurred during the assessment. This, again, is an indication that feedback is crucial and plays a pivotal role in fostering mathematics proficiency. Learner D indicated that feedback is good for tracking their progress. This is also prominent for the teacher so that they can change their instructional strategies. Lastly, Learner E stresses that feedback is good because it helps them to be more engaged in the lesson. This shows that feedback assists learners to pay attention and stay focused. According to a framework of social constructivism (Vygotsky, 1978) feedback is essential as it advances learners' mathematical and critical thinking. This, therefore, means that teachers should ensure that they give timely feedback after every assessment task that is administered.

The empirical findings from the interview session indicate that balancing assessment tasks by giving feedback influences a learner's engagement positively. The findings align with the findings of Shaikh et al. (2023) from their study, which indicated that feedback encouraged learners' interactions.

Theme 3: Reduced anxiety and stress

Reduced anxiety and stress were emphasised during a reflection session as one of the influences of balanced assessment tasks on learner engagement. During this session learners indicated that they become less stressed when teachers allow them to ask questions during an assessment task. Their comments were as follows:

“...I feel ok about asking my teacher questions during an assessment task.....”
(Learner A)

“I am not afraid of asking my teacher questions when I am writing a classwork, it also helps me to ease my frustration when working on a problem...” **(Learner B)**

“I feel free about asking questions and I would prefer that we are all allowed to do so because some of us get stressed when working on a math problem...” **(Learner C)**

“When my teacher allows me to ask questions during an assessment tasks I prefer to ask her in privacy it helps me with my anxiety...” **(Learner D)**

Learner A feels good about asking questions during an assessment task. This indicates that the learner is comfortable about asking questions for clarity. Learners B, C, and D indicated that



asking questions during assessment tasks assists in easing their stress, anxiety, and frustration. As a result, these learners are less prone to experiencing mathematics anxiety, which naturally enhances their engagement in the lesson. Balancing assessment tasks by allowing learners to ask questions influences learners' engagement in mathematics.

These findings concur with the findings of Attard and Holmes (2021), who found that learner disengagement in mathematics mostly occurred when learners were deprived of an opportunity to ask questions during an assessment task. More so, the findings indicate the need to consider creating a more supportive learning environment. According to Purnomo et al. (2021) effective engagement is essential, as learners with positive emotions and attitudes toward mathematics are more inclined to persist through challenges and succeed in the subject.

Theme 4: Fostering metacognitive skills

During another reflection session, learners indicated that they preferred working in groups and with peers. This is an indication that the assessment tasks that allow student collaboration in mathematics foster learners' metacognitive skills. This was evidenced by the comments they made:

"I prefer working with others its good because we learn from each other ..." (**Learner B**)

"it is good to work in collaboration with others we become competitive and challenge each other ..." (**Learner C**)

"... working in collaboration with other learners is interesting because I get to teach others what I have learned" (**Learner D**)

"... I like working in collaboration with other learners because we become more focused and we enjoy ourselves learning becomes fun" (**Learner E**)

From the remarks above, it was apparent that collaboration influences learners' engagement in an assessment task. Learner B prefers working with others, while learner E states that learning is fun when a teacher allows learners to work in groups and with peers. This suggests that peer assessments and group work are valuable for enhancing learners' understanding and boosting their engagement in the assessment process. Learner D thrives in collaborative settings, where peer teaching naturally takes place. This approach aligns with Vygotsky's (1978) constructivist theory, which emphasises the idea that learners gain knowledge through interaction with one another. This is, therefore, an indication that balancing assessment tasks by allowing learners to work together has a good influence on learner engagement in mathematics.

These findings concur with the findings Torrejos, (2024) that including peer assessment within a balanced assessment strategy helps learners develop metacognitive skills. This allows learners to reflect on their own learning and to learn from one another. Furthermore, learners' participation in the assessment allows learners to ascertain their learning progress and helps teachers make decisions about teaching processes.



Limitations of study

The study reported on the influence of balanced assessment tasks on learners' engagement in mathematics. Through the study, the influences of balanced assessment tasks on learners' engagement were discovered. Although the study has outlined the influences of balanced assessment tasks on learners' engagement, the study does not provide the influence of balanced assessment tasks on learners' achievement in mathematics. Furthermore, the study focused only on learner data through qualitative methods. Quantitative methods could enrich the study by presenting quantitative data to supplement the qualitative data. As this was a small-scale study centred on the experiences of learners and was conducted at a single school, the findings are not broadly generalisable. Consequently, we suggest that future research addresses these limitations to provide a more comprehensive understanding.

RECOMMENDATIONS

Based on the findings of this study, we recommend that participatory action research studies be conducted to explore and address challenges related to balancing assessment tasks. Such research could provide deeper insights into effective strategies for designing assessments that foster learner engagement, while also accommodating diverse learning needs and abilities. By involving educators, learners, and other stakeholders in a collaborative process, these studies can contribute to the development of more equitable and impactful assessment practices in mathematics education. In addition, a quasi-experimental study could be conducted to examine the impact of implementing balanced assessment tasks on learners' academic performance. Such studies could also offer valuable frameworks for designing and structuring balanced assessment tasks in mathematics, thereby ensuring they effectively enhance both learning outcomes and engagement.

CONCLUSION

This study addresses a critical issue in mathematics education: the influence of balanced assessment tasks on student engagement. By examining the research question, *"What is the influence of balanced assessment tasks on learners' engagement in mathematics?"* this paper identifies several positive impacts, including increased motivation; enhanced feedback and progress tracking; reduced anxiety and stress, and the promotion of metacognitive skills.

Based on these findings, the study recommends adopting innovative approaches to mathematics assessment to foster greater learner engagement. Specifically, balanced assessment tasks should be designed to provide meaningful and equitable learning experiences. Teachers should be equipped with a solid understanding of social constructivism theory, which will support the effective implementation of balanced assessments. Lesson studies are strongly suggested as a practical tool to deepen teachers' understanding of these tasks. Furthermore, frequent professional development programmes are necessary to empower educators to address the complexities inherent in mathematics assessment and ensure the successful application of balanced practices.



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CONNECTING VAN HIELE'S THEORY WITH UBUNTU PHILOSOPHY IN TEACHING GEOMETRY IN PRIMARY SCHOOLS

Zoleka Gula and Zingiswa Jojo

University of the Western Cape

Rhodes Univeristy

This qualitative study explores how Indigenous Knowledge Systems (IKS) and Ubuntu values can empower mathematics teachers in enhancing geometry teaching in primary schools. This work is grounded by theoretical foundations of Ubuntu philosophy and van Hiele's theory of geometric thought. The study addresses the research question: How can primary school mathematics teachers be empowered to integrate Ubuntu and IKS in the teaching of geometry? Using criterion sampling in a phenomenological case study design, three experienced Grade 5 mathematics teachers in using indigenous artefacts from rural schools in the Alfred-Nzo East District, Eastern Cape, South Africa, were purposively selected from a population of 60 intermediate phase mathematics teachers. Three lessons for each participant were observed, and they were interviewed on how they integrated IKS and used Ubuntu principles. Findings indicate that teachers emphasized cooperative learning through IKS and used real-life contexts to facilitate understanding of geometric concepts. Despite challenges such as overcrowded classrooms, limited resources, and varying teacher preparedness, Ubuntu values played a critical role in fostering inclusivity, peer-assisted learning, and collaboration. The study recommends regular professional development workshops on cooperative learning through Ubuntu and IKS integration to strengthen teachers' pedagogical and subject-specific competencies in mathematics education.

Keywords: Indigenous knowledge systems, Ubuntu, geometry education, cooperative learning, primary school mathematics, professional development.

INTRODUCTION AND BACKGROUND

Teachers can apply the wisdom of *Ubuntu* as an ancient African philosophy to create more humane and productive mathematics classrooms. Emphasis should be on interconnectedness, collaboration, and mutual respect in education. *Ubuntu* is often expressed as "I am because we are," reinforcing the idea that knowledge is collectively built through shared experiences (Gade, 2012). This study adopts *Ubuntu* philosophy as a pedagogical lens to strengthen cooperative learning in mathematics, particularly in the teaching of geometry in rural primary schools. Despite South Africa's curriculum reforms promoting cultural inclusivity, many mathematics classrooms remain dominated by Western content and teacher-centred instruction, with limited integration of IK or *Ubuntu* values (Mutekwe, 2015; Naicker, 2021). There is limited empirical evidence on how these philosophies can be practically applied in teaching geometry. This gap necessitates studies that connect *Ubuntu* and IK with concrete mathematics teaching frameworks like van Hiele's theory.

Ngubane and Makua (2021) highlight *Ubuntu*'s five pillars, namely: solidarity, co-existence, compassion, respect, and dignity as essential values in fostering inclusive and supportive





learning environments. In the context of South African mathematics education, "solidarity" can refer to collective efforts and support to improve math outcomes, addressing challenges like underperformance, inequality, and the need for a strong mathematical foundation for all students and the country's future (Leepo, 2015). Co-existence, according to Modetse et al. (2023), incorporates the role played by mathematics in interdisciplinarity and the links between mathematics and various situations related to everyday life, professional needs, citizenship, and understanding of the world.

Karpman and Roldan (2021) assert that compassion in mathematics education involves fostering empathy and understanding for students' emotional experiences and struggles with mathematics, while creating a supportive and inclusive learning environment. Respect and dignity are implicated in the Sustainable Development Goal (SDG 4), which strives to ensure inclusive and equitable quality education and the promotion of lifelong learning opportunities for all. Equity refers to the principle of fairness, which implies making sure that personal and social circumstances like gender, socio-economic status, or ethnic origin are not a barrier to accomplishing educational potential. With *Ubuntu* values, primary school mathematics teachers should be prompted to create opportunities for peer learning and collaborative problem-solving.

Despite recognition of these values in policy and discourse, a clear research gap remains. Few empirical studies have explored how *Ubuntu* and IK can be practically integrated into the teaching of geometry in primary schools, especially in rural contexts. This study aims to address that gap by examining teachers' lived experiences and how these philosophies enhance geometry instruction.

LITERATURE REVIEW

Geometry is not only foundational for developing spatial reasoning and critical thinking (Alex & Mammen, 2016), but it also underpins learners' broader mathematical proficiency. In South African primary schools, however, geometry instruction faces challenges including overcrowded classrooms, limited resources, and inadequate teacher training. Learners often encounter difficulties in recognising, classifying, and reasoning about geometric figures due to abstract delivery methods and language barriers. This has resulted in persistent underperformance, particularly in under-resourced and rural contexts where culturally disconnected pedagogies prevail (Spaull, 2019).

IKS supports contextualised learning by linking cultural practices such as beadwork, basket weaving, and indigenous games, with mathematical concepts like symmetry, measurement, and pattern. Such cultural artefacts, including *ukhukho* and *ingceke*, enable learners to visualise 2D and 3D shapes through familiar objects. As Matindike and Ramdhany (2024) argue, integrating indigenous knowledge promotes cultural pride and fosters sustainable, equitable learning. Ntuli (2022) emphasizes IKS's relevance to the African Renaissance, advocating for its recognition in formal curricula to empower communities and bridge cultural gaps. Breidlid (2009) similarly positions indigenous knowledge as a holistic alternative to dominant Western paradigms, calling for the rehumanisation of education through place-based knowledge.



The integration of IK into mathematics education has emerged as a meaningful strategy for improving geometry instruction in rural primary schools. The challenges faced by these schools, such as overcrowded classrooms and limited teaching resources, necessitate innovative, contextually grounded approaches. Susilawati et al. (2017) argue that teaching geometry should enable learners to see the connections between geometric properties and their everyday experiences. Rural communities possess a wealth of indigenous knowledge that can be leveraged to enhance mathematical understanding. Hilbert and Cohn-Vossen (2021) describe geometry as an investigative branch of mathematics that links history, culture, and design, making it a rich space for integrating real-world applications through culturally embedded practices.

Mathematics education in South Africa has long been criticised for its reliance on Eurocentric instructional models, often neglecting African cultural contexts (Mutekwe, 2015). African philosophy, as described by Sogolo (1990), encourages the integration of folklore, traditions, and indigenous practices into education to make learning more relevant. This aligns with efforts to decolonise the mathematics curriculum by drawing from learners' cultural realities. Dynamic teaching strategies that engage learners through active participation reflect these principles, reinforcing the importance of experiential learning. For instance, painting traditional rondavels with geometric patterns or classifying and measuring agricultural produce during harvest are practical examples that embed mathematics in learners' lived experiences (Couto & Vale, 2014).

Geometry plays a central role in mathematical development, supporting learners' logical reasoning and spatial problem-solving skills (Alex & Mammen, 2016). Culturally responsive frameworks, such as Ubuntu and IKS, support geometry learning by embedding it in familiar contexts. Naicker (2021) describes Ubuntu as affirming humanity through interconnectedness. In mathematics classrooms, Ubuntu encourages cooperative learning where learners explore geometric ideas together, building both conceptual understanding and cultural identity.

Spatial reasoning is recognised as a foundational skill for advanced mathematical thinking (Clements & Battista, 2012; Sarama & Clements, 2014). Providing learners with hands-on and interactive experiences supports their development of spatial awareness and geometric understanding from an early age. Susilawati, Suryadi, and Dahlan (2017) further recommend informal engagement with geometry prior to formal instruction. This underscores the relevance of using Ubuntu values and IKS to support learners' progression in geometry.

Several studies have explored learners' van Hiele levels of geometric thinking (Tan et al., 2015), as well as geometry instruction in African classrooms (Tabassum et al., 2024). Few studies explore how *Ubuntu* and van Hiele's theory can be jointly applied in geometry instruction. This study builds on this gap by showing how *Ubuntu* values (respect, collaboration) reinforce van Hiele's structured progression, particularly through cooperative tasks aligned to instructional phases.



THEORETICAL FRAMEWORK

This section draws on two theoretical frameworks, van Hiele's theory of geometric thinking and *Ubuntu* philosophy, to guide the design, implementation, and interpretation of the study. These theories were selected because they provide complementary insights into both the cognitive and socio-cultural aspects of teaching geometry in primary schools.

Van Hiele's Theory describes how learners progress through five hierarchical levels of geometric thinking: (1) Visualization – recognition of shapes by appearance. (2) Analysis – focus on properties of shapes. (3) Informal Deduction – logical reasoning about relationships between properties. (4) Formal Deduction – use of theorems, axioms, and proofs. (5) Rigour – advanced understanding of geometric systems. The theory also outlines five instructional phases: information, guided orientation, explication, free orientation, and integration (van Hiele's, 1986; Clement & Battista, 1992) In this study, van Hiele's theory was used to inform lesson planning and structure observation tools. Teachers were observed introducing geometry through concrete artefacts before guiding learners toward abstract reasoning, aligning with the van Hiele levels and instructional phases. This ensured developmentally appropriate instruction that matched learners' cognitive readiness.

Ubuntu philosophy, expressed as “*Umuntu ngumuntu ngabantu*” (a person is a person through others), places emphasis on interconnectedness, collaboration, mutual respect, and shared humanity (Letseka, 2012; Waghid, 2014). These values promote inclusive and learner-centred teaching. In this study, Ubuntu informed the social and relational aspects of the learning environment. Teachers encouraged peer learning, respectful group interaction, and the sharing of IK and resources. Learners were observed discussing geometric problems in groups, helping each other, and valuing input from elders and their own cultural backgrounds, key markers of *Ubuntu*-informed pedagogy.

Van Hiele's theory provided a structured cognitive pathway for developing geometric understanding, while *Ubuntu* philosophy supported a collaborative and culturally responsive classroom climate. The integration of the two allowed for: (1) Culturally embedded resources (e.g., *ingceke*, beadwork) to be introduced at the visualization level. (2) Group-based activities during guided orientation and explication, in line with Ubuntu's emphasis on shared learning. (3) Learners advancing their understanding through dialogue, cooperation, and mutual respect, aligning *Ubuntu* values with van Hiele's instructional phases.

Thus, the study framed geometry teaching as both a cognitive and relational process, underpinned by developmentally appropriate instruction (van Hiele) and culturally embedded cooperative learning (*Ubuntu*).



METHODOLOGY

This study employed a qualitative research approach within a phenomenological case study design to explore how IK and *Ubuntu* philosophy could be used to enhance geometry teaching in rural primary schools. The phenomenological approach was chosen to capture the lived experiences of mathematics teachers as they applied indigenous artefacts and cooperative learning strategies in their teaching practices.

Participants were selected through purposive sampling. From a population of 60 intermediate-phase mathematics teachers in the Alfred Nzo East district of the Eastern Cape, three experienced Grade 5 mathematics teachers were chosen based on their willingness to participate and their familiarity with local indigenous practices relevant to geometry instruction. Data were collected using lesson observations and semi-structured interviews. Each participant taught three geometry lessons observed by the researcher, followed by reflective interviews about their classroom practices. The Kotoka (2012) observation tool was utilised as an instrument to record actions in the classroom. This tool looks at the teachers' teaching styles, in reviewing learners' previous knowledge, teaching aids used, simplification of difficult concepts and the relevance of examples used in teaching geometry.

Data analysis followed Braun and Clarke's (2006) six-phase thematic analysis approach. These phases include: (1) familiarisation with the data, (2) generating initial codes, (3) searching for themes, (4) reviewing themes, (5) defining and naming themes, and (6) producing the report. After transcribing the interview data and observational notes, initial codes were generated inductively based on recurring patterns across participants' responses.

The study used open thematic coding, allowing patterns to emerge from the data without imposing predetermined categories. Themes were then refined and grouped according to the research questions, with the theoretical framework (*Ubuntu* philosophy and van Hiele's theory) used to interpret and structure the final themes. This helped ensure that the findings remained grounded in the participants' voices while being analysed through a culturally responsive lens.

Triangulation of data sources enhanced the credibility of the findings by cross-verifying patterns. Reflexivity was maintained through reflective journaling by the researchers after each school visit to minimise bias and document analytical decisions.

The study obtained permission and ethical clearance (reference 2020/06/10/37351109/13/AM) from the institution's ethics committee and participants' prior data collection. In this study, data were analysed thematically, following the stages from coding, categorising, and the development of themes (LeCompte & Goetz, 1982; Peel, 2020). To ensure confidentiality, teachers from the three selected schools were assigned these codes for the duration of the study. The pseudonyms PA, PB, and PC were used throughout the study to represent Participants A, B, and C.



RESULTS

Four themes emerged from the data analysis that has to do with teaching geometry in primary schools using indigenous knowledge. These were: (1) indigenous artefacts and learner engagement, (2) aligning geometry content with curriculum and situated learning, (3) Fostering effective teaching pedagogies, and (4) use of indigenous artifacts to teach geometry. Each of these themes included categories that showed how the participants' use of indigenous knowledge facilitated the teaching of mathematical relationships, interactions and activities.

Indigenous artefacts and learner engagement

PA taught mathematics to a Grade 5 class of 103 learners, where overcrowding made participation difficult due to limited space and excessive chairs. To ensure effective learning, she adopted a creative and empathetic approach, integrating indigenous examples into her geometry lessons. The activities she used to assess learners' prior knowledge were essential in engaging them and building on what they already knew.

During one lesson, PA instructed learners:

PA: Take a string from the material I requested you to bring to the class. Use a string you have to make imbeleko yekati.

Learners became excited and engaged in animated discussions with their peers. Those without materials sought help from classmates, demonstrating Ubuntu values without direct teacher instruction. As the activity progressed, PA noticed that many learners struggled and guided them further:

PA: Ask a person next to you to assist you in making imbeleko yekati.

Despite working in pairs and groups, some learners still found the task challenging. PA identified a learner who had successfully created *imbeleko yekati* with multiple shapes and asked:

PA: Can you please come up front and teach other learners step-by-step how to come up with this kind of imbeleko yekati?

By inviting a learner to demonstrate, PA reinforced collaborative learning and problem-solving, aligning with the Ubuntu philosophy.

Extract 5.2 illustrates how the teacher incorporated peer learning in the classroom.



Figure 1: Peer learning on making *imbeleko yekati*

The participant then asked the learners to mention the 2-D shapes they had formed while making *imbeleko yekati*. Some learners approached the teacher, demonstrating their sense of freedom during the activity. Instead of only focusing on *imbeleko yekati*, they shared other shapes they could create, such as *unyawo lwenkukhu* (chicken foot).

Encouraging their creativity, PA responded:

PA: Please carry on with what you can make using a string and mention the shapes you can find.

To further support learning, PA asked one learner to use a grid board to draw an example of *imbeleko yekati* created by a classmate for the entire class to observe. As learners raised their hands to indicate recognition of shapes, PA directed them:

PA: One learner at a time must come to mark the shape and also tell us the name of that shape.

Figure 1 illustrates the various shapes identified by the learners in relation to *imbeleko yekati* for teaching different 2-D shapes. This hands-on approach demonstrated the participant's awareness of the importance of active engagement in learning. It also served as a peer-based feedback strategy, making the lesson more interactive and enjoyable for the learners.

Aligning geometry content with curriculum and situated learning

PA integrated real-world examples to help learners see the relevance of geometry in everyday life, promoting situated learning. For example, she used a soccer ball drawing from a Life Skills lesson to teach 2-D shapes. This approach connected IK and real-world experiences, enhancing learners' understanding of geometric concepts. Figure 2 shows the picture PA used to relate geometry to real-life contexts.



Figure 3: *Relating mathematics with real-life context*

The teacher asked learners orally:

PA: *“What are the shapes you see on the chalkboard?”*

Initially, the learners responded based on the chalkboard itself rather than focusing on the drawings. PA clarified that they should look at the drawings to identify the shapes. After the learners responded, PA explained:

PA: *“People practice mathematics in daily activities. The sports field is an example of how mathematics is used in real-life contexts. In a ball, there are different shapes used to decorate it.”*

By using the Life Skills drawing on the chalkboard, PA fostered interdisciplinary connections, linking geometry to other subjects and helping learners understand the relevance of geometry in different contexts. PA also integrated geometry with other subjects, using the Grade 6 NS & Tech project. She asked:

PA: *Look at the different Houses and mention the shapes you see.*

The learners identified various shapes, and PA further asked them to mention the different 3-D shapes of the house and justify their answers:

PA: *Look at these houses again and give the name of the 3-D for each.*

This approach demonstrated PA’s awareness of cross-curriculum integration, where geometry is connected with other subjects to enhance learners' understanding.



Figure 4: *Integrating mathematics with other subjects through IKS*

Fostering effective teaching pedagogies

During classroom observations, I noticed that the participants used various strategies to cater to diverse learning needs, offering different teaching approaches, materials, and assessments.



For example, PA used mental activities to reinforce topics and connect prior knowledge with new concepts. She engaged learners by using objects like a ball to introduce 3-D shapes, encouraging them to identify geometric objects and relate them to real-life contexts. PA also referenced a ball drawing from the Life Skills lesson, asking learners to name the object.

She then demonstrated rolling and sliding using a ball and tin. Figure 5 shows a learner holding the ball and tin used during the demonstration.



Figure 5: Use of real-life objects to teach 3-D shapes

PA's teaching strategy scaffolded learning, helping learners gradually build their understanding. This was evident when she used a string game for learners to create *imbeleko yekati* and investigate different shapes through guided discovery. Additionally, PA encouraged cooperative learning by promoting teamwork and peer support.

In her overcrowded classroom, some learners were seated in groups of four. PA instructed them:

PA: You will work in groups for the activity, but please do not move around to avoid wasting time. Form groups with the people behind or in front of you. If you prefer, work with those at your desk as you are seated in threes and fours.

PA then directed them to take their DBE Workbooks and complete the activity using the materials they had:

PA: Take your workbook and open to page 113. Make the objects using the materials you have. After you have finished, answer the questions based on each object.

PA instructed the learners to discuss their answers before writing them down, emphasizing that every group member should fully understand the responses. This approach promoted discussion and peer learning. She said:

PA: Please discuss before doing anything. Ensure that your group members are satisfied with your responses by supporting answers to convince them.

She further emphasized:

PA: No one should write answers without discussing and coming to a common agreement with members. Encourage other members to share their views and respect



one another's opinions, even if they do not make sense. Convince one another politely with reasons.

By encouraging discussion before writing, PA reinforced Ubuntu values of mutual respect and collaborative decision-making:

PA: Please respect one another in your groups and respect their views even if they do not make any sense. Convince one another politely by giving reasons why you are not agreeing with his or her answer. Build confidence in one another.

Through this, PA's approach aligned with cooperative learning strategies, promoting an inclusive, collaborative environment that supported deeper mathematical understanding.



Figure 6: Show learners working as groups to make models.

Through the activity of making 3D objects with concrete materials, learners had the opportunity to investigate the relationship between edges, faces, and vertices. They were also asked to present their responses in a table, which showcased their work in PA's class.

When asked during the semi-structured interview why she chose group work instead of individual assignments, PA responded:

PA: The learners will be free around their peers and will be able to discuss while learning from one another.

Regarding her selection of examples for the lesson, PA explained:

PA: I choose to employ situations that the students are accustomed to. There are both social and economic means of subsistence in this society. They construct different home houses as they did for the NST project and also play indigenous games like string or rope games and different sports. I felt it relevant to introduce my learners to properties of 2-D and 3-D shapes using examples like imbeleko yekati, soccer balls, and different house models, as I assumed every learner knew what was depicted in the photographs. I also compared the cylinder with tin and marshmallow sweets the learners had.



This reflects the teacher's awareness of using indigenous artifacts to enhance primary school learners' understanding of geometry. PA expanded the scope of geometry teaching to ensure learners connected with real-life, familiar contexts.

Use of indigenous artifacts to teach geometry

PB taught mathematics in a rural, under-resourced school. To introduce the lesson, PB posted a picture of a traditional wooden trailer (*isileyi*), which immediately sparked discussion among the learners. When asked why they were talking, one learner responded, *"I thought it was because of the pictures you posted, Sir."*

PB encouraged learners to comment on the images, and they identified *isileyi*, explaining its use for transporting maize, pumpkins, beans, and other harvests from gardens (*emasimini*). The teacher then asked:

PB: *Do you know (wooden shell structure) udladla? What is the difference between udladla and isileyi?*

Learners shared their understanding. One explained, *"Udladla is used for storing maize until it dries, while isileyi transports the harvest from the garden to the home."* Another added, *"Udladla is fixed in one place, but isileyi can be moved."*

PB then guided learners to analyze the geometric shapes in these structures, transitioning them from recognizing shapes to describing their properties.

PB: *What shapes are used to make udladla or isileyi?*

Learners identified shapes such as rectangles, cubes, cylinders, and, in some cases, triangles. PB further deepened their thinking:

PB: *Have you observed the angles formed when making udladla or isileyi?*

This question guided learners toward a more analytical approach as they began noticing geometric properties they had previously overlooked. With smiles and head shakes, they honestly admitted, *"No."* One learner spontaneously exclaimed, *"Yho, sir! I never thought that udladla or isileyi is also the practice of mathematics at home. I just noticed now when you asked about the angles formed in the objects."*

This realization underscores how Ubuntu-inspired discussions help learners recognize the mathematical significance of cultural artifacts.



Extract 4: The example of *isileyi* to teach geometry in a real-life situation

The teacher then displayed pictures of reed mats (*ukhukho*) and baskets (*ingceke*), sparking excitement and laughter among the learners.

PB: *Why are you so excited?*

One learner responded, *"I'm wondering why you're posting pictures of these things, Sir."*



Extract 5: Ukhukho to relate geometry with IK

The teacher used the posted pictures to enhance learners' understanding of geometric concepts.

PB: *What shape is used to make ukhukho?*

Learners mentioned rectangles and squares. One learner added that *ukhukho* features rhombus patterns created with string for decoration.

To introduce measurement, PB asked:

PB: *What measuring instruments are used to make one of these objects?*

Some learners laughed, and one responded, *I have never seen any measuring instrument used. I think they just estimate.* Another learner added, *"My grandmother uses a special wooden ladder when making ukhukho. I think that helps her measure the sizes."*



By integrating indigenous objects like *isileyi* and *ukhukho*, PB demonstrated how IK supports mathematical understanding. To explain length, he asked:

PB: *How long is a metre?*

He then instructed a learner to step forward and demonstrate, reinforcing experiential learning and spatial awareness.



Figure 7: *Relating measurements with IK*

In PB's classroom, learners worked in groups for a measurement activity.

PB: *Form groups by turning your desks to face those behind you.*

This instruction, rooted in Ubuntu philosophy, encouraged cooperative learning.

PB: *Work in groups to complete the practical activity. Everyone must participate, no playing around."*

This collaborative approach reinforced learning through hands-on engagement. Some tasks required a tape measure instead of a ruler, such as measuring the height of a door and a wall. Before starting, PB asked a learner to demonstrate by pointing to the height and length of the classroom door (Figure 8).

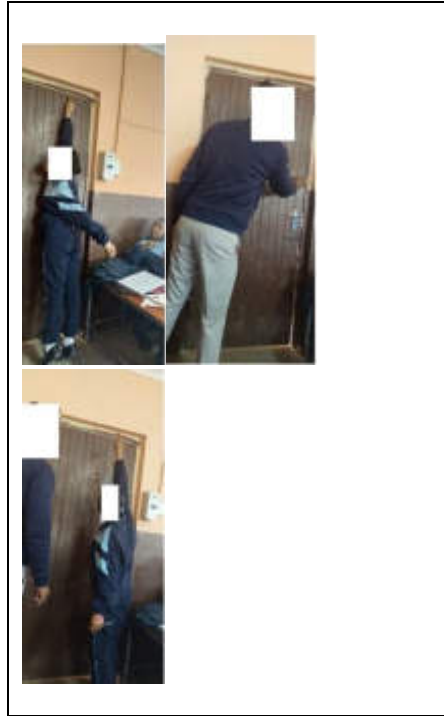


Figure 8: *Relating measurements with things around the classroom*

As learners prepared for the activity, one asked, “Sorry Sir, what are we going to use to measure the door and wall since you gave us rulers?” In their groups, learners were hands-on, with some recording while others measured. Figure 9 shows learners using a tape measure to take measurements in the classroom.



Figure 9: *Relating measurements with real-life context*

When asked about the significance of indigenous artifacts, PB explained:

PB: I like these artifacts because they easily introduce my learners to estimations and measurement of distance, and they help revise shapes and capture learners' attention.

In reflecting on his teaching, PB noted:

PB: Through what learners already know, they become interested and actively participate. The strategies work because learners discuss shapes related to indigenous artifacts they use at home. This helps draw their interest to the topic.



PB's reflection highlighted how using familiar materials enhances learners' motivation and understanding of geometry. When asked about choosing relevant indigenous artifacts for lessons, PB advised:

PB: One should observe how learners play and think of familiar things. Then, you can ask the right questions about those artifacts to lead them to understanding mathematical concepts.

This suggests that PB recognizes the value of using indigenous knowledge to improve learners' comprehension of geometry, leveraging the artifacts and games learners are already familiar with.

During observations, it was noted that PC also explained several geometric ideas to learners using well-known indigenous artefacts. Once in the classroom, the teacher decided to divide the learners into groups.



Extract 6: Beadwork, honeycomb and ulusu to relate geometry with IKS and nature

The teacher used images of indigenous food, such as honeycomb and tripe (*ulusu*), along with artifacts to connect mathematics to learners' everyday experiences. The pictures excited the learners, with some murmuring enthusiastically and others associating them with traditional ceremonies (*umgidi*).

To engage them further, the teacher instructed:

PC: I know you are familiar with these pictures, but I want you to take a sheet of paper and quickly draw at least two shapes you can identify. You may draw more if time allows. Discuss with your group before drawing.

This activity encouraged learners to recognize geometric shapes in familiar cultural contexts while fostering discussion and collaboration.

Learners actively participated by drawing shapes they identified in the pictures. By guiding them to compare similarities, differences, and relationships in shape sizes, PC helped them transition from recognizing geometric figures to analyzing their properties. The activity introduced concepts like symmetry, internal and external angles, and geometric properties, reinforcing their development in geometric thinking.



When researchers asked why the teacher used beadwork, traditional attire, honeycomb, and tripe in the lesson, TC explained:

PC: My intention was to teach geometric concepts through indigenous artifacts and familiar natural objects. Children often wear traditional attire and beadwork during Heritage Day celebrations and traditional ceremonies. They also eat traditional foods like ulusu. In rural areas, honeycombs are common, and learners often see them in their surroundings.

By incorporating indigenous artifacts like beadwork, honeycombs, and traditional attire, PC created a culturally responsive learning experience that deepened learners' engagement with geometric concepts.

When researchers asked whether indigenous knowledge is important for teaching and learning mathematics, PC responded:

PC: Yes, indigenous knowledge is important because it allows learners to relate mathematics to real-life activities. Shapes are used daily in designing traditional attire and making beadwork. Additionally, geometry is found in nature, making it easier for learners to connect with mathematical concepts.

When asked about the geometric concepts he could teach using nature and indigenous artifacts, PC responded:

PC: I can teach many geometric concepts such as lines, shapes, angles, measurements, patterns, tessellation, and transformation.

The researchers further inquired if there were additional reasons for selecting the pictures beyond connecting mathematics to indigenous knowledge. By fostering collaboration and teamwork, PC integrated Ubuntu principles of mutual respect, collective learning, and shared knowledge in the classroom. He replied:

PC: Yes, I wanted to promote Ubuntu values in the classroom while encouraging reasoning and critical thinking. The activity fostered teamwork as learners shared their views and had the freedom to express their understanding.

Through peer interaction, PC fostered an environment where learners engaged freely, reinforcing Ubuntu values such as empathy, collaboration, and shared learning.

PC: It is important for learners to experience mathematical concepts in real-life contexts. Beadwork and traditional sewing enhance critical thinking, reasoning, and hands-on learning. These activities require attention, listening to instructions, and asking questions for clarity.

By integrating cultural elements into mathematics lessons, PC emphasized the significance of real-world applications in geometric learning.



Researcher: *Were you promoting any specific values by using these pictures? How would you justify teaching through them?*

Recognizing the role of indigenous knowledge in fostering reasoning and problem-solving, PC responded:

PC: *This activity reinforces Ubuntu philosophy by promoting critical thinking and reasoning. Learners share ideas in a supportive manner, fostering participation and collaboration.*

This approach encouraged learners to embrace Ubuntu values through respectful discussions and cooperative learning.

DISCUSSION

This study highlights the integration of van Hiele's theory, *Ubuntu* philosophy, and IK in primary school geometry teaching. Learners engaged in structured activities that supported their geometric thinking while being immersed in cultural and linguistic contexts. Using indigenous games, artifacts, and real-life applications made mathematics more relatable and meaningful. Aligned with van Hiele's Level 0 (Visualization), *learners recognized geometric shapes in cultural artefacts, and later progressed to Level 1 (Analysis)*, where they identified and described properties of shapes (van Hiele's, 1986). This transition was observed as learners moved from simply naming shapes to articulating attributes such as faces, edges, and symmetry. For instance, learners identified symmetry in beadwork and classified 3D shapes in traditional huts, scaffolding their reasoning towards deeper geometric understanding (Clements & Battista, 2012 and Susilawati et al., 2017).

Van Hiele's instructional phases were also evident in the teaching approach, teachers guided learners through information and orientation phases by using local artefacts, then moved to explication by prompting learners to describe the properties in their own words. One learner spontaneously exclaimed, *"Yho, sir! I never thought that udladla or isileyi is also the practice of mathematics at home. I just noticed now when you asked about the angles formed in the objects.* These responses reflect active learner engagement at the visual and descriptive levels of van Hiele's geometric thinking.

Ubuntu philosophy shaped the cooperative nature of these learning environments. Values such as solidarity, shared responsibility, and respect were promoted as learners worked in groups and helped one another during mathematical tasks (Naicker, 2021). Teachers emphasised collaboration in classroom discussions, encouraging learners to share ideas and solve problems in groups, thus embodying the communal ethics of *Ubuntu* (Ngubane and Makua, 2021).

IK provided meaningful learning experiences, with artifacts like *ingceke* (woven baskets) and *udladla* (grain storage huts) illustrating geometric principles such as pattern, shape, and volume. This reflects the van Hiele model's emphasis on using concrete, familiar experiences



as the starting point for abstract reasoning. For example, in PA's lesson, learners realised that household structures embodied mathematical ideas.

This communal exploration of knowledge aligns with *Ubuntu's* emphasis on co-construction of meaning, where knowledge is shared, negotiated, and constructed collectively rather than imposed unilaterally (Couto & Vale, 2014; Susilawati et al., 2017). Ubuntu thus complements van Hiele's progression model by creating relational learning environments that support geometric development.

The use of isiXhosa alongside English facilitated conceptual understanding by allowing learners to express their ideas without language barriers. This supports van Hiele's claim that language is key in helping learners move from visual recognition to conceptual abstraction (Van Hiele, 1986). Teachers alternated between the two languages during explanations and discussions, promoting both comprehension and inclusion.

Hands-on activities involving measurement estimation, such as estimating a meter using physical steps, further embodied van Hiele's principle of linking geometry to movement and real-world tasks. This aligns with *Ubuntu* values of learning through doing together and helps learners anchor abstract concepts in lived experience (Modetse et al., 2023).

This study shows how theoretical principles from van Hiele's model and ethical values from *Ubuntu* philosophy can work synergistically to support effective geometry teaching. Culturally embedded artefacts, cooperative learning structures, and the use of indigenous language all contributed to a learning environment where learners were both cognitively and socially engaged. This alignment of pedagogy, culture, and theory validates the use of *Ubuntu* strategies for enhancing geometric understanding in primary schools.

Limitations of the Study

This study was conducted in three rural primary schools within the Alfred-Nzo East District of the Eastern Cape, South Africa. While this setting provided rich, context-specific insights into the integration of Ubuntu and Indigenous Knowledge Systems (IKS) in geometry teaching, the small, purposive sample limits the generalisability of the findings to broader populations.

Time constraints during classroom observations also posed a limitation, as school timetables and teaching schedules restricted prolonged engagement. Additionally, the research relied primarily on teacher perspectives and classroom observation, meaning learner voices were only indirectly represented.

Despite these limitations, the study offers valuable contributions by illustrating how Ubuntu-informed pedagogy and Van Hiele's geometric theory can support culturally responsive mathematics teaching.



CONCLUSION

The findings reveal that using indigenous artefacts, games, and cultural practices creates a rich learning environment where learners actively engage with geometry through familiar experiences. The study highlights the importance of Ubuntu philosophy in promoting collaborative learning, where learners support one another and share their knowledge. Teachers played a critical role in facilitating peer-assisted learning and incorporating real-life examples to help learners visualize geometric concepts. The use of indigenous knowledge allowed learners to develop problem-solving skills and critical thinking abilities while connecting their learning to their everyday environments. Furthermore, the study emphasizes the role of language in geometry instruction. Teaching in both English and home language enabled learners to express their ideas more clearly and grasp mathematical concepts more effectively. By integrating indigenous knowledge and culturally responsive teaching strategies, teachers created an inclusive learning space that values learners' backgrounds and lived experiences. The study recommends that teacher training programs focus on equipping teachers with skills to integrate Ubuntu principles and indigenous knowledge into mathematics teaching. Additionally, curriculum developers should consider incorporating real-world applications and indigenous contexts into the mathematics syllabus to bridge the gap between theoretical concepts and learners' lived realities.

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THE ROLE OF TECHNOLOGY IN ENHANCING MATHEMATICS TEACHING: PERSPECTIVES FROM SOUTH AFRICAN PRE-SERVICE TEACHERS

Rosina Nkadi Ngoako and Pule Kereng Gilbert

University of South Africa

Technology in mathematics education serves as a valuable resource to enhance teaching and learning through interactive experiences. This enables lecturers to equip pre-service teachers with the essential foundation to incorporate teaching methods like Technological Pedagogical Content Knowledge confidently. This qualitative study uses classroom observations and open-ended questionnaires to explore pre-service teachers' preparedness to integrate technology into mathematics instruction. A questionnaire was administered to a sub-sample of ten pre-service mathematics teachers, who responded to assess their knowledge and self-perceived competencies. At the same time, feedback on observations reflected the actual implementation at the analytical levels. The questionnaire offered valuable insights into pre-service teachers' attitudes, skills, and challenges regarding integrating Information and Communication Technology into their teaching practices. The findings revealed pre-service teachers' readiness to integrate technology into mathematics instruction, bringing practical lessons from the South African educational setting. The findings shed light on how teacher education programmes can improve teacher preparation and how they can incorporate the practical use of technology in teaching. Targeted professional development initiatives should be implemented to address pre-service teachers' attitudes, skills deficits, and challenges in integrating technology, ensuring preparedness and proficiency in utilising technology to improve mathematics teaching.

Keywords: *Technology integration, information and communication technology (ICT), pre-service teachers, teaching practice, TPACK.*

INTRODUCTION

Teachers' competencies in teaching mathematics play a crucial role in learners' conceptual understanding and achievement. The same can be said about teachers' competencies in integrating technology into teaching because 21st-century skills and the fourth industrial revolution (4IR) have since evolved to respond to economic and societal problems in different countries (Lee et al., 2018). The Department of Basic Education (DBE, 2001) identified some curriculum and instructional barriers to learning, some of which are processes and methods used in teaching, including learning materials and equipment. In addressing these barriers, DBE (2001) suggests that the teaching and learning process be flexible enough in all types of schools to accommodate learners' needs.

Various technological tools and teaching strategies are now used to support learning and enhance collaboration among learners, with the most popular being WhatsApp, because teaching based only on traditional methods has become obsolete (Rhaman et al., 2021). This



calls for teachers to incorporate more structured and comprehensive practices that align with technology integration for meaningful learning. Technology integration in teaching and learning enables learners to learn quickly because learning experiences are concrete (Ajani & Govender, 2023). It also assists learners in seeking information and extends their collaboration outside of school using digital tools.

Although technology supports and enhances learning, some factors hinder its integration into teaching. For instance, teachers' beliefs and perceptions can affect their willingness to integrate technology into their teaching (Abel, Tondeur & Sang, 2022). Pedagogy is the art, science, or act of teaching (Hardman, 2019). It involves teachers' ideas and beliefs, and what Shulman (1986) describes as content knowledge that informs what and how to teach. Suppose teachers do not have positive beliefs about using technology. In that case, they may not see the importance of using information and communication technology (ICT) in teaching, and this can deprive learners of the opportunity to be exposed to innovative methods that can increase collaboration and meaningful learning. Mirzajani, Mahmud, and Luan (2015), in their literature review on obstacles that prevent the use of ICT in Malaysian pre-service teachers' educational courses, found that insufficient training and a lack of technological knowledge and skills were among the factors that prevented them from infusing ICT in their courses and called for the need to train pre-service teachers in ICT to prepare them for the effective use of ICT in teaching.

Since the outbreak of COVID-19, many researchers support the idea of infusing technology in teaching and learning (Ajani & Govender, 2023; Rhaman et al., 2021; Sabiri, 2020; Sanusi, Oyelere, & Omidiora, 2022), and many believe that ICT integration in teaching and learning has a significant impact on facilitating classroom pedagogy (Abel et al., 2022; Andyani et al., 2020; Vandeyar, 2020). In South Africa, the Gauteng province took a leading role in providing Grade 12 learners with tablets to encourage and promote the use of technology in learning; however, Ajani and Govender (2023) argue that the provision of ICT resources is not a guarantee that technology will be integrated with teaching and learning; instead, the training of teachers in ICT and grasping how to blend the pedagogical content knowledge (PCK), technological knowledge (TK), and technological pedagogical knowledge (TPK) are essential in supporting technology integration in teaching and learning. Engelbrecht, Borba, and Kaiser (2023), in their study of "Will we ever teach mathematics again in the same way we used to before the pandemic?" also support technology integration in mathematics teaching. In this study, 'technology' refers to digital tools and platforms such as laptops, projectors, educational software, and internet-based resources to enhance mathematics instruction.

Choice of relevant technological tools for teaching mathematics in classrooms

Multimedia and technology seem to be the most favourable way of engaging learners in learning. Tools like blogs, YouTube, videos, and gaming software are the most popular and helpful in providing meaningful learning opportunities (Taylor & Parsons, 2011). Teachers should have the knowledge and technological skills to assist them in selecting teaching and learning technologies for curriculum delivery (Ajani & Govender, 2023). Ajani and Govender



further argue that, when appropriate technologies are chosen, teachers must adjust their teaching approaches and align them with the chosen technologies, which has a bearing on learner performance since technology use improves engagement with the learning content.

Teachers who plan instructional activities well can effectively teach and learn (Burgess et al, 2020). Such activities include the use of teaching and learning aids, which both teachers and learners can use respectively to foster interactive learning. Since the outbreak of COVID-19, recent literature has been at the forefront of research on the usefulness of technology in teaching (Almufarreh & Arshad, 2023; Ching & Roberts, 2020; Onyema, 2020; Tachie, 2019; Turugare & Rudhumbu, 2020). For the past decade, teachers have been encouraged to use technology in instruction delivery to model concepts and bring real-life experiences into classrooms (Bingimlas, 2009; Buabeng-Andoh, 2012; Tondeur et al, 2012). For instance, in mathematics, teachers can use technological tools such as whiteboards, computers, and projectors while teaching functions in modelling graphs to remove the rigid structure associated with graphs. For example, showing the effect of graph of $f(x) = x^2 - 1$ and $h(x) = (x - 1)^2 - 1$. It can be beneficial to enhance learners' understanding of the effect of the parameters of p and q in graphs of parabolas. Therefore, integrating technology into teaching enhances understanding (Misirli, 2016). Furthermore, when using technological tools in teaching mathematics, teachers can promote learner interest and interaction (Buentello-Montoya, Lomeli-Plascencia, & Medina-Herrera, 2021; McDonald, 2016).

The ability to choose appropriate technologies for teaching requires the necessary technological knowledge (TCK) and TPK. If teachers lack TCK and TPK, choosing and infusing technology into their teaching will be burdensome. Ajani and Govender (2015) opine that teachers' professional development in integrating technology in teaching and learning should be ICT-based. Sanusi et al (2022) agree with this notion by saying that teachers should be capacitated for the adoption and maximisation of the use of ICT for effective teaching and learning.

Integrating technology in mathematics teaching

Several researchers, such as Attard et al. (2020), Derakhshandeh et al. (2023), Hardman (2019), and McBee et al. (2023), discussed the importance of ICT in teaching and its impact on learner attainment. Some studies associate the use of technology in teaching with constructivist pedagogy. However, the role of the teacher is still crucial in modelling instructional activities because learners are often unable to construct knowledge independently (Hardman, 2019). According to Vygotsky's (1978) Zone of Proximal Development (ZPD), teachers play a crucial role in structuring guidance to mediate learning activities. Integrating technology in teaching and learning requires expert knowledge; teachers should know different technological tools (TK) and how to use them to enhance learning (Ajani & Govender, 2023). Technology is a mere tool, but how it is used is important in making learning experiences effective. Practical, meaningful, and varied teaching strategies, like technology, can promote high-level thinking and equip learners with 21st-century skills (Rhaman et al, 2021). Training institutions are responsible for developing and producing a digitally literate workforce that can use their competencies when teaching and further develop them in their learners. This paper argues that,



when only traditional approaches are used in teaching, learners become passive and are not actively engaged as they are when innovative teaching methods are used. To engage learners meaningfully in learning requires teachers to integrate machines in teaching because it is what the present generation of learners uses in their interactions outside the school environment.

Ways of integrating technology in teaching to develop learner interaction

Learner engagement begins with the teachers' instructional approach (Rhaman et al, 2021), and it is one of the many constructs that have been at the centre of discussion in mathematics education research. ICT in teaching promotes learner interest and engagement (Engelbrecht, Llinares, & Borba, 2020; Ji, Han, & Ko, 2023) and can extend to self-regulation, reducing discipline problems in class. Technology integration is of more benefit to primary school learners because they are still inquisitive and inexperienced in technology. Because learning occurs in different forms, learners must be exposed to different ways of learning, including technology.

During COVID-19, some researchers (Jordan, 2023; Pimmer & Rambe, 2018; Sabarudin, Marzuki, & Khairunnisak, 2020) encouraged the use of social media such as WhatsApp and YouTube to allow learners to interact with the learning content, including giving the learners activities and assessments to do at home. This proved a successful method for teacher-learner and learner-learner collaborations to enhance learning. WhatsApp encouraged learners to seek information and construct knowledge outside the classroom, promoting self-regulated learning. Baguma et al (2019) argue that using social media in learning supports learners' development of higher-order thinking skills because learners are forced to construct knowledge independently rather than depending on their teachers.

The role of training institutions in preparing pre-service teachers for teaching

It is the role of training institutions to train teachers and ensure that they enter the school system and are fully prepared for the teaching profession, including the use of technology in teaching (Wang, Schmidt-Crawford, & Jin, 2018). The Department of Higher Education and Training (DHET) requires that newly qualified teachers should have, among others, highly developed knowledge and skills in numeracy, literacy, and information technology (DHET, 2015). Teacher training takes the form of theory and practical sessions. For practical sessions, student-teachers go for teaching practicums to observe, learn, and practice the art of teaching. Theoretically, students are trained on different subject contents and pedagogies. Most teaching programs, especially in content subjects, infused technology as part of their education courses, exposing students to different technological tools and software in teaching. However, Wang et al. (2018) argue that "merely having pre-service teachers complete these courses may not be enough for the knowledge transfer and application of technology integration in their future classrooms" (p. 234). In mathematics modules, in particular, software like Microsoft Word, Excel, GeoGebra, Sketchpad, and others are included in the training. More teachers have started seeing the importance of technology in teaching and agree that it should be included in the pedagogical and content knowledge they need to acquire. As Ajani and Govender (2023) put it, knowledge and skills teachers acquired many years ago may no longer be relevant to



today's classroom practices because of the ever-changing and dynamic nature of the education system. Universities not only provide theoretical training to pre-service teachers; they also conduct professional development programs for in-service teachers. However, technology integration in teaching is not emphasised and is not part of the aspects to be assessed during teaching practicums. Engelbrecht et al (2023) argue that universities should invest in the professional development of teachers now, more than ever, to update on effective pedagogical methods, including the use of technologies, because education is now moving in a direction where students are becoming involved in what and how they are taught. It depends on the school. Not all schools are ready for technology; they do not have the resources or limited resources. So, teachers use what they have: Chalk and Talk.

Conceptual underpinnings of technology integration in teaching and learning

Technology-enhanced learning software and tools occur where technology is the major driver for these improvements at the cost of pedagogy. The literature shows the missing equilibrium between pedagogy and technology in the constantly advancing technology-enhanced learning territory. Subsequently, e-learners struggle to recognise the pedagogical value of such e-learning objects or gadgets. Some researchers relied on Mishra and Koehler's (2006) TPACK (Technological Pedagogical Content Knowledge) model regarding technology integration for teaching. Moreno et al. (2019) view TPACK as an efficient and reliable framework that can guide research about integrating technology in the classroom. The TPACK framework provides valuable guidance for integrating educational technology effectively, as dictated in the CAPS (Curriculum and Assessment Policy Statement) document in the South African context.

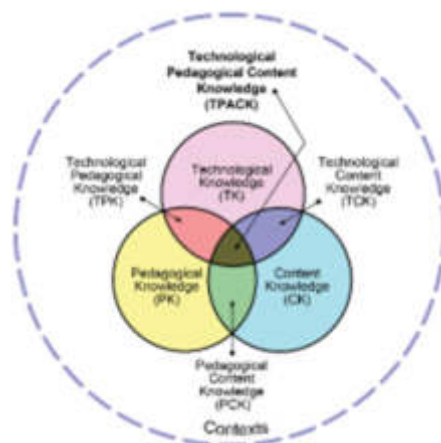


Figure 1: TPACK Framework adopted from <http://tpack.org>. Reproduced by permission of the publisher, © 2012 by tpack.org.

According to TPACK, effective ICT integration arises when these three aspects intersect and inform one another. Pre-service teachers should balance their TK (technological knowledge), PK (pedagogical knowledge), as well as CK (content knowledge) to advance informed choices about how and when to practise technology in teaching, in the end enhancing learning practices for learners.



Teaching pedagogies that foster learner interaction and engagement should be stimulated due to their role in enhancing learning and, consequently, learner performance. An excess of review literature exists about the importance of ICT integration in teaching and learning (Moreno et al., 2019; Samantra & Acharya, 2022; Wang et al., 2018). However, there seems to be a dearth of research about the actual integration of ICT in teaching and learning, especially in using qualitative methods.

Wang et al. (2018) purport that the most frequently used data collection methods in review studies that investigated pre-service teachers' development of TPACK were self-report; and further argue that more often, there is potential for error in self-reported data. This study aimed to explore pre-service teachers' skills and competencies in integrating technology into the teaching and learning of mathematics, using open-ended questionnaires and classroom observations. To attain this purpose, the following research questions were addressed:

- What skills and competencies do pre-service teachers possess to enable them to integrate technology in the teaching and learning of mathematics?
- How do pre-service teachers use their skills and competencies to integrate technology in the teaching and learning of mathematics to promote learner interest and engagement?

RESEARCH METHODOLOGY AND DESIGN

A qualitative case study design was followed, and classroom observations and open-ended questionnaires were used as data collection methods (Creswell, 2015). Yin (2018) defines a case study as an empirical inquiry that investigates a contemporary phenomenon within its real-life context. Rule and John (2011) view it as a systematic and in-depth investigation of a particular instance in its context to generate knowledge. The interpretive paradigm acknowledges that reality is socially constructed (Cohen, Manion & Morrison, 2018) to understand pre-service teachers' technological skills and competencies and how they use them in mathematics teaching and learning. This assisted in relaying the study recommendations that can influence future social and educational activities. In questionnaires, pre-service teachers' skills and competencies are gathered to determine how much they know and can apply. Recorded classroom observations were used to determine pre-service teachers' application of technological knowledge and skills in teaching.

Participants

A purposive sample of ten fourth-year pre-service teachers who participated in teaching practice was selected. The sample consisted mainly of pre-service teachers doing a Bachelor of Education (B.Ed.). Of the ten participants involved in the study, six were from the Free State province, and four were from Mpumalanga province in South Africa. The participants were sampled because they taught mathematics during their teaching practice, which would help the researcher discover how they integrate technology in mathematics teaching and learning.



Data collection instruments

This research used classroom observations and open-ended questionnaires as data collection instruments. The questionnaire was the main instrument, complemented by classroom observations to corroborate information from the questionnaires. The questionnaire helped to get first-hand information about the technological skills and competencies possessed by participants. It required participants to mention their competencies and how they managed to teach technology-integrated lessons during their teaching practice. Classroom observations were mainly conducted to witness and affirm the consistency of data collected from questionnaires, and to see how pre-service teachers integrated technology in teaching and learning. A classroom observation schedule was used in conjunction with the checklist. The latter assisted the researcher in checking if all the criteria listed in the classroom observation schedule were observed or included in pre-service teachers' lessons.

Questionnaires

Open-ended questionnaires consisted of a combination of short and long questions. Short questions required yes/no answers, and they were asked to verify if participants have technological tools such as laptops, smartphones, tablets, and so on that can be used during teaching, and whether they used the tools in teaching. Long questions required participants to explain how they used technological tools in their lessons, and they were used to identify pre-service teachers' skills and competencies, including challenges and perceptions regarding technology integration. Ten pre-service mathematics teachers responded to the questionnaires. The data was then analysed and triangulated with that obtained from classroom observations to provide a robust cross-validation of findings.

Classroom observations

The researchers used classroom observation tools to determine participants' integration of technology in their mathematics teaching and learning. The following subheadings were used to record data gathered from recorded lessons in the classroom observation schedule:

Topic, Grade, Teaching methods/Approaches, ICT integration, and briefly describe how the lesson went. For ICT integration, the researchers wanted to see if participants used any technological tool or software in their lesson delivery. The description of participants' classroom management was to detail how lessons were taught using technological tools, including learner engagement. These were taken from participants' recorded lessons and written lesson plans.

Data presentation and analysis

According to Maamin, Maat, and Ikhsan (2020), teacher skills and teaching practices are some of the constructs that form teacher quality, and both were found to have a significant association with learner achievement. This study intended to investigate pre-service teachers' skills and competencies in integrating technology into teaching. In addition, the researchers wanted to



explore how pre-service teachers utilise their technological skills to enhance mathematics teaching during their teaching practice. Both objectives are linked to teacher skills (knowledge of technological tools and how to use them) and teaching practices (infusing technology in teaching). After analysing data collected from lesson observations and questionnaires, the researchers found that most pre-service teachers do not use technology in their mathematics lessons. The following section is a presentation of data analysis from participants' questionnaires and classroom observations.

Participants' questionnaires

Participants responded to short and paragraph-type questions in open-ended questionnaires. Short questions required yes/no answers about having their computers/laptops, and whether they used them during their teaching practice lessons.

When asked how they use technology in their teaching, participants responded by saying:

Participant D: *"I used my laptop and projector to explain explanations to learners, and learners were excited to see the learning material projected on the screen. I have learnt that integrating technology in class is fun and makes the learners eager to learn"*.

Participant G: *"By sometimes letting the learners be taught by seeing videos or even showing them some concepts, such as 3D shapes on a laptop"*.

Participant A: *"I am going to integrate technology by using videos to further explain what the whole lesson is about"*.

Most participants agreed by answering "yes" that they had some knowledge of integrating technology into teaching. In responding to the question asking if they were able to integrate technology into teaching, participants D and J responded as follows:

Participant D: *"Yes, it makes mathematics more enjoyable"*.

Participant J: *"Yes, I feel like it makes things easier"*.

Although most of the participants agreed on the importance of technology integration for teaching, participants said,

Participant H: *"If teachers could have been thoroughly trained to use technology, it could have been better"*.

Participant D: *"Technology should be used as part of learning for the benefit of the class"*.

Participant G: *"I think technology tools must be used in everyday learning to enhance learners' understanding of the contexts taught"*.

Participant H indicated that there were still some challenges regarding technology integration



in teaching and learning. Participant I said,

Participant I: *“The school does not have technological equipment we can use; technology is good, but then we do not have it in schools” ...*

Participant E: *“The schools are not exposed to any technological materials to teach learners”.*

Classroom observations

Altogether, ten mathematics lessons were observed, and from these ten, one was in Grade R, two in Grade 3, another two in Grade 4, one in Grade 5, one in Grade 6, one in Grade 10, and one was in a Technical and Vocational Education and Training (TVET) college for N3 (third level of the National Certificate Vocational offered at Technical and Vocational Education and Training colleges in South Africa). Teachers were named using letters of the alphabet, see Table 1.

From all ten mathematics lessons that were observed, it was found that eight participants followed the standard “chalk and talk” approach (see Table 1). Only two managed to use technological tools in their lesson delivery. Two teachers who integrated technology were Teachers D and G, teaching 2-D shapes in Grades R and 3, respectively. Both played a video song about 2-D shapes from teachers' laptops, which learners watched and listened to. The teaching strategies of both teachers were almost similar. They allowed learners to watch the video and thereafter asked questions based on it. The 2-D shapes that were played in the song were a triangle, a square, a rectangle, and a circle, as shown in Figure 2:

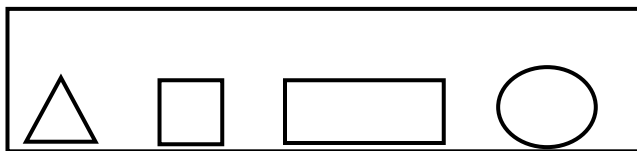


Figure 2: 2-D shapes taught in a Grade 3 class

Teacher D said to her learners:

“I am going to play you a video”,

Immediately, the learners stood up and wanted to move towards the teacher’s laptop. The teacher then said,

“You do not stand up, move, remain where you are, and only listen with your ears”.

As the video played, learners were attentive, and they all looked toward the teacher’s laptop. After playing a video, she asked learners to mention what they heard from the video, and those who answered the questions responded well.



Table 1: Summary of observed lessons for ICT integration in mathematics teaching and learning

Teacher	Grade (Level)	Topic taught	ICT integration	Method	Classroom management
A	3	Patterns	No	Show and Question-and-answer & Tell	Teacher introduced the topic to learners and showed different patterns on the board. She then asked them to describe the patterns, and later gave them activities.
B	6	2-D Shapes	No	Telling & Discussion	Teacher asked learners to name 2-D shapes they know. Thereafter she described the properties of 2-D shapes on the board.
C	4	Length	No	Practical, Telling & Discussion	Teacher explained the topic, solved a few problems on the board showing learners conversion of the units of length. Then asked learners to measure objects in the classroom. Then asked to estimate and compare their answers to their estimated values. Learners participated well.
D	3	2-D Shapes	Yes	Question-and-answer & Telling	Teacher started with daily routine of learners counting from 1-100, and then asking learners to come to the board to write the day's date by plucking cards on the board. The teacher then played a video song about 2-D shapes from the laptop, and asked learners to tell what they saw and heard. Then explained the topic and different 2-D shapes. Learners participated and responded well to questions.
E	4	Whole numbers	No	Question-and-answer & Telling	Lot of explaining and used questions occasionally
F	N3	Limits	No	Question-and-answer & Telling	Lot of explaining and used questions occasionally
G	R	2-D shapes	Yes		The teacher then played a video song about 2-D shapes from the laptop, and asked learners to tell what they saw and heard. Then used information given by learners to explain about 2-D shapes. Learners participated and responded well to questions.
H	5	Breaking down method	No	Question-and-answer & Telling	Teacher asked learners to give place values of numbers written on the chalkboard, and thereafter demonstrated the breaking down method. Activities were then given.
I	6	Order of operations	No	Question-and-answer & Telling	Teacher introduced the topic and explained to learners what BODMAS means. Then solved examples to show learners the order of operations to follow when solving problems involving BODMAS, and then gave learners activity.
J	10	Rates	No	Question-and-answer & Telling	Teacher asked questions about prior knowledge, then asked learners to explain "rate". Teacher solved few examples on the board and gave learners activities from the activity bank.

DISCUSSION OF FINDINGS

Teacher knowledge, skills, and competencies of technology integration in Teaching

The findings revealed that most pre-service teachers who participated in this study possess laptops to assist them in completing assessments, including searching for information on the Internet. This was revealed when participants were asked about their knowledge of using technology in teaching, where most of them indicated that they had laptops and knew how to infuse technology in teaching, except for two participants who indicated otherwise. These participants did not infuse technology into their lessons, and it becomes evident that their expressions demonstrate a lack of adequate TCK skills to infuse technology into teaching. However, Participants D and G, who were able to integrate technology in their lessons in grades R and 3, respectively, were evidence that they could search the internet for information to use in their teaching by using videos to teach 2-D shapes.



In answering research question 1. *What skills and competencies do pre-service teachers possess to enable them to integrate technology in the teaching and learning of mathematics?* The study found that on the one hand, eight participants possess perceived skills and competencies since they only mentioned what they can do without actually doing it. On the other hand, only two participants (D and G) displayed their actual skills and competencies about technology integration in their teaching.

Teacher perceptions regarding technology integration in teaching

Some participants went to the extent of mentioning the benefits of using technology in teaching, for instance, Participants D and G who were both primary school teachers and taught 2-D shapes said, “*and learners were very excited to see the learning material projected on the screen*”; and “*it makes mathematics more enjoyable*”. The fact that Participant D’s learners jumped from their chairs on the mentioning of playing a video, is an indication of the level of enthusiasm the learners displayed in wanting to see what the video was all about, showing the impact technological use in teaching can have on promoting primary school learners’ interest and curiosity. This agrees with Herro, Visser, and Qian’s (2021) sentiments when they said educators believe that technology integration in teaching is essential.

Infusing technology into lessons

Infusing technology in teaching for some pre-service teachers seems to be an activity yet to be conceptualised, since almost all participants in this study did not have their lessons integrated with technology. Despite Participant J presenting a lesson that did not infuse technology in his teaching, he commented by saying, “*I feel like it makes things easier*”, giving the impression that he does use it. Participants’ utterances indicate that technology use promotes learner interest and effectiveness in teaching and learning (Aliyu et al., 2021). When Participant D’s learners stood up at the mention of playing a video, this indicated the extent to which technology use can arouse primary learners’ interest in learning. When learner interest can be captured, especially at the beginning of the lesson, learning can be enhanced, and teachers might not encounter discipline issues during lessons because learners are keen to learn (Sabarudin, Marzuki, & Khairunnisak, 2020). In answering the research question:

2. How do pre-service teachers use their skills and competencies to integrate technology in the teaching and learning of mathematics to promote learner interest and engagement?

The researchers found that pre-service teachers could search for additional teaching materials, such as videos, and project them for learners to watch. This not only augmented teachers’ information about the content taught but also assisted in promoting learner interest and increased learners’ enthusiasm to learn.

Challenges in pre-service teachers’ attempts to infuse technology into teaching

It is worth noting that there are still challenges faced regarding technology integration in teaching in many countries, for instance, in Lesotho (Turugare & Rhudumbu, 2020), Nigeria (Mormah & Bassey, 2021), and South Africa (Ajani & Govender, 2023). Pre-service teachers



in this study revealed that they experienced a lack of resources to integrate technology into teaching. It was expected that eight participants who knew how to integrate technology in teaching would do so in their lessons, either using schools' resources or their own devices, as they mentioned that they did have laptops. Using their own devices in teaching could have indicated pre-service teachers' willingness to improvise and infuse technology in teaching. However, of the eight that did not infuse technology in their teaching, two indicated that they could not do so because of a lack of technological resources in schools, indicating challenges hindering them from using technology. Nevertheless, even those who did not infuse technology in teaching agreed that it does assist in enhancing teaching and learning.

Eight participants did not integrate technology into their lessons. These results show pre-service teachers' gross neglect of using innovative methods like technology integration to enhance mathematics teaching and learning. This demonstrates that despite participants indicating that they had the technological skills and competencies to integrate technology into teaching, they were still not doing it. The participants' failure to integrate technology in teaching agrees with Baguma et al (2019) when they said the low use of technology is attributed to teachers' resistance to changing their teaching methods. Other participants continued with their usual "chalk and talk" methods of teaching, which are now becoming obsolete, using the chalkboard as the primary teaching, and learning resource, and the teacher as the main resource and knowledgeable person.

It should be a concern to training institutions especially those offering online teaching and learning that in this era of technological advancement in many spheres such as education, business and commerce, pre-service teachers who will be entering the education system, do not show a willingness to embrace innovative teaching methods like infusing technology in mathematics teaching. Pre-service teachers themselves indicated that because they were studying online, they used technology in their studies to download additional resources, attend asynchronous lessons, and complete online assessments (Samantray & Acharya, 2022). Nevertheless, their lessons still do not infuse technology. DHET (2015) requires pre-service teachers to acquire TCK. However, there is no reason to acquire this kind of knowledge if it is not put to good use. This depends on the training received at the university and the school.

CONCLUSION

Successful technology integration also depends on the availability of resources and support provided by schools. Emerging technologies such as AI and robotics underscore the urgency of equipping learners with relevant mathematical and technological skills to remain competitive in future job markets. This study contributes to understanding the gap between theoretical training and practical technology implementation in mathematics education among pre-service teachers. Artificial intelligence and robotics are technologies that threaten labour markets, which will soon be challenged by an abundance of competencies that are no longer required (Lee et al., 2018). As a result, technology integration in teaching, especially in mathematics, can no longer be ignored. Furthermore, the Department of Basic Education (DBE) has introduced new subjects in schools, Robotics being one of them. Mathematical



knowledge is required in building and using technological tools; it becomes very strange that mathematics learners are left behind when it comes to the infusion of technology in their learning. Therefore, this study recommends that since DHET requires beginner teachers to acquire technological skills and competencies to use in teaching, training institutions include assessment and observation of TCK or TPACK in pre-service teachers' lessons during their teaching practice sessions to ensure that they are relevant to the present curriculum and community needs (Mutambara, 2023).

The findings in this paper are not representative of the whole population since it was a case study of only one training institution. Therefore, the results cannot be generalised. However, the results should encourage technological integration not only for preservice teachers but also for in-service teachers.

Ethical considerations

This study adhered to all ethical research guidelines and did not involve any direct interaction with human or animal participants.

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Disclaimer

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INCLUSIVE, EQUITABLE, AND LINGUISTICALLY RESPONSIVE MATHEMATICS EDUCATION IN MULTILINGUAL SOUTH AFRICAN CLASSROOMS

Kereng Gilbert Pule

University of South Africa

This systematic review addresses the urgent need to overcome linguistic and cultural barriers in South Africa's multilingual mathematics classrooms. Many learners face a sudden transition from mother-tongue instruction to English as the language of learning and teaching in Grade 4, a shift that often undermines their mathematical understanding and exacerbates educational inequities. The review is motivated by persistent achievement gaps affecting learners from African language backgrounds, which call for transformative pedagogical approaches. The primary aim is to explore inclusive, equity-driven teaching strategies that leverage learners' linguistic and cultural identities as assets for learning, thereby improving engagement and comprehension in mathematics. The study highlights why linguistically responsive mathematics teaching is both an educational and social imperative, situating the problem in the South African context of historical language hierarchies and current policy challenges. The review employs a qualitative systematic methodology to synthesise findings from classroom-based studies on multilingual mathematics education. Drawing on theoretical frameworks of Ubuntu philosophy, social constructivism, and cognitive load theory, it examines how communal values, collaborative discourse, and cognitive supports inform effective practices. The evidence indicates that incorporating learners' home languages into mathematics teaching significantly enhances conceptual understanding, participation, and learner confidence. Inclusive strategies such as code-switching, translanguaging, and culturally relevant pedagogy were found to improve problem-solving and self-efficacy while reducing language barriers that hinder learning. These findings carry strong implications for teaching, learning, and policy suggesting that embracing linguistic diversity and cultural context in mathematics education can drive more equitable outcomes in South African classrooms.

Keywords: transformative pedagogical approaches, culturally relevant pedagogy, code-switching, translanguaging

INTRODUCTION AND MOTIVATION

South Africa's multilingual classrooms encounter a significant challenge: the sudden transition from home-language teaching to English in Grade 4. This transition is particularly challenging for learners who speak African languages, as they find it difficult to comprehend mathematics in a foreign language. Early training in English-language mathematics may lead to learners misinterpreting fundamental terminology and diminishing their confidence, which in turn results in reduced engagement and achievement.

Practices from the apartheid era that privileged English and Afrikaans in educational



institutions while marginalising indigenous languages intensify these issues. Numerous learners continue to have challenges in learning due to inequitable resources and monolingual curricula, decades post-apartheid. Consequently, mathematics classrooms continue to be linguistically disjointed, perceiving learners' home languages as hindrances rather than tools for comprehension. The disparity between language policy and practice requires a re-evaluation of mathematics teaching to align with learners' linguistic realities.

This work is driven by the necessity to render mathematics instruction inclusive and equitable amidst South Africa's linguistic variety. The Language in Instruction Policy (LiEP) advocates for multilingualism; yet, the execution of mathematical instruction has been irregular and inadequate. Conventional uniform pedagogies have proven inadequate for bilingual and multilingual learners, leading to disparities in mathematical proficiency (Adler & Ronda, 2023). This article analyses how the integration of learners' languages and cultures enhances mathematical understanding and engagement.

It contends that diversity ought to serve as a learning asset instead of an obstacle by emphasising inclusivity, equity, and multilingualism. To identify and evaluate effective multilingual mathematics instructional strategies, including code-switching, translanguaging, additive bilingual education, and culturally responsive pedagogy. The report analyses these strategies using rigorous theoretical frameworks and empirical evidence to demonstrate how to create classrooms conducive to the academic success of all learners. The introduction illustrates the necessity of linguistically sensitive and culturally affirming mathematics instruction for addressing structural inequities and improving learner outcomes in South Africa.

THEORETICAL AND PHILOSOPHICAL FRAMEWORK

Ubuntu Philosophy

Ubuntu provides an Afrocentric ethical framework for this study's inclusive education strategy. Ubuntu (I am because we are) emphasises interconnection, mutual respect, and the community's role in individual well-being. This philosophical approach to education emphasises that learning is a group process that enhances each learner's understanding. An Ubuntu-informed approach to multilingual mathematics would recognise each learner's language and culture as part of the classroom community's knowledge base. If a learner explains an idea in their native tongue, it enriches the group's comprehension rather than deviating from the norm.

Ubuntu's emphasis on communal learning coincides with inclusive pedagogies that promote cooperation and peer support, creating a successful community. Ubuntu philosophically supports diversity in teaching by framing the classroom as a community where teachers and learners learn from one another (Praeg, 2020). This Ubuntu worldview supports integrating linguistic and cultural diversity into mathematics teaching. It emphasises collaborative accomplishment above individual competition, encouraging equity and recognising all learners' backgrounds.



Ubuntu-inspired mathematics teachers may encourage group problem-solving sessions where learners discuss ideas in their preferred language and share their results with the class. Such practices promote Ubuntu's communal peace and value every learner's voice. Research reveals that this concept helps learners feel validated and at home in the classroom by reflecting on their identities (Adler & Ronda, 2023). Ubuntu also supports social justice in education by arguing that helping marginalised learners benefits the broader community. Ubuntu philosophy holds that "no learner is an island," and that embracing our shared humanity and linguistic heritage can enrich education for all.

Social Constructivism

Social constructivism emphasises language and social interaction in knowledge formation, providing a vital theoretical perspective. This Vygotsky-inspired view holds that learners learn through discourse, collaboration, and scaffolding in their cultural and language context. Social constructivism suggests that multilingual mathematics learners learn better when they discuss and negotiate meaning in their preferred language. Learners can externalise and refine their mathematical understanding using language. They can better understand mathematics discussions by using their home languages, which match their cognitive frameworks and past knowledge.

Social constructivism holds that code-switching and multilingual discussion in a linguistically responsive classroom help learners learn mathematics through shared, socially mediated experiences. Social constructivism in multilingual settings uses the Zone of Proximal Development (ZPD), where learners can learn more with support. Using different languages as mediation tools helps teachers work within students' ZPD. A teacher may first engage learners in their home language to guarantee a foundational grasp of an issue, then progressively introduce English mathematical vocabulary.

This guided transition helps learners relate new concepts to established language and cognitive schemas, demonstrating Vygotsky's principle of shifting from the known to the new with expert support (Vygotsky, 1978). South African classroom studies show that encouraging conversation so the language learners understand the material best increases engagement and minimises the fear of speaking incorrectly (Setati & Adler, 2022). Thus, social constructivism endorses the use of learners' native languages for mathematical sense-making and provides a framework for peer-interactive learning activities. Using this idea, teachers can build multilingual learning environments where learners co-construct knowledge and fulfil their potential through collaborative, language-rich engagement.

Cognitive Load Theory

Cognitive Load Theory (CLT) explains why linguistically responsive mathematics instruction is advantageous. Sweller (1988) defined CLT as intrinsic (inherent to the subject being learnt) and extraneous (imposed information). Mathematics already appears difficult due to intricate problem-solving and abstract reasoning. Learners must divide their mental effort between language and mathematics when instruction is offered in a language they do not speak. In



practice, a learner solving a word problem in an unfamiliar language may struggle more with translation than mathematics, overburdening their working memory.

South African research shows that this language barrier adds a substantial strain to learners' mathematical processing (Botes & Mji, 2010). CLT advocates minimising superfluous load to optimise learning. In multilingual classrooms, this can be done by letting learners choose their preferred language to acquire new concepts. By doing so, learners can concentrate on mathematical thinking rather than translating instructions and terminology.

Multilingual CLT is supported by empirical evidence. Studies demonstrate that letting learners debate and internalise mathematics concepts in their local language reduces cognitive load and improves comprehension and retention. Malindi et al. (2023) discovered that multilingual instruction strategies engaged learners better in problem-solving since they did not have to alter mental contexts between languages. In contrast, English-only learners with inadequate English skills sometimes face cognitive overload, causing confusion or inability to retain foreign language mathematics techniques.

From the CLT viewpoint, such challenges are caused by an overwhelming extraneous load, not mathematics skills. Thus, language-sensitive instruction, such as bilingual explanations, multilingual glossaries, or visual aids with mother-tongue labels, reduces the cognitive load on learners. In conclusion, Cognitive Load Theory supports the idea that mathematics instruction should accommodate learners' language proficiency by reducing cognitive strain, allowing them to focus more on mathematical concepts and improving learning outcomes.

METHODOLOGY

A comprehensive search of numerous academic databases found empirical research, policy reports, and scholarly articles that satisfied inclusion criteria. Studies had to be conducted in South African schools, focus on mathematics teaching and learning in multilingual classrooms, investigate one or more culturally and linguistically inclusive pedagogical approaches, and report measurable impacts on learner engagement, achievement, or teacher practice.

Qualitative and quantitative investigations provided diverse evidence. Titles and abstracts were screened against inclusion criteria many times, followed by full-text evaluations to ensure relevance and rigour (Sapire & Essien, 2021). To enable systematic comparison across sources, data extraction techniques recorded each study's background, methodology, sample characteristics, and major findings. This systematic methodology guaranteed that the review was based on solid data and balanced how different tactics perform in real classrooms.

Two levels of analysis followed the study collection. First, the literature was categorised by the main themes investigated: code-switching, translanguaging, additive bilingual programs, culturally relevant education, or multilingual resources. To identify common learning effects, this categorisation allowed all code-switching experiments to be compared. Secondly, a theme synthesis identified common patterns, outcomes, and problems across varied studies. Key themes from inductive analysis were better conceptual comprehension, learner participation,



and teacher training for inclusive practices. Context-specific factors, such as whether rural vs. urban school settings affected approach efficacy, were also considered (but full context analysis is limited by each study). Each source's quality and dependability were assessed by research methodology and sample size. Triangulating quantitative data (like test score gains) with qualitative insights (like learner and teacher testimonies) strengthened the conclusions. The methodology used systematic literature selection and thematic analysis to develop a nuanced evidence foundation. This method ensures that the findings and recommendations are valid and applicable to multilingual mathematics education policy and practice.

Author's Position

In this research, the author advocates for linguistic and cultural diversity as educational strengths rather than impediments. The author believes that traditional monolingual teaching methods are incompatible with classroom realities and contribute to inequality after studying South African schooling. This worldview is shaped by personal education and inclusive scholarship. Following the Ubuntu idea, the author argues that each learner's cultural background and mother tongue contain valuable knowledge that can improve the collective learning experience. The author openly favours a mathematics education paradigm shift away from English domination as the only path to academic success. Instead, the author believes that introducing African languages and cultures into mathematics training will improve multilingual learners' knowledge and enrich the subject for all learners. A prior study (Setati & Adler, 2022) shows that affirming learners' identities in the classroom boosts engagement and performance. The author's goal is to promote equity and social justice in education using educational methodologies.

The author's philosophical and professional background influences findings interpretation. Social constructivism and social justice view education as a way to empower marginalised populations and break down systematic barriers. This indicates an intentional preference for educational innovations that empower under-represented linguistic groups and promote classroom democracy. The author has taught mathematics in multilingual contexts, giving him practical insight into teachers' and learners' daily struggles. The author's background helps him understand the data: he's seen learners' faces light up when a concept is conveyed in their home language, which mirrors the empirical findings. The author prioritises equity while maintaining objective analysis; all statements in this evaluation are supported by the literature. By revealing this positionality, the author emphasises academic integrity and reflexivity. The practitioner-advocate approach should highlight the human effect of statistics and explain the study's suggestions practically. According to the author, culturally and linguistically inclusive mathematics teaching is both pedagogically sound and morally important for educational fairness in South Africa.

LITERATURE SYNTHESIS AND FINDINGS

Code-Switching

The literature suggests code-switching, or switching languages during instruction, can improve



knowledge in multilingual mathematics classrooms. Effective teachers employ code-switching to simplify complex concepts and guarantee all learners can follow along. Maluleke (2019) describes how teachers switching between English and an African native language helped learners understand mathematics ideas that were difficult in English. By introducing a difficult idea in learners' first language and then repeating it in English, teachers provide two pathways for comprehension, improving learners' understanding.

This methodical alternation helps translate vocabulary and lets learners explain mathematical thinking in their preferred language. Learner confidence in problem-solving increases as they use their whole language range. The approach follows cognitive theories and reduces cognitive load by lowering the mental effort needed to decode a second language while learning mathematics. When the language of teaching is difficult, code-switching to a familiar language gives rapid help, minimising meaning loss at important learning times.

Besides improving comprehension, code-switching boosts learner participation. Learners are more inclined to participate in class discussions rather than hide for fear of making English blunders when they ask questions or discuss ideas in their home language. Sepeng (2013) found that learners were more eager to try difficult word issues when they could first talk them through in their language, which often led to insights they could subsequently express in English.

Malindi et al. (2023) found that this approach increased problem-solving skills because learners used their home language to reason and then switched to English to express their reasoning, cross-checking their understanding in two languages. Teachers indicated that code-switching created a more inclusive classroom by showing respect for learners' cultural and linguistic history, which enhanced their confidence and sense of belonging (Setati & Adler, 2022). However, the literature warns that code-switching without careful integration might lead to disconnected lessons. Teacher consistency and purposeful code-switching in instances where learners seek explanations or links to prior knowledge are crucial. The review found code-switching to be an effective, evidence-based strategy that overcomes language gaps and promotes equitable mathematics learning.

Translanguaging

Recent research emphasises translanguaging as a dynamic pedagogical method that goes beyond code-switching by integrating different languages into a single learning experience. Translanguaging encourages learners and teachers to use all linguistic resources to make meaning, such as discussing a problem in one language, reading information in another, and writing an answer in both. This method improves conceptual understanding in multilingual mathematics classrooms. Guzula and Abdulatief (2024) found that systematic translanguaging, such as group work where learners freely mixed languages to solve arithmetic problems, improved mathematical reasoning and idea explanation.

Learners can internalise things in their home language before expressing them in school, which strengthens their learning. Through translanguaging, learners can access complicated ideas in



the language that best fits their past knowledge. They use familiar linguistic frames to understand the “why” of mathematical topics rather than just memorising processes in English (Sapire & Essien, 2021). Translanguaging improves mathematical literacy because bilingual learners can read, interpret, and discuss arithmetic tasks better.

Classroom research shows that translanguaging boosts learner confidence and participation. When allowed to utilise their native tongues, “quiet” or “struggling” learners in an English-only context frequently become more active contributors. In a Grade 9 mathematics class, translanguaging allowed learners who spoke isiXhosa at home to lead problem-solving discussions by translating essential phrases between English and isiXhosa (Guzula & Abdulatief, 2024). This helped them understand the content and gave them academic status. Teachers reported that translanguaging tactics including bilingual group reporting and multilingual mathematics glossaries helped learners and teachers communicate and ensure conceptual understanding.

Translanguaging can also scaffold cognition: a teacher may ask a difficult question in English but urge learners to debate their thoughts in their native language before answering. This approach helps learners organise their thoughts and speak more clearly in the second language. The review finds that such methods improve mathematical discussions and prevent misunderstandings. Translanguaging challenges language separation in education and requires teacher expertise and a good attitude towards multilingualism (Garcia & Wei, 2014). The data strongly imply that systematic translanguaging is an effective equitable strategy that allows multilingual learners to fully participate and flourish in mathematics using all their linguistic skills.

Culturally Relevant Pedagogy

Culturally relevant pedagogy (CRP) in mathematics education links curriculum to learners’ cultures, lives, and communities. CRP makes mathematics more meaningful and accessible to learners from diverse ethnic groups, making it essential to inclusive practice, according to the review. In a South African classroom, this could mean using examples, parallels, or issue contexts from learners’ lives and traditions. A teacher might teach geometry using traditional beading patterns or pose a community issue statistical challenge. Such techniques recognise and exploit learners’ home and community knowledge, coinciding with Ladson-Billings’ (1995) academic performance, cultural competency, and critical consciousness.

Seeking cultural reflection in the curriculum helps learners connect new mathematical concepts to familiar contexts, improving comprehension and retention. A case study described a class where the teacher introduced algebraic functions using Zulu basket weaving patterns, an indigenous practice. Learners were engaged and able to visualise abstract algebraic relationships through a cultural lens (Setati & Adler, 2022). This shows how culturally relevant pedagogy can make mathematics fun and approachable. CRP in mathematics also promotes diverse ways of knowing, recognising that different culturally impacted approaches to a subject are valid. This encourages richer classroom dialogue rather than one-dimensional teaching.



Culturally relevant instruction regularly improves learner outcomes and belonging. Learners are more inclined to participate and persevere in difficult mathematics problems when they identify their cultural narratives or community difficulties. Taylor and Vinjevoold (2019) found that learners in culturally appropriate classrooms were more interested in mathematics and thought it mattered to them, which improved performance and attendance.

Culturally relevant teaching also promotes Ubuntu by approaching the classroom as a community where each learner's background contributes (Praeg, 2020). Breaking prejudices and power relations helps dispel the idea that only Western contexts or examples are adequate for mathematics learning. Importantly, CRP challenges learners and teachers to address social justice. Mathematics education may teach learners to critically evaluate and question society by including real-world concerns like economic inequality and social data from their areas. CRP teachers are encouraged to reflect on their prejudices and learn about their learners' cultures to improve instructional responsiveness (Moschkovich, 2021). The review notes that culturally responsive teaching may involve more class planning and professional development, but learners become more engaged, academically successful, and proud of their identities. CRP is seen as a commitment to educational equity that ensures all learners value what they learn and achieve on their terms.

IMPLICATIONS FOR MATHEMATICS TEACHING, LEARNING, AND RESEARCH

Teaching

The literature review offers several major implications for bilingual mathematics teaching. Teacher methods should recognise and include learners' home languages and cultures. This implies abandoning the "one-language-fits-all" approach and adopting code-switching, translanguaging, and culturally relevant examples in training. This transition requires teachers to rethink their role as facilitators who bridge languages and mediate comprehension rather than authoritative sources in only one official language.

Professional development gives teachers the skills and confidence to teach inclusively. Training programs should teach teachers how to arrange lessons in English and/or Setswana/Sepedi/isZulu and how to develop examinations that allow learners to explain responses in their preferred language. Studies demonstrate that guidance and practice in these areas boost teachers' success in diverse classes. Taylor and Vinjevoold (2019) discovered that multilingual strategy workshops made teachers feel more competent and enhanced learner involvement. Continuing professional learning on diversity is vital for meaningful transformation in teaching.

The review found that present curricula often do not give teachers enough flexibility or assistance to adapt to multilingual environments (Adler & Ronda, 2023). Education stakeholders must rethink curriculum design to incorporate linguistic and cultural linkages. Textbooks or teacher guides with multilingual terminology lists, local cultural difficulties, or collaborative learning exercises may be helpful. Teachers may use formative examinations that



allow multilingual responses or oral explanations to measure mathematics knowledge rather than English competence. Importantly, teachers should foster an inclusive classroom. They should establish a secure atmosphere for learners to take intellectual risks and express questions without language discomfort. Teachers' deliberate use of inclusive practices directly addresses South African education's longstanding inequalities, according to the literature.

Learning

The findings affect mathematics learning for learners. They develop academically, emotionally, and socially when schools use inclusive, linguistically responsive practices. Learners may obtain a greater conceptual understanding of mathematics. Multiple studies show that learners who utilise their local language while learning mathematics can better understand and retain complex concepts. Learners who discuss mathematics difficulties in Tshivenda with peers may catch nuances or make connections that they would miss in English, which improves their ability to solve problems and apply concepts in new contexts. Learners improve their bilingual mathematical literacy by mastering mathematical concepts in both languages.

Learners find learning more relevant with inclusive techniques like culturally relevant pedagogy. They are more motivated to learn mathematics when it is applied to real-world problems or community challenges. In such situations, learners may be more interested in mathematics and proactive in learning. When individuals identify with the subject, they take ownership of their learning, which may improve study habits, questioning, and willingness to face difficult tasks (Moschkovich, 2021).

Inclusive, equitable tactics help learners feel like they belong, which is crucial to academic success. Learners are more inclined to participate in conversations, collaborate, and ask for help in classrooms that emphasise accents and native languages. As learners participate more, they practise and reinforce their skills, improving performance and confidence. They learn to listen to other perspectives and may translate or explain concepts to needy peers, which strengthens their empathy.

Inclusive mathematics classes may also boost resilience, according to a study. Learners acquire resilience and adaptability by facing the obstacle of learning a second language with guidance. In addition to structural adjustments, Learners should coach and comfort pupils to assist them overcome unfavourable self-perceptions. In a supportive, linguistically rich environment, mathematics becomes more than numbers and formulas; it becomes an inclusive journey where every learner can confidently build knowledge, supported by their language and culture, and become a more capable and self-assured mathematician.

Research

This review's findings highlight multiple opportunities and requirements for additional research in multilingual mathematics education. One implication is the need for longitudinal studies that investigate the long-term effects of inclusive and linguistically responsive pedagogies on learners. It is essential to investigate whether learners who acquire mathematical



skills through code-switching or translanguaging in primary education maintain these advantages in subsequent grades and higher education. Longitudinal studies could examine if early exposure to an inclusive approach mitigates achievement gaps over time or results in increased enrolment in advanced mathematics courses.

Furthermore, longitudinal data may investigate the enduring impact on learners' bilingual competencies and whether sustaining academic proficiency in two languages offers benefits in their subsequent educational pursuits or professional careers. Another area for research involves scaling and context adaptation: investigating how these strategies can be effectively implemented across various regions, school types (urban versus rural, well-resourced versus under-resourced), and grade levels. For instance, how does successful translanguaging manifest in a large, diverse urban classroom compared to a small rural classroom with limited resources? Comparative studies may provide insights into the adaptability of inclusive methods and identify essential support structures, such as community involvement or policy support, that enhance success across different contexts (Adler & Ronda, 2023).

The rapid advancement of technology implies that research must investigate innovative tools to enhance multilingual mathematics teaching. Digital platforms and educational software may be designed to facilitate the translation of mathematics content or to integrate multiple languages seamlessly within exercises. Future research should investigate the efficacy of these tools, specifically their impact on reducing language barriers and the nature of interactions between learners and teachers. The review's conclusion suggested the potential of AI-powered translation and other digital tools as emerging resources in multilingual classrooms.

Systematic investigations in this area would be beneficial, particularly in identifying optimal strategies for integrating technology with human instruction. Furthermore, research must inform policy: empirical evidence regarding the translation of language-in-education policies into classroom practice (or their failure to do so) is essential for guiding policymakers in the refinement of these policies. Despite South Africa's supportive language policies in theory, research such as Sapire and Essien (2021) can be further developed to identify the most significant policy-practice gaps and explore potential solutions for bridging them.

Cross-disciplinary research represents a promising avenue for exploration. Collaborations among linguists, cognitive scientists, and mathematics teachers may provide a comprehensive understanding of multilingual learning processes, potentially impacting the structure of teacher training programs (Moschkovich, 2021). Incorporating learner perspectives in research via qualitative studies of learner experiences will ensure that future innovations are aligned with the needs and viewpoints of learners.

CONCLUSION AND RECOMMENDATIONS

This systematic review highlights that inclusive, equitable, and linguistically responsive mathematics education has significant potential to address persistent disparities in South Africa's multilingual classrooms. The article critically examines various studies, demonstrating that when teachers use learners' home languages and cultural contexts, mathematics learning



becomes more accessible and effective for learners who might otherwise struggle. Linguistic and cultural diversity serve as significant assets that enhance understanding, engagement, and learner confidence.

The theoretical frameworks of Ubuntu and social constructivism offer philosophical and pedagogical justification for these practices, emphasising that learning is a communal, meaning-making process best facilitated by respecting each learner's identity and voice. Cognitive Load Theory provides a cognitive basis by illustrating that the use of familiar language minimises extraneous mental effort, enabling learners to concentrate on understanding mathematical concepts.

The findings collectively support a shift in mathematics education that aligns with global initiatives aimed at decolonising and democratising education. Successful classrooms adapt to the multilingual reality of learners rather than enforcing a rigid monolingual norm, resulting in enhanced academic outcomes and promoting social justice and empowerment (Setati & Adler, 2022; Praeg, 2020). The evidence provided demonstrates that significant and lasting enhancements in mathematics achievement for historically marginalised learners can be achieved by rethinking educational practices with an inclusive perspective.

The recommendations advocate for systemic support, suggesting that education departments should develop learning materials in multiple languages and that universities should modify teacher education curricula to incorporate modules on multilingual education. This review proposes a restructured mathematics education system in which inclusivity and equity are fundamental components of practice. Thus, South Africa can progress towards classrooms where all learners, irrespective of their linguistic backgrounds, have the opportunity to excel in mathematics and contribute effectively to the scientific and socio-economic advancement of their communities.

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ROLE OF INCLUSIVE EDUCATION IN PROMOTING EQUITY IN MATHEMATICS EDUCATION

Rankweteke Puleng Edwin

University of South Africa

The paper discusses the role of inclusive education in achieving fair and equity in mathematics education within the South African learning settings. With increased multiculturalism and diversity in classrooms, there is a need to integrate inclusive teaching and learning practices that will offer opportunities to learners from different genders, backgrounds, cultures, ethnicities, races, physical abilities, and socioeconomic status to thrive in the learning environment. For inclusive mathematics education, there is a need for stakeholders (teachers, learners, school administrators, parents, and society) to be welcoming and supportive of the diverse learning needs of all learners in the shared general mathematics classroom. The global educational practice has seen countries move towards inclusive practices. However, it is unclear the extent to which South African mathematics classrooms and other learning settings have embraced inclusive practices. The paper applies qualitative interview approach to investigate the importance of inclusive education, the role it plays in creating equity and fairness, and the obstacles to inclusive practices from the context of mathematics teachers in Limpopo Province, South Africa. The findings show that while teachers in Limpopo embrace the idea of inclusivity in mathematics education, more needs to be done to overcome challenges such as poor attitudes and inadequate resources if fairness and equity is to be achieved in mathematics learning environments.

INTRODUCTION AND BACKGROUND

The focus of this study was to explore the role of inclusive education in the teaching of Mathematics in high schools. It was hoped that the study would help both the researcher and the participants gain a deeper understanding of inclusive education policies and related teaching strategies. It focuses on understanding the impact that inclusive education has in promoting equality of learners in the learning environments. The paper's scope is mathematical attainment for all learners, which is a fundamental concern for all interested in understanding mathematics teaching and learning. The foundation of inclusive education is the recognition of human diversity and entails "supporting the belonging and full participation of all people together" (Cologon, 2014, p. 4). For inclusive mathematics education, there is a need for stakeholders (teachers, learners, school administrators, parents, and society) to be welcoming and supportive of the diverse learning needs of all learners in the shared general mathematics classroom (Goos et al., 2020). Thus, inclusive education covers learning difficulties or special needs but is not a synonym for it.

According to UNESCO (2009), inclusion is a description of what means to be crucial as well as be included in a society. Inclusion, in the context of mathematics teaching and learning, describes a way of teaching, in which all students are taught in a regular classroom (Cornwall



& Graham-Matheson, 2012). However, the emergence of Open Distance e-learning (ODEL) means that inclusion covers all forms of learning in different settings and not necessarily classroom activities (Manase, 2024). The contemporary environment has increasingly grown multicultural and more diverse and the need to ensure fair and equal education is now more important than ever (Manase, 2024). Inclusive education acknowledges the intrinsic worth and significance of each learner and attempts to establish a conducive atmosphere wherein all learners, irrespective of their diverse backgrounds and abilities, are provided with equitable opportunities to thrive and achieve academic success (Mulovhedzi, 2021).

From the perspectives of teachers, inclusive education can foster equity and celebrate diversity. By understanding the experiences, challenges, and perceptions of educators, valuable insights can be gained to enhance the implementation of inclusive practices and contribute to more inclusive and equitable educational systems (Ehsan, 2018). Inclusive education's comprehensive approach values and respects the unique qualities of every learner, considering things like learners' cultural background, language, gender, ethnicity, race, and economic status (Ghosh & Galczynski, 2014). The goal of inclusive education is to foster an inclusive and embracing culture within educational institutions, where every student experiences a sense of belonging, actively participates in their learning, and has their individual needs and strengths catered to (Francis et al., 2016). Teachers play a pivotal role in actualising inclusive education, as they are key agents who facilitate the development of inclusive practices and create a supportive and inclusive classroom environment. In this context, inclusivity refers to creating an enabling environment for learners with differences such as socioeconomic status, physical ability, intellectual abilities, race, and challenges such as geographical distance to thrive in the learning environment.

Research gap

The contemporary world is increasingly experiencing a recognition of the significance of inclusive education in promoting fairness and equity in learning environments. Despite this, there is a research gap in the context of South Africa. As such, there is a scarcity of studies conducted specifically focusing on the role of inclusive education in teaching and learning mathematics in the South African educational context. This research gap necessitates empirical investigations to explore the current status of inclusive education in South Africa, examining the impact of marginalised learners and obstacles to successful implementation. Additionally, there is a need to explore the experiences and perspectives of mathematics teachers in relation to inclusive education together with the societal, cultural, and institutional factors affecting the implementation of inclusive education. Addressing these research gaps will help advance inclusive practices in South Africa and offer evidence-based recommendations to foster fairness and equity in the country's educational system especially in the teaching and learning of mathematics.

Research aim and objectives

The study aims to explore the role of inclusive education in promoting equity in mathematics education. The objectives of the study are:



- To explore the importance of inclusivity in mathematics education from the perspectives of mathematics teachers.
- To investigate the role of inclusive education in reducing disparities among learners in mathematics learning contexts.
- To explore the experiences of teachers with inclusive education in mathematics learning settings.

Motivation for the study

Many societies across the globe are growing increasingly diverse and unequal. Within and across countries, migration due to war, conflicts, climate change, and poverty (to name a few), are shifting the nature of contemporary societies. Schools in general and mathematics classrooms in particular, reflect societal diversity and inequities as well as contain the possibilities to transform them. Providing access to quality mathematics education to learners from different cultural perspectives and knowledge backgrounds, diverse racial, ethnic, religious, linguistic, gender, socio-economic status, and sexual orientations, is an important endeavour. Within this context, it is not surprising that there has been a growing focus on equity-related research in mathematics education practices, theories, curricula, and policies. These aspects act as motivation for this study which focuses on the role that inclusive education can play to promote equality among learners in recognition of their uniqueness and diversity in mathematics classrooms.

In studying mathematics, students can experience different levels of challenges based on their unique diversities including their socioeconomic status, physical abilities, intellectual strengths, cultures, races, geographical location, and gender – among others. This study is motivated by the need to explore practices that can help to reduce these disparities and create a fair and equal platform for learners to succeed in mathematics education.

THEORETICAL FRAMEWORK

The theories applied in the study are system theory and social-constructionist theory. Niklas Luhmann's system theory is sometimes used in research on inclusive education (Baraldi & Corsi, 2017). People socially construct institutions through their communication with each other. Their theory on social construction is rooted in Marx's proposition that a person's consciousness is determined by their sociality, which also includes what most people know as reality (Möller & Siri, 2023). The concept of the social system is meaningful for investigating how inclusion and exclusion in education occur. A social system is created through social communication. Communication consists of information, messages, and understanding and is performed through linguistic and physical actions (Möller & Siri, 2023). Once a system is created, it communicates meaning. Meaning acts as the internal node around which communication in the system is centred, and it defines what types of communication and ideas are included in the system and what types are excluded (Rapp & Corral-Granados, 2024). For example, activity in a classroom can communicate many different meanings and, therefore, consist of many different social sub-systems. In the context of mathematics, teachers can foster



an environment where learners respect each other's uniqueness and embrace them to enhance knowledge acquisition rather than create conflicts. The teachers can create sub-systems where they can help learners who have difficulties cope while not making others feel left out.

The environment puts pressure on a system by asking it to include different types of meaning. Because a system is built on a particular meaning, it cannot incorporate contradictions. Therefore, the system reacts by ignoring the pressure or differentiating into several sub-systems, which again strengthens the argument that educational systems consist of many social sub-systems (Rapp & Corral-Granados, 2024). It is suggested that social systems are socially constructed not only by human beings but also by norms and institutions such as laws, policies, or general popular opinions. The latter understanding stems from neo-institutional theory, which maintains that ideas travel and are translated into educational systems, objectified into various types of policies, and used to focus on inclusion in education (Rapp & Corral-Granados, 2024). In mathematics teaching and learning, educators can develop policies that ensure all learners are treated fairly and equally, considering their diversity. This implies that policies developed would also focus on creating an enabling environment for all learners to thrive academically. The conceptual framework in Figure 1 is ideal for this study as it recognises the existence of different social subsystems in the learning environment, which are caused by virtue of the diversity amongst the learners. Thus, the strategies developed by teachers to ensure inclusivity in mathematics education constitute the institutional environments affecting the social constructed social systems.



Figure 1. *Conceptual Framework (Rapp & Corral-Granados, 2024)*

LITERATURE REVIEW

Inclusive education has gained traction in the global education landscape as a model for promoting equity and diversity amongst learners in schools. It recognises that each learner is valuable. It aims to create a setting where all learners, notwithstanding their diverse abilities and backgrounds, have fair and equal opportunities to learn and succeed (Campbell, 2021). Inclusivity in mathematics education means creating a learning environment where all students feel valued, supported, and have access to quality instruction. The key aspects of inclusive mathematics teaching and learning: Creating a safe and welcoming environment, differentiated instruction, diverse learning experiences, culturally responsive teaching, collaboration and



others. Various studies have focused on inclusivity in mathematics and the influence it has in creating equity and fairness for learners.

Some studies have shown how technology integration in mathematics teaching and learning can help reduce disparities among learners. According to Kormos (2018), online materials, digital resources, and virtual learning opportunities assist in bridging the resource gap faced by many urban communities. Consequently, students are capable of accessing educational content notwithstanding their location and beyond their immediate environment, which leads to increased access to mathematical resources. De Vita, Verschaffel, and Elen (2014) found that technology can also offer accommodation and assistive tools for students with disabilities, thereby ensuring equal access to mathematical learning experiences (Carstens, Mallon, Bataineh, & Al-Bataineh, 2021). Despite the impact of technology, various challenges need to be addressed to optimise the potential of technology in mathematics education. Challenges such as limited resources and funding often hinder the integration of technology in schools (Aruleba & Jere, 2022).

Landa, Zhou, and Marongwe (2021) found that collaborative efforts among stakeholders are necessary to advocate for increased funding and support for technology initiatives in South African schools. Since technology can help bridge these gaps, equitable access to technology and digital resources must be ensured for all students (Mhlongo, Mbatha, Ramatsetse, & Dlamini, 2023). Technology integration coupled with strategies such as culturally responsive teaching practices and inclusive curriculum development can help engage and support underrepresented groups in mathematics (Moreno-Armella & Santos-Trigo, 2015).

Online collaborative projects and competitions can promote collaboration and teamwork in urban mathematics classrooms. According to Nousiainen, Kangas, Rikala, and Vesisenaho (2018), online platforms provide opportunities for students to engage in collaborative problem-solving activities, communicate, share ideas, and learn from one another. This fosters a sense of community within the classroom and broadens students' perspectives on mathematics. It also ensures that learners who are not high performers in mathematics get assistance from the more knowledgeable learners. Students can also engage in online math competitions such as math Olympiads, math challenges, and online math clubs, which helps in further improving performance for learners facing difficulties in mathematics (Muntean, Bogusevski, & Muntean, 2019).

Technology can also create opportunities for learners with disabilities to thrive in the learning environment. For instance, learners with visual impairment typically face social and emotional challenges in the learning environment. They face social isolation, reduced self-esteem, and increased dependency on others (Salleh & Zainal, 2010). Encouraging a supportive and inclusive school environment and counselling services can help address these students' social and emotional needs; teachers and parental support can serve as protective factors against socio-emotional challenges such as a lack of social skills (Manitsa, Barlow-Brown, & Livanou, 2024).

Several studies in South Africa and beyond have highlighted various barriers to inclusive



education. They include school-level (institutional) barriers and cultural-level barriers. At the school level, inclusive education has in recent years been vastly embraced as an ideal model for education, both in South Africa, regionally, and internationally (Maher, 2009). However, this acceptance of inclusive practices may not necessarily translate to the actual occurrences in classrooms. The success of inclusive practices depends on the actions and behaviours of teachers as well as school administrators. It also depends on the investments by school management into practices that promote inclusivity.

Frederickson and Cline (2015) identified various challenges such as a lack of awareness among teachers, limited resources, and inadequate professional development opportunities. Studies have found that while teachers often report that they agree with the idea of inclusion, they believe that the needs of learners with unique needs such as those with disabilities are best met in separate classrooms (Campbell et al., 2003). Bornman and Rose (2021) noted a general lack of resources and support in addition to negative attitudes towards learners with unique needs, which contributes to the slow adoption of inclusive practices in South Africa.

A shift has been experienced in modern teacher education in South Africa that has focused on training teachers to accommodate diversity in the classroom (Oswald & Swart, 2011). New inclusive policies in South Africa have challenged the existing schemas among teachers regarding best practices for learners with disabilities (Oswald & Swart, 2011). Ensuring that educators are not only supportive of inclusion policies but also willing to challenge outdated beliefs and practices continues to be a considerable barrier to inclusive education. Teachers' attitudes toward inclusion might become more positive if, along with training, they were to receive the appropriate service support for their learners with disabilities.

Cultural-level barriers to inclusion education have been experienced in the South African context. Polat (2011) argues that improved infrastructure and resources may be necessary but are not sufficient to achieve inclusive education and that a change in attitude among school professionals and the community towards learners with unique abilities can enhance the adoption of inclusive education in low-income countries such as South Africa. If a community expresses disregard and prejudice towards people with disabilities, then discriminatory practices will continue to be propagated even in schools. Shaukat et al. (2019) investigated the attitudes and beliefs of teachers towards inclusive practices and found that while many teachers recognized the importance of inclusive education, they faced challenges in translating their beliefs into practice due to various contextual factors. It examined the professional development needs of teachers concerning inclusive education, emphasizing the importance of targeted training programs to enhance teachers' knowledge and skills.

A study in the KwaZulu-Natal province of South Africa focusing on children, parents, teachers, and aid workers to determine the extent of adoption of inclusive education revealed that banishment of learners with disabilities was perceived as a barrier by all participant groups (Maher, 2009). In the study, teachers blamed negative societal attitudes toward disability for the stigmatisation of learners with disabilities within ordinary schools and considered this a justification for maintaining separate schools (Maher, 2009). Further, parents and children in



this same study stated that learners may be safer in special schools for children with disabilities due to the intolerant attitudes of other children and school staff (Maher, 2009).

Similarly, Masasa et al. (2005) found that caregivers of children with disabilities who lived in the Western Cape province of South Africa expressed similar fears concerning the mistreatment of their children in ordinary schools, with 72% of the respondents stating that they believed their children with disabilities were better off in special schools. These findings indicate the degree to which a change in societal attitudes towards tolerance of learners with unique abilities as well as diverse backgrounds is required to achieve inclusive education.

The benefits of inclusive education have been highlighted in studies. A study by Rivera & McKeithan (2021) looked into how students with disabilities fare academically and socially in inclusive settings. The results showed positive outcomes, including academic progress and improved social interactions. Additionally, Koller et al. (2018) investigated the experiences of students with disabilities in inclusive classrooms, emphasizing the positive impact of inclusive practices on their self-esteem and overall wellbeing. Foreman and Arthur-Kelly (2017) identified effective inclusive pedagogical approaches and classroom strategies. They stressed the significance of differentiated instruction, teacher collaboration, and personalised support to address the diverse needs of students.

Equity is central to the principles of inclusive education, ensuring that every learner has equal opportunities for success, regardless of their background or characteristics. Equity in inclusive education focuses on eliminating barriers and providing fair and just opportunities for all students to succeed (Hardy & Woodcock, 2015). It requires addressing systemic inequalities and providing targeted support to marginalized groups, including students from low-income backgrounds, racial and ethnic minorities, and those with disabilities. Inclusive education plays a critical role in promoting equity by creating inclusive environments that prioritize individual needs, foster belonging, and ensure access to quality education for all learners (Hardy & Woodcock, 2015). This can be achieved through online platforms that encourage students to interact and share ideas that relate to mathematics problem-solving, as indicated by Nousiainen et al. (2018) – among other interactive strategies.

Culturally responsive pedagogy is a critical component of promoting equity within inclusive education. The study by Samuels (2018) emphasizes the significance of incorporating students' cultural backgrounds and experiences into the curriculum, instruction, and assessment practices in mathematics education. Culturally responsive pedagogy recognizes and affirms diverse cultures, values, and ways of knowing, empowering students from various backgrounds and promoting educational equity. Language diversity is another aspect of diversity that requires attention in inclusive education. Frederickson and Cline (2015) emphasise the significance of valuing and supporting students' home languages while developing their proficiency in the language of instruction. Inclusive education recognises language diversity as an asset and ensures that linguistic and cultural backgrounds are acknowledged, respected, and integrated into the educational experience. In the context of mathematics education, teachers can strive to present mathematical concepts using numbers and figures with minimal focus on language to



allow easy understanding for English-as-Second-Language (ESL) students. Where necessary, teachers can include translations for such students, which can be achieved through online platforms.

METHODOLOGY

The research design adopted in the study is a qualitative phenomenology approach. The approach is suitable for the study since the aim is to explore the experiences of mathematics teachers with inclusive education practices. These experiences will be based on the teachers' point of view, as they will describe their perceptions, hence, phenomenology (Shafiei et al., 2019). The qualitative nature emanates from the subjective experiences of teachers that will be provided through qualitative structured interviews (Hall et al., 2016). The research population is the mathematics teachers within four public schools (two rural schools and two urban schools) in Limpopo Province, particularly Moletlane Circuit in Capricorn District. Purposive sampling was used as the focus is on schools that have integrated inclusive practices within their mathematics classes. The schools that were identified were those that had embraced inclusive education practices including but not limited to the use of technology and creative strategies to promote mathematics learning. The district board was contacted to show schools that had embraced this practices and consent sort from the school heads. This sampling approach is suitable as it allows for the identification of participants who fit the criteria (Campbell et al., 2020). A sample size of 15 teachers ($N = 15$) was targeted although interview responses from only five of them was considered as a sample in this paper.

The data collection tool for the study is a qualitative open-ended interview that was intended to unearth the subjective experiences of the participants with inclusive education. The tool is considered because it allows the participants the opportunity to be expressive of their experiences without much restriction (Shafiei et al., 2019). The interviews were conducted virtually via a WhatsApp to minimise the costs involved and to the convenience of the participants. A sample showing questions from the interview schedule is shown in Table 1 below. The interview questions together with an informed consent form were shared through a WhatsApp call and recorded by the researcher with the consent of the participants. Ideally, the interview sessions lasted 15-20 minutes per participant.

Table 1: Sample Interview Questions

1. Do you believe that inclusive education is important? If so, why?
2. Does inclusive education help to promote fairness and equity in mathematics education? If so, how?
3. What obstacles did you experience in achieving inclusive education?

The interview outcomes were analysed thematically focusing on three areas: (1) the significance of inclusive education in mathematics; (2) the role of inclusive education in promoting fairness and equity in mathematics education, and (3) obstacles to inclusive education in mathematics learning settings.



SAMPLE DATA AND FINDINGS

The interview outcomes of five participants were considered for the analysis. Regarding the interview Question 1, all five participants believed that inclusive education was important in mathematics learning. Participant 1 mentioned, *“I think it is appropriate to consider all learners in their uniqueness and appreciate how they are. If we value everyone, then performance will improve.”* This response helps to highlight the role of inclusive education in mathematics, particularly making all learners feel valued in their learning environment, which contributes to improved performance. Participant 3 on the other hand stated, *“We have to keep up with changes in the world. We have to accommodate everyone in the education system.”* This response implies that the role of inclusive education is keeping abreast with changes in global education systems towards inclusivity of learners given the global multiculturalism and diversity. Participant 4 highlighted the role of inclusive education in promoting equality by indicating, *“The more we create practices in mathematics classroom to accommodate different learners, the fairer and more equitable our classrooms become and the more tolerant.”* Participant 1 also indicated, *“Inclusive practices create equal opportunities for everyone”*. These two responses indicate that inclusive practices promote fairness and equity among learners by allocating equal opportunities.

All five participants acknowledged that they experienced obstacles to inclusive education. Participant 5 indicated, *“I think our attitude is still not good for inclusive education to thrive. We teachers have cultural reservations towards inclusive practices.”* Participant 2 stated, *“We do not have enough resources in schools to have adequate inclusive programs.”* These two responses indicate that cultural attitudes and structural (school-level) barriers negatively affect inclusive practices in mathematics education.

IMPLICATIONS OF OUTCOMES FOR MATHEMATICS TEACHING AND LEARNING

The findings have significant implications for mathematics teaching and learning as they highlight the significance of inclusive education in the context of modern-day mathematics learning settings that are increasingly diverse and multicultural. It also highlights the influence of inclusive education in achieving equality within the student body. Finally, it helps to show the existing challenges that affect the attainment of an operational and effective inclusive framework.



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FROM ERROR TO INSIGHT: ANALYSING STUDENT RESPONSES TO FINANCIAL MATHEMATICS QUESTIONS AT NC(V) LEVEL 2

A.S. Ngoveni^a, M.A. Ngoveni^b and G.T. Mphuthi^b

^aTshwane University of Technology ^bUniversity of South Africa, South Africa

This study investigates the types of errors exhibited by NC(V) Level 2 students in response to a multi-part financial mathematics question from a preparatory examination. Seventeen student scripts were purposively sampled from a South African TVET college and analysed using a qualitative document analysis approach. Guided by Cognitive Load Theory, the study examined how multi-step tasks involving simple and compound interest, instalments, and cost comparisons may overwhelm students' working memory. The analysis identified two major categories of errors: conceptual errors, such as misinterpreting financial terminology and using incorrect values (e.g., instalments as principal), and procedural errors, including incorrect substitutions, omitted steps, and inappropriate operations. These patterns suggest that students often struggle not due to carelessness but due to cognitive overload and limited problem-solving fluency. The findings highlight the need for instructional strategies that reduce cognitive burden and build students' capacity to apply financial mathematics in real-life contexts.

Keywords: Cognitive Load Theory, Conceptual Errors, Document Analysis, Financial Mathematics, Procedural Errors, TVET Education

INTRODUCTION

In South African Technical and Vocational Education and Training (TVET) colleges, persistent underperformance in financial mathematics is a major educational concern. This poor performance is often linked to students' limited financial literacy, weak foundational knowledge, and a disconnect between classroom instruction and real-life financial contexts (Ngoveni, 2025; Makonye, 2020). Financial mathematics is a core component of the National Curriculum (Vocational), yet many students struggle to master fundamental concepts such as budgeting, interest calculations, and evaluating financial decisions (Ngoveni, 2025).

Underlying these performance challenges are specific conceptual and procedural errors, which significantly affect students' ability to apply mathematical reasoning to financial problems. Conceptual errors arise when students misunderstand core ideas, such as mistaking instalment amounts for principal values or failing to distinguish between simple and compound interest. Procedural errors, on the other hand, occur when students correctly understand the concept but misapply formulas, skip steps, or incorrectly substitute values (Khalo & Bayaga, 2015; Hernández-Cortes et al., 2023; Ngoveni, 2025).

Despite growing evidence of these error types, the term "error analysis" is rarely introduced early or emphasised in financial mathematics research, especially in vocational settings. Yet, studies show that structured frameworks like Newman's Error Analysis (NEA) offer valuable diagnostic tools to locate where in the problem-solving process students go wrong, across



stages such as reading, comprehension, transformation, process skills, and encoding (Ngoveni, 2025; Putri et al., 2023). While NEA is useful for categorising and identifying error types, this study applies Cognitive Load Theory (Sweller, 1988) as its primary theoretical lens. CLT provides a deeper explanation of why students make these errors by accounting for the mental effort required to solve multi-step financial problems. When the complexity of a task exceeds a student's working memory capacity, cognitive overload may result in both conceptual misunderstandings and procedural breakdowns (Sweller, 2010).

This paper responds to the research question: How do conceptual and procedural errors made by NC(V) Level 2 students in financial mathematics reflect cognitive overload, and what instructional insights can be drawn from these errors?

THEORETICAL UNDERPINNINGS

This study is conceptually framed by Cognitive Load Theory (CLT), originally developed by Sweller (1988). CLT asserts that learners have a limited working memory capacity, and when the demands of a learning task exceed this capacity, cognitive overload occurs, impeding learning and performance. Financial mathematics, particularly in vocational education contexts, often involves complex, multi-step procedures that place significant cognitive demands on students. Tasks such as selecting the correct interest formula, substituting appropriate values, and interpreting contextual financial information all compete for working memory resources.

CLT distinguishes between three types of cognitive load:

- Intrinsic load: The inherent difficulty of the task, such as understanding compound interest or interpreting budgeting scenarios.
- Extraneous load: Cognitive burden caused by poorly designed instruction or confusing problem structures.
- Germane load: Mental effort dedicated to learning and constructing new schemas.

In this study, student errors, both conceptual (e.g., misinterpreting the meaning of "instalment" or "variance") and procedural (e.g., incorrect substitution, incomplete steps), are interpreted through the lens of CLT. For example, failure to distinguish between the total amount and the monthly repayment may result from extraneous load created by ambiguous question wording, while procedural slips in using the interest formula could reflect an overwhelming intrinsic load.

To make this framework analytically useful, the study adapted elements from Newman's Error Analysis (NEA) to identify the stage at which each error occurred, whether due to a misunderstanding of the question (reading/comprehension), misapplication of formulas (transformation/process), or incorrect final presentation (encoding). While NEA provided the diagnostic structure, CLT offered the explanatory lens for understanding why such errors might occur, especially when the multi-step nature of the financial task increased the likelihood of overload.

The linking NEA's error stages with types of cognitive load, the analysis aims to go beyond simply listing errors to interpreting them as indicators of specific cognitive challenges. This



integrated framework supports the development of instructional strategies that reduce extraneous load, simplify intrinsic load where possible, and increase germane load through scaffolding and conceptual clarity.

Student Challenges in Financial Mathematics

Financial mathematics is a critical component of mathematics education, as it equips students with the skills to make informed decisions about money, budgeting, savings, interest, and long-term financial planning. It plays a foundational role in developing personal financial literacy, which is increasingly recognised as essential for economic participation and responsible citizenship (Căprioară, 2019; OECD, 2016). In the South African context, financial mathematics is a compulsory topic in the National Certificate (Vocational) curriculum, aiming to provide students with tools to engage meaningfully with real-life financial situations (Ngoveni, 2025; Makonye, 2020). Despite this, local and international studies suggest that many students struggle to connect classroom instruction with practical financial reasoning, particularly in environments with weak mathematical foundations or limited exposure to apply problem-solving (Ngoveni, 2025; Makonye, 2020; Căprioară, 2019; OECD, 2016).

In a study conducted in Kosovo, Yildirim and Vardari (2020) found that university students across disciplines scored an average of just 39.7% on financial mathematics literacy assessments, reflecting a fundamental gap between theoretical instruction and its practical application, particularly in budgeting and interest calculations. These findings point to a broader trend: while financial mathematics is present in the curriculum, students struggle to transfer classroom knowledge into real-world financial reasoning.

This disconnect is particularly evident in tasks involving compound interest, loan terms, or instalment calculations. Pakhomova (2022) attributes these difficulties to the abstract and symbolic nature of financial instruction, noting that learners often lack experience with contextual or scenario-based learning. To bridge this gap, the integration of business simulations and narrative-based tasks is recommended, which fosters deeper conceptual understanding and application.

Souza et al. (2024) echo these concerns, arguing that students fail to see the relevance of financial mathematics taught through isolated procedures. Their findings advocate for open-ended, problem-solving tasks that demand strategic exploration and contextual decision-making. Similarly, Vinnikova et al. (2020) emphasise that a one-size-fits-all teaching approach overlooks the diverse financial experiences and backgrounds of students. Their research supports pedagogies that strengthen conceptual foundations and promote long-term retention.

Căprioară et al. (2019) add that many financial mathematics curricula prioritise procedural fluency, mastery of formulas and operations, while neglecting the decision-making and reasoning required for personal financial literacy. Their textbook analysis found limited emphasis on real-world problem-solving, which restricts students' ability to apply mathematical tools in unfamiliar or complex situations.



Conceptual and Procedural Errors in Financial Mathematics

The difficulties discussed above are often rooted in identifiable conceptual and procedural errors, which emerge when students attempt to apply mathematics in financial contexts. Conceptual errors involve misunderstandings of key ideas, such as confusing instalments with total amounts or failing to distinguish between simple and compound interest. Procedural errors occur when students understand the concept but make mistakes in the steps—such as misapplying a formula, performing incorrect operations, or omitting necessary calculations (Khalo & Bayaga, 2015; Ngoveni, 2025; Makonye, 2020).

These error types are especially prevalent in vocational education settings, where students often experience challenges in decoding financial language, interpreting compound tasks, and following through multi-step problems. Ngoveni (2025) and Motseki and Luneta (2024) found that NC(V) and TVET students regularly exhibit such errors—misinterpreting question instructions, overgeneralising formulas, and selecting inappropriate values—particularly under assessment conditions involving word problems and applied scenarios.

Several studies highlight that these errors are systematic rather than random, arising from cognitive overload and instructional gaps rather than mere carelessness. For example, Putri et al. (2023) reported that high school students frequently skipped steps or substituted incorrect values in compound interest problems, while Motseki and Luneta (2024) observed that TVET students struggled to distinguish between mathematically relevant and irrelevant contextual details. These patterns reflect common breakdowns in conceptual understanding and procedural execution.

Several studies use frameworks such as Newman's Error Analysis (NEA) to classify mathematical mistakes across stages of problem-solving (e.g., reading, comprehension, transformation). While NEA offers insight into where errors may occur, this study is not based on NEA as a coding or analytical tool. Instead, it draws exclusively on Cognitive Load Theory (Sweller, 1988) to explain *why* such errors arise, particularly in the context of multi-step financial problems that may exceed students' working memory capacity (Sweller, 2011). CLT allows for an interpretation of both conceptual errors (e.g., confusing financial terms or relationships) and procedural errors (e.g., incorrect formula application) as outcomes of cognitive overload rather than simple carelessness or gaps in knowledge.

This literature highlights the importance of analysing student errors not simply as surface mistakes but as diagnostic indicators of deeper cognitive and instructional challenges. By situating the analysis of conceptual and procedural errors within the framework of CLT, this study contributes to the growing body of work aiming to improve mathematical instruction in financial contexts, particularly in vocational education environments.

METHODOLOGY

This study followed a qualitative document analysis approach to explore the types of conceptual and procedural errors that NC(V) Level 2 students exhibit when solving a multi-part financial mathematics question. Document analysis was chosen because the focus of the study was on interpreting written student responses, rather than studying participants directly through interviews or classroom observation. The aim was to systematically examine students'



mathematical thinking as reflected in their written solutions and to interpret patterns of error through the lens of Cognitive Load Theory.

The data consisted of 17 student scripts from a 2021 preparatory examination conducted at a public TVET college in Limpopo, South Africa. The scripts were selected purposively, with the intention of capturing a range of responses that could reveal different types of errors. The selected question from the examination focused on compound interest, total repayment, and cost comparison, areas that typically involve multi-step reasoning and are known to be cognitively demanding for students. The choice of this specific question was deliberate, as it integrated real-world financial concepts and required students to apply multiple operations across several stages of calculation and interpretation.

Student responses were analysed using an error categorisation scheme informed by existing literature on mathematical errors within financial contexts. The analysis focused on identifying and interpreting two main types of errors. Conceptual errors were those that reflected misunderstandings of financial terminology, incorrect selection of formulas, or confusion regarding the relationships between quantities, for instance, misinterpreting monthly instalments as the principal amount. Procedural errors, on the other hand, occurred when students demonstrated an understanding of the underlying concept but made mistakes in its execution. These included inserting incorrect values into formulas, omitting essential steps in calculations, or applying inappropriate mathematical operations.

Each script was read carefully and coded manually. The classification of errors was guided by Cognitive Load Theory, which provided a theoretical lens for interpreting why these mistakes occurred. For instance, when students skipped intermediate steps or selected inappropriate strategies, these responses were considered indicative of cognitive overload, a result of excessive intrinsic or extraneous cognitive load in managing the complexity of the task. This framework helped distinguish between errors due to a lack of understanding and those related to the cognitive demands of the problem.

To ensure trustworthiness in the analysis, several strategies were employed. First, a clear audit trail was maintained, documenting the process by which codes were developed, applied, and refined throughout the data analysis. This provided transparency and traceability in how interpretations were reached. In addition, a subset of student scripts was independently reviewed by an experienced mathematics educator to verify the consistency and accuracy of the error classifications. Finally, where appropriate, student responses were cross-referenced with relevant curriculum standards and examiner expectations to confirm that the interpretations aligned with the intended learning outcomes and assessment criteria.

Although the data was drawn from a 2021 assessment, the focus of the study was not on evaluating performance at a specific point in time, but on identifying recurring patterns of error that remain relevant in the ongoing challenge of improving financial mathematics instruction in the TVET context.

FINDINGS

This section analyses student responses to Question 4.2 from the 2021 NC(V) Level 2 financial mathematics trial examination. The question comprised five sub-items requiring students to



extract financial information, perform simple and compound interest calculations, and evaluate the most cost-effective repayment option. Given its real-world context and multi-step structure, the question offered valuable diagnostic insight into students' conceptual understanding, procedural fluency, and reasoning processes in applied financial mathematics.

Question 4.2 Constance wants to buy the following television set from her local furniture shop. Constance does not have the cash to buy this television set, but she has three options available to her.

OPTION A: She can choose to enter into a hire purchase agreement with the furniture shop and pay R500 for 24 months with no deposit required.



OPTION B: She can borrow the money from her local bank, which charges her 16% simple interest per annum for a period of 2 years.

OPTION C: She can borrow the money from a microlender and pay back 18% compound interest per annum for a period of 2 years.

- 4.2.1 What is the cash price for the television? (1)
- 4.2.2 How much would Constance pay after 24 months if she chooses Option A? (2)
- 4.2.3 How much would she pay back per month if she chooses Option B? (3)
- 4.2.4 How much would she pay back per month if she chooses Option C? (3)
- 4.2.5 Which of the three options would be the best one for Constance? (1)

Error Analysis of Questions Involving Calculations

Question 4.2.1

Question 4.2.1 required students to identify the cash price of an item from a given advertisement. This was considered a direct retrieval task, and all students were expected to earn full marks. However, this expectation was not met across the entire cohort.

Out of 17 students in the 2021 group, 13 correctly identified the cash price as R8999.00. However, four students did not answer correctly, highlighting an unexpected gap in basic information extraction.



Among the incorrect responses, two students, 7/2021 and 11/2021, submitted R9000.00 instead of the exact amount. Their written responses indicate that they rounded the cash price up despite the absence of any instruction. This represents a conceptual error, where students appear to have applied everyday practices of estimation or approximation in a mathematical context that demanded precision. Such rounding, though minor in appearance, deviated from the task's requirements and resulted in the loss of marks.

7/2021 

11/2021 

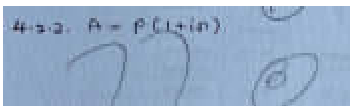
Question 4.2.2

Question 4.2.2 required students to calculate the total amount payable when an item is bought in monthly instalments. The task involved a straightforward multiplication of a fixed monthly payment by the total number of months in the agreement. This was designed to test students' ability to interpret and apply numerical data from a real-world financial scenario.

Of the 17 students, 11 arrived at the correct answer of R12,000, either by directly stating the result or by performing the calculation $R500 \times 24$.

Among the six students who did not reach the correct answer, there was notable variety in the types of errors observed:

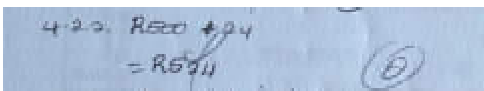
Student 12/2021 attempted to use the simple interest formula, typically used for calculating accumulated values on loans or savings, rather than a basic multiplication. The response suggests difficulty in recognising the nature of the transaction and misinterpreting the required method, likely due to reading the question inattentively or overgeneralising from previously learned procedures.

12/2021 

Student 14/2021 multiplied R500 by 2 instead of 24, indicating a possible confusion between months and years. This response suggests a misunderstanding of the time frame, potentially caused by a default assumption of annual terms when processing interest-based questions.

14/2021 

Student 17/2021 added 24 to R500, resulting in R524, rather than multiplying the two values. This reflects a procedural error stemming from an incorrect operation selection—addition instead of multiplication, despite both numbers being presented in the task.

17/2021 

These varied errors demonstrate a combination of conceptual and procedural misunderstandings, with some students misreading the question structure, others applying incorrect formulas, and several failing to interpret numerical data within its proper context.

Question 4.2.3



In Question 4.2.3, students were expected to apply the simple interest formula to calculate the total amount payable after two years and then divide that total by 24 months to determine the monthly payment. This two-step calculation required the correct formula use and a clear understanding of the context of the payment schedule.

This question posed a significant challenge for the 2021 cohort. Only two students out of 17 managed to score a mark, with one student achieving partial credit (2 marks) and one earning full marks (3 marks). The low success rate highlighted critical gaps in students' understanding of the formula's application and the result's contextual interpretation.

Six students correctly identified the simple interest formula, $A = P(1 + in)$, but failed to substitute the correct value for the principal (P). This error affected the entire calculation, demonstrating that the correct formula alone was insufficient if not paired with proper input values.

Among the responses, students 12/2021 and 15/2021 illustrate another recurring error. Both students correctly applied the formula and calculated the total amount A, but left their final answer as the total amount without completing the second step, dividing by 24 months to find the monthly payment. This indicates a partial understanding of the question, where students engaged with the computational procedure but missed the contextual requirement for interpreting the result.

These findings reveal a pattern of procedural breakdowns and incomplete interpretations of multi-step problems, especially when context-specific reasoning was required after formula application. This breakdown reflects a failure to integrate multiple steps, a sign of intrinsic cognitive load exceeding students' processing limits.

12/2021

$$\begin{aligned}
 \text{Ans: } A &= P(1 + in) \\
 &= 1000 \left(1 + \frac{10}{100} \times 2\right) \\
 A &= 1200
 \end{aligned}$$

15/2021

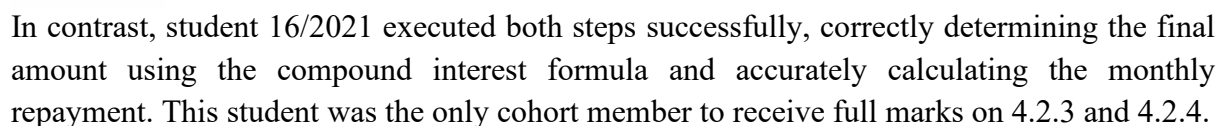
$$\begin{aligned}
 \text{Ans: } A &= P(1 + in) \\
 A &= 1000 \left(1 + \frac{10}{100} \times 2\right) \\
 A &= 1200
 \end{aligned}$$

Question 4.2.4

Question 4.2.4 required students to calculate the final repayment amount using the compound interest formula and divide this total by 24 to determine the monthly payment. This question is built on the previous one (4.2.3) by introducing an exponential component, making the mathematical structure slightly more complex but following the same two-step reasoning process.

As with the previous question, only two students out of 17 scored marks. One of these students earned two marks, while the other achieved full marks for correctly calculating the final amount and the monthly repayment.

Student 15/2021 correctly applied the compound interest formula and arrived at the accurate final amount (A) of R12, 530.40, but did not proceed to divide the amount by 24 to determine the monthly payment. This mirrors the student's response to Question 4.2.3 and suggests a consistent pattern of incomplete execution of multi-step problems and correct formula use, but failure to interpret the task fully. This confirms that while procedural recall of the formula was evident, intrinsic and extraneous loads prevented full task execution.



A common trend among other students was the incorrect substitution of the principal value (P) into the compound interest formula. This was demonstrated by student 5/2021, who used R500, the monthly instalment, rather than the actual cash price or loan amount, resulting in a fundamentally flawed calculation. Although the structure of the formula was correctly written and understood, the incorrect input led to a meaningless output.

These responses illustrate a continued challenge with multi-step financial calculations, particularly in recognising which values are contextually appropriate when applying standard formulas.

4.13 Option B

$$A = P\{1 \leq Z \leq 3\}$$
$$= \Phi(3) - \Phi(1) = 0.7443 - 0.2420 = 0.5023$$

4.14 Option C

$$A = P\{1 \leq Z \leq 3\}$$
$$= \Phi(3) - \Phi(1) = 0.7443 - 0.2420 = 0.5023$$

16/2021

例 23: $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 2 \end{pmatrix}$
 $A^{-1} = \frac{1}{|A|} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$
 $A^{-1} = \frac{1}{1} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$
 $A^{-1} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$

$$B = P(C + L)^D$$

$$= 80 + (1 + 0.1)^8$$

$$= 80 + (1.1)^8$$

$$= 80 + 2.14358881$$

$$= 82.14358881$$

$$P = 6.347$$

5/2021

Question 4.2.5

This question required students to evaluate the three financial options presented in the earlier parts of the question and determine which one would offer the most significant financial benefit to the buyer, Constance. This task involved interpreting and comparing total repayment values across simple interest, compound interest, and instalment plans to identify the most cost-effective choice.

Despite being worth only one mark and requiring a straightforward selection, only six out of 17 students responded correctly. This result highlights a widespread challenge among students in interpreting numerical results within a decision-making context.



While earlier sub-questions focused on procedural calculations, this question tested students' ability to engage in comparative reasoning and apply financial judgment based on prior computations. The low success rate suggests that even when students successfully performed calculations, many struggled to synthesise that information to make an informed, final decision. Although non-computational, the requirement to compare total costs introduced a cognitive challenge for students with fragmented or incomplete earlier calculations.

DISCUSSION

The findings of this study show a consistent pattern of procedural conceptual errors across several sub-questions in the financial mathematics trial examination. Even though Question 4.2 was designed to test students based on realistic financial scenarios, its multi-part, context-based structure seemed too overwhelming to many students. Of the 17 students, only two achieved near full or full marks on the compound and simple interest tasks (Questions 4.2.3 and 4.2.4). On the other hand, less than half of the students correctly completed Question 4.2.5, which required them to decide based on the results of the previous solutions. These trends emphasise the critical obstacles students' exhibit regarding their ability to apply financial knowledge in real-life contexts. Several authors echoed similar concerns in their findings found in the existing literature (Pakhomova, 2022; Souza et al., 2024; Khalo et al., 2022).

According to Cognitive Load Theory (CLT), the findings from this study indicate that the challenges faced by students may emanate not only from knowledge deficiencies but also from cognitive overload due to the task's structure and complexity. Sweller (1988, 2011) posits that problem-solving tasks requiring multiple sequential steps, like formula selection, value identification, total computation, and result interpretation, may readily exceed a student's working memory capacity. This is particularly true when the students lack foundational fluency. This was apparent in multiple responses when students accurately identified a formula (e.g., $A = P(1 + in)$) but employed incorrect values (e.g., using R500 instead of the cash price as the primary). These errors signify deficiencies in the transformation and processing phases (Paas et al., 2003), wherein students were unable to establish a meaningful connection between the context and the requisite technique.

The frequent omission of the final step, dividing the total amount (A) by 24 months, further illustrates intrinsic and extraneous cognitive load interfering with problem completion. While students may have understood individual components of the problem, the need to carry out operations across multiple steps appeared to exceed their capacity to integrate and retain task elements, consistent with findings by Motseki and Luneta (2024) and Putri et al. (2023). These authors similarly found that TVET students often misapply interest formulas or skip final steps due to cognitive strain, not necessarily a lack of effort or carelessness.

Conceptual misunderstandings were also apparent in the misinterpretation of financial terms, such as the substitution of the monthly instalment value for the principal amount in both the simple and compound interest calculations. This reflects a failure of contextual comprehension, a key barrier to applied mathematical literacy noted in prior studies (Makonye, 2020; Căprioară et al., 2019). The incorrect rounding of the cash price by students 7/2021 and 11/2021 may also suggest an overreliance on intuitive, everyday approximations rather than precise mathematical thinking, as previously discussed by Lusardi and Mitchell (2023) and Vinnikova et al. (2020).



Question 4.2.5, which required comparative reasoning rather than calculation, produced poor outcomes despite its simplicity. This reinforces the idea that students may complete mathematical procedures in isolation but struggle to synthesise results into a coherent decision, particularly when under cognitive pressure (Sweller, 2011). It also supports findings by Căprioară et al. (2019) and Souza et al. (2024), who argue that current curricular approaches often neglect decision-making and interpretation as essential components of financial literacy.

The findings underscore the need to consider not only what is taught but also how it is taught, particularly in contexts like TVET, where students bring diverse prior knowledge and may not have developed the fluency required to manage compound, real-world tasks. While many students could recall formulas, they lacked the ability to apply them flexibly within an unfamiliar context, suggesting insufficient development of both procedural fluency and strategic competence, as framed by Kilpatrick et al. (2001).

This was apparent in multiple responses when students accurately identified a formula (e.g., $A = P(1 + in)$) but employed incorrect values (e.g., using R500 instead of the cash price of R8999 as the primary amount). The real-world nature of the task, combined with its multi-step structure, significantly increased both intrinsic and extraneous cognitive load, making it more difficult for students to retain and process information effectively.

CONCLUSION

This research explored the procedural and conceptual errors made by NC(V) Level 2 students while responding to a multi-part question about financial mathematics. Consistent difficulties in the interpretation of contextual information were revealed. These included the execution of multi-step procedures, the selection of appropriate formulas, and the drawing of financial comparisons. This study showed that these errors reflected more profound challenges related to the task's complexity and the cognitive processing limitations reflected in the Cognitive Load Theory. This study contributes to the body of knowledge in the TVET colleges, especially in financial mathematics. It provides a clear understanding of various types of errors that students make using the cognitive perspective and highlights the need to support high-load mathematics tasks.

Limitations

The study involved only one TVET college and 17 NC(V) Level 2 engineering students, which limits the generalisability of the findings. Another element is the focus on the written trial examination answer scripts alone. Another factor included is that the TVET sector accommodates students from a diverse demographic space. Factors such as prior schooling, language proficiency or socio-economic background might have influenced the students' performance.

RECOMMENDATIONS

The cognitive demands associated with the multi-step questions and the observed error patterns highlight the need to design instructional materials to reduce cognitive load and strengthen conceptual understanding. During the teaching of this section, instructors may break down complex problems into manageable questions, which will assist learners in grasping sequential



steps. This breakdown will help model how to interpret and apply financial information. This will further assist students in contextualising the reasoning behind using a particular formula. The financial concepts should also be clarified using real-world problems to ease students' understanding and application.

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VALIDATING THE RESEARCH INSTRUMENTS: UNDERSTANDING PRE-SERVICE MATHEMATICS EDUCATORS' TECHNOLOGICAL CONTENT KNOWLEDGE (TCK) OF EUCLIDEAN GEOMETRY

Nkosinathi Ndlovu and Deonarain Brijlal

Durban University of Technology

Poor performances by preservice teachers in South Africa continue unabated due to a variety of reasons. It is often assumed that the students entering a university have the relevant preliminary knowledge to proceed with the university curriculum. However, when we teach or assess students, we find that students have gaps in their school knowledge. In order to understand the technical content knowledge of preservice teachers, we decided to explore such understanding in Euclidean Geometry. This paper reports on a research instrument designed for pre-service teachers who are being prepared to teach mathematics at high schools in South Africa. This paper reports on a pilot study conducted to validate research instruments designed to assess pre-service mathematics educators' Technological Content Knowledge (TCK) in Euclidean Geometry using GeoGebra. Conducted at a university of technology in South Africa, the study informed the refinement of questionnaire and a computer-based geometry task. Key findings from the pilot led to revisions that enhanced the instruments' reliability and alignment with the TPACK framework. The validated tools will be used in the main study exploring technology integration in mathematics education.

Keywords: pre-service teachers, research instrument, pilot study

INTRODUCTION

Transition from high school to university is particularly challenging for the preservice mathematics teacher. Mathematics teaching includes showing abstract ideas concretely and solving challenging problems (Steadly, Dragoo, Arafeh & Stephen, 2008). As part of the mathematics curriculum, the geometric domain continues to be an essential topic in mathematics teaching at all educational levels (Niyukuri, Nzotungicimpaye & Ntahomvukiye, 2020).

Due to the abstract nature of geometry, students often find it difficult to grasp, highlighting the need for innovative teaching approaches (Crossfield & Bourne, 2017). Dynamic geometry software, such as GeoGebra, is widely available and used to enhance conceptual understanding through interactive learning (Dockendorff & Solar, 2018). GeoGebra, a free and open-source tool (Sarmiento & Luna, 2017), is widely used to improve mathematics teaching and learning at all educational levels (Zengin, 2018).

Educators often face challenges in integrating technology into mathematics instruction due to limited competency, exposure, and experience, as seen in Malaysia with tools like GeoGebra (Musa et al., 2021). Similarly, preservice teachers in England lack confidence in using



technological devices effectively (Saralar-Aras & Firat, 2021), making it difficult to integrate pedagogical content knowledge into technology-based activities (Kiricilar & Yildiz, 2018).

In order to ensure that the research instruments address the research question of the main study we carried out a pilot study to validate the precision of the research instruments. The research questions for the main study are: a) What Technological Content Knowledge (TCK) of Euclidean geometry do pre-service educators possess using a GeoGebra software?, b) To what extent do pre-service educators understand Technological Content Knowledge (TCK) of Euclidean geometry using a GeoGebra software?, and c) Why do pre-service educators use Technological Content Knowledge of Euclidean geometry using a GeoGebra software in a lesson In order to answer these research questions we designed data capturing research instruments which were administered in a pilot study. In this paper we ask: *How valid are the research instruments designed to explore the understanding of pre-service mathematics educators' Technological Content Knowledge (TCK) of Euclidean Geometry?*

LITERATURE REVIEW

A pilot study serves as a preliminary investigation designed to refine research instruments and procedures prior to conducting the main study (Polit, Beck, & Hungler, 2001; Teijlingen & Hundley, 2001). Pilot studies are commonly used in quantitative research to test instruments and calculate sample sizes, especially in randomized controlled trials (Lancaster, Dodd & Williams, 2004). They are also essential in qualitative research, serving as preparation for the main study (Collingridge & Grantt, 2008). Pilot studies contribute to enhancing the trustworthiness of the results and ensuring transparency in the audit trail (Bryne, 2001). Pilot studies are rarely articulated in research literature, and mostly they are confined to justifying methodological choices, particularly in quantitative research (Teijlingen & Hundley, 2001). In education research, key questions about the role of pilot studies remain unresolved (Fraser et al., 2018), more especially among novice researchers (Harding, 2013)". Malmqvist et al. (2019) asserted that "conducting and reporting on pilot studies is a neglected part of the research process". Based on this argument, this paper presents authors' reflection on the pilot study done in completing a PhD mixed method dissertation on understanding pre-service mathematics educators Technological Content Knowledge (TCK) of Euclidean geometry.

METHODOLOGY

A pre-study was conducted before the main research to trial and adjust the research tools and methods. This process aimed to validate the proposed methodology and assess feasibility (Kim, 2010). Using a mixed-methods approach, the pre-study included both quantitative and qualitative methods. The pilot study helped clarify procedures and identify the most appropriate methods for answering the main study's research questions. In this study, a pre-study was conducted using a questionnaire administered to pre-service mathematics educators to explore their knowledge of Euclidean geometry and TCK in teaching with GeoGebra. Modifications and recommendations are discussed later. The qualitative approach in this study involved a GeoGebra task and observations to explore pre-service educators' TCK of Euclidean geometry in a GeoGebra lesson. These tools were piloted with twelve fourth-year Bachelor of Education



mathematics pre-service educators. The pilot tested the research questions, allowed practice in observing participants, and provided logistical insights that facilitated a more systematic approach to data collection and analysis.

Sample size and Selection of participants for the pilot study

Sample size recommendations for pilot studies vary, with some not requiring calculations. However, the pilot sample should mirror the target population, maintaining the same inclusion and exclusion criteria (Thabane et al., 2010). Graser (2019) recommends a pilot study with at least one participant or 10% of the expected sample size. Since pilot studies are crucial for the main study, researchers must approach participants with confidence and ensure all key measures are addressed to prevent unnecessary resource expenditure.

Pilot studies primarily assess feasibility, not test hypotheses (Janghorban, Latifnejad & Taghipour, 2014), so sample size is based on participant recruitment and study design rather than statistical power. For the questionnaire, ten third-year pre-service educators were purposefully selected. The pilot revealed issues with question flow and structure. The GeoGebra task pilot involved two groups of four pre-service educators, with participants providing pseudonyms at the end. The pilot also highlighted challenges in predicting responses, allowing the researcher to refine questioning techniques and probing strategies.

Questionnaire

The first phase of data collection used a closed-ended questionnaire with eight questions, which was validated for content validity before data collection. According to Creswell (2007), content validity refers to how well each item measures its intended concept and should be assessed by subject matter experts using a quantitative approach.

With the assistance of an expert scholar in mathematics education research, the GeoGebra task and questionnaire were checked for validity. The goal was to ensure that each item accurately measured its intended concept. After the pilot study, the questionnaire was revised to better align with the study's objectives. The reviewed questionnaire was expanded from eight questions to ten questions (see Table 1).

GeoGebra written task

The question that arises mostly in mathematics education research is how technology may support students' learning of mathematics. Dynamic software such as GeoGebra aids learners to create and manipulate geometry and algebraic representations – features that support problem-solving and reasoning (Falcade et al, 2007; Preiner, 2008). This study is concerned with pre-service educators' technological content knowledge (TCK) of Euclidean Geometry in a GeoGebra-based lesson. Hence, a written GeoGebra task was designed for pre-service educators to elicit their TCK of Euclidean geometry. The task focused on a topic in Euclidean Geometry and was designed by the researcher. During the pilot study, eight pre-service educators were selected to respond to the task. These pre-service educators met the selection criteria and would be excluded from the main study.



During the validation process, the question items of the GeoGebra task were checked if they measure what they are supposed to measure. An expert in the field advised on linking the GeoGebra task with the TPACK constructs. To elicit pre-service educators' TCK of the TPACK, I used the screen recorder during their engagement with the task on a computer. The design of the task was informed by the objective of the study: to understand the pre-service mathematics educators use GeoGebra to show their TCK of Euclidean Geometry. As such, the task was designed to address the CK, TK and TCK of the TPACK framework. Pre-service educators' possession of Euclidean geometry knowledge and technological knowledge of GeoGebra constitute their TCK of Euclidean geometry. Hence, using a GeoGebra task as tool in this study aided the researcher this component to pre-service educators. Table 1 below represents the alignment of the GeoGebra task with the TPACK framework constructs:

Question	Content Knowledge (CK)	Technological Knowledge (TK)
1.1	Knowledge of the shape of the circle ABC	Using GeoGebra to construct circle ABC with centre O.
	Knowledge of the concept "angle" and "circumference"; the relationship between angle measure at the circumference and the angle measured at the centre.	Using GeoGebra to measure $\hat{ACB}$ at the circumference, and $\hat{AOB}$ at the centre
	- Knowledge of the "chord" and its construction within the circle. - Knowledge of the "radius" and its construction within the circle	- Using GeoGebra to construct the chords AC and BC. - Constructing radii AO and OB using GeoGebra.
2.1	Visualising diagram a) and b) as belonging to the same theorem	Proving the theorem using GeoGebra through measuring angles.
3.1	Knowledge of the relationship between base angles of an isosceles triangle (angles at the semi-circle).	Proving $\hat{C}_1$ and $\hat{A}_1$ which are angles at the semi-circle.
4.1	- Knowledge about a cyclic quadrilateral - Opposite angle of a cyclic quadrilateral	Constructing a cyclic quadrilateral with GeoGebra.

As mentioned already, the question items of the task were examined for construct validity to determine whether pre-service educators are measuring what they are supposed to measure. The rigorous scrutiny of the tasks also provided an opportune moment to develop and refine the final task to be used in the main study. The analysis of data from the GeoGebra computer-based task was done using Microsoft Excel. The responses on this piloted task contained right or wrong answers (see Table 2).



Video Recording

The data was collected through observations and analysis of video recordings. Recordings of pre-service educators took place as they engaged in the task at the computer laboratory. This was done to capture all the non-observant activities among pre-service educators. According to Flick (2006), videos are a valuable tool in collecting data since they catch facts and processes that are too rapid or too intricate for the human eye. Likewise, when pre-service educators engage on the task, they were video recorded. This assisted in analysing their behaviour, social interaction and the impact that GeoGebra had on their thought processes and solutions to the Euclidean geometry problems.

DISCUSSION AND ANALYSIS OF DATA

The presentation and analysis of the results from the pilot study are discussed below:

Analysis of results from a questionnaire

Ten pre-service mathematics educators completed the questionnaire, which included two sections: Section A (demographics) and Section B (13 items on Euclidean Geometry and GeoGebra integration). Items 1–4 assessed content knowledge (CK), Items 5, 7, and 8 evaluated pedagogical knowledge, and Items 9–13 focused on GeoGebra integration skills. The following is the analysis of pre-service educators' responses to question items requiring their content knowledge (CK) of Euclidean Geometry:

Question 1: In your secondary school experience as a learner, were you taught Euclidean Geometry?		
Responses:	Yes = 100%	No = 0%
Question 2: If your answer to Question 1 is yes, how did you find learning Euclidean Geometry?		
Responses:	Easy = 39%	Difficult = 61%

Figure 1: Pre-service mathematics educators' responses to item 1 and item 2

Figure 1 shows that all pre-service mathematics educators responded to Items 1 and 2. While all had been taught Euclidean Geometry in high school, 61% found it difficult, indicating limited content knowledge. This supports findings by Ogbonnaya and Bogoshi (2024) on inadequate instruction in Grade 11 geometry. Based on the TPACK framework, participants lacked content knowledge (CK), consistent with existing literature (Mukuka & Alex, 2024; Tachie, 2020). These items will be retained for the main study as they align with the research questions and support instrument reliability. The question items 3 and 4 measured pre-service mathematics educators' experiences with learning Euclidean Geometry at a university level. The results of responses to these items are presented below:



Question 3: As a university student, did you learn about Euclidean Geometry in the Mathematics Education Module?		
Responses:	Yes = 100%	No = 0%
Question 4: If your answer to Question 3 is yes, how did you find learning Euclidean Geometry?		
Responses:	Easy = 87%	Difficult = 13%

Figure 2: Pre-service mathematics educators' responses to item 3 and item 4

Contrary to Mukuka and Alex (2024), findings show that pre-service educators find Euclidean Geometry easier to learn at university. This item will be retained, as it offers relevant data for the main research question. In question item 5, pre-service mathematics educators were required to indicate if they could teach Euclidean Geometry in FET Phase. The results of the response to this are presented in figure 3 below:

Question 5: Based on your knowledge of Euclidean Geometry, are you confident to teach it in FET Phase?	
Responses:	
Very confident = 55%	
I can teach it with difficulties = 38%	
Not confident to teach it = 0%	

Figure 3: Pre-service mathematics educators' responses to item 5

Figure 3 indicates that 55% of pre-service mathematics educators reported confidence in teaching Euclidean Geometry, while 38% indicated they could teach it with some difficulty. However, two participants did not respond to this item. As a result, the item was excluded from the main study due to incomplete responses and its overlap with pedagogical content knowledge, which will be addressed in the third phase of data collection. Instead, Item 5 was revised into a content-based question to assess pre-service educators' knowledge of high school Euclidean Geometry and to verify the accuracy of responses provided in Item 4.

Question 6 aimed to identify the digital resources pre-service mathematics educators would use to teach Euclidean Geometry. However, only 48% of respondents completed this item. The low response rate suggested that the extensive list of options may have caused confusion, resulting in the question being skipped either intentionally or unintentionally. In response, the item was revised to improve clarity and ensure better response rates.



Question 6: Which software can you use to teach Euclidean Geometry?	
Autograph	☐
GeoGebra	☐
Geometers' Sketchpad	☐
All of the above	☐
None of the above	☐

Figure 4: Pre-service mathematics educators' responses to item 6

Question 7 required pre-service mathematics educators to demonstrate their knowledge of Euclidean Geometry by selecting options corresponding to the forms of knowledge outlined by Mishra and Koehler (2006). However, 81% of the participants did not select any options. This suggests that the question may have been confusing, as the pre-service educators had not been familiarized with the TPACK framework or its components as defined by Mishra and Koehler (2006).

In Question 8, respondents were asked whether they had been adequately equipped to teach Euclidean Geometry within their Mathematics Education course. The results indicated that 81% of respondents felt prepared to teach the subject. However, the researcher was perplexed by the 19% who reported not receiving training to teach Euclidean Geometry despite being at the same level of study. Consequently, this question was replaced with a follow-up to Question 7, where respondents were asked whether they were capable of using GeoGebra software on a computer.

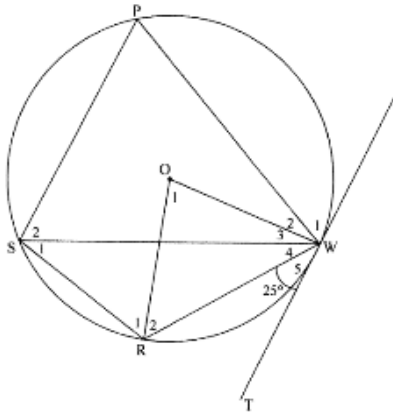
Question 9 asked respondents whether they could teach mathematics using GeoGebra software, and all the participants (100%) responded that they could. As this research is concerned with pre-service mathematics teachers' Technological Content Knowledge (TCK) of Euclidean Geometry in a GeoGebra lesson, the question was reworded to ask about respondents' perceptions of using GeoGebra to construct Euclidean geometry diagrams. In the same manner, Question 10 was redundant since respondents who indicated they could teach mathematics using GeoGebra in Question 9 would presumably have been trained to use it. Question 10 was thus changed to address respondents' views regarding applying GeoGebra to prove Euclidean geometry theorems.

Modification of a questionnaire after a pilot study

During the pilot study, changes were made to the questionnaire items. The following table below shows the changes made to the questionnaire:



Table 1: Changes made to the Questionnaire items

Question items	Changes made
Question 1: In your secondary school experience as a learner, were you taught Euclidean Geometry?	Question 1 was changed: In your secondary school experience as a learner, <u>did you learn</u> Euclidean geometry?
Question 5: Based on your knowledge of Euclidean Geometry, are you confident enough to <u>teach</u> it in FET Phase? [1] Very confident [2] I can teach it with some difficulties [3] Not confident to teach it.	Question 5 was changed: In the diagram below, PWRS is a cyclic quadrilateral in the circle, centered at O. $\triangle PSW$ is an equilateral triangle. TW is a tangent to the circle at W. Radii OR and OW are drawn. $\hat{W}_5 = 25^\circ$.  Determine, with reasons, the size of angle R1.
Question 6: Which of the following resources do you think you can use effectively to teach Euclidean Geometry? [1] Smartboard [2] Whiteboard and Data Projector [3] Geometers' Sketchpad [4] GeoGebra [5] None	Question 6 was changed: Which software do you think you can use to teach Euclidean Geometry? Autograph ☐ GeoGebra ☐ Sketchpad ☐ None ☐
Question 7 was added: Have you been equipped with GeoGebra software in Mathematics Module? Yes ☐ No ☐	



Question 8 was added: If your answer to Question 7 is yes, mention the topic in Mathematics Module where you were equipped with GeoGebra _____	
	Question 9 was changed: Can you use GeoGebra specific applications to construct Euclidean Geometry diagrams? Yes ☐ No ☐
	Question 10 was changed: Can you prove Euclidean Geometry theorems on a computer? Yes ☐ No ☐

Reasons for additions, changes and deletion of Question items

Question 1 had to be modified to reveal more insight on whether pre-service mathematics educators learn Euclidean geometry.

Question 5 has changed to a secondary school-based content knowledge of Euclidean geometry to reveal more information about pre-service educators' content knowledge of Euclidean geometry.

Question 6 was changed to include options on different mathematics dynamic software to gather insight on whether pre-service educators know GeoGebra software as a tool for effective teaching of Euclidean Geometry.

Question 7 was changed to check if pre-service mathematics educators have been equipped with GeoGebra in their Mathematics Education course.

Question 8 was changed to have information on pre-service educators' ability to use GeoGebra on a computer.

Question 9 was changed to obtain pre-service educators' perceptions about GeoGebra specific tools for constructing Euclidean Geometry figures.

Question 10 was changed to obtain pre-service educators' insight about knowledge of proving theorems with GeoGebra software?



ANALYSIS OF RESULTS FROM A GEOGEBRA COMPUTER-BASED TASK

Eight third-year pre-service educators from a four-year Bachelor of Education program participated in the pilot study. Enrolled in a Mathematics Education course covering various topics, including Euclidean geometry, they completed a GeoGebra-based task on Euclidean geometry using computers with GeoGebra software. The details of the questions and data analysis are provided below.

Questions 1, 2, and 3 probed the student's understanding of "the center angle is twice the circumference angle" with emphasis on visualization. Diagrams determined the calculation of the central angles to assess technological knowledge (TK) and knowledge of the subject (CK) of Euclidean geometry, with responses graded between 0 to 4. Scores are presented in the table below.

Table 2: Scoring of participants' responses within TK and C

Stude nt		Rubric score $\left(\frac{mark}{4}\right)$										$\left(\frac{mark}{3}\right)$	%
		QUESTIONS											
	Q 1.1			Q 2.1			Q 2.2						
	(a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)	(d)	(e)		
S1.	0	0	0	0	0	2	0	0	2	2	2	8	24
S2.	0	0	0	0	0	2	0	0	2	2	2	8	24
S3.	0	0	0	0	0	2	0	0	2	2	2	8	24
S4.	4	4	4	4	4	2	4	4	2	2	2	34	100
S5.	3	3	0	3	3	2	0	0	2	2	2	20	59
S6.	0	0	0	0	0	2	0	0	2	2	2	8	24
Mean (x)	0,29	0,29	0,16	0,29	0,29	0,5	0,16	0,16	0,5	0,5	0,5	2.5	0,39

A general observation in Table 1 across the question items shows that participants cannot use GeoGebra to measure AOB using GeoGebra. Some participants displayed incorrect answers to the GeoGebra-based questions and others did not attempt answering these question items. In Question 1.1 (a) to (c), pre-service educators were asked to measure angles at the centre using GeoGebra. However, they failed to measure these angles. When asked to make a conjecture about the relationship between angles ACB and AOB, they concluded that angle AOB is 25° because "angle at the centre is twice the angle at the circumference". The pilot study revealed that pre-service educators lacked technological knowledge (TK) in using GeoGebra to measure angles, despite having theoretical knowledge of Euclidean Geometry. Although they made correct conjectures about angle relationships, they struggled to apply this knowledge



practically with GeoGebra. The analysis of responses from the pilot study indicated that the question items were effective, with an adequate distribution of responses across the task. No changes were made to the task, and the results confirmed the feasibility of the main study, which is expected to meet its primary objectives.

CONCLUSION

This paper presents the preliminary results of the pilot study conducted to validate the research instruments. The analysis identified areas for improvement, with the pilot study assisting in refining data collection tools. Students' engagement with mathematical models was encouraging. Revisions to the questionnaire led to modifications that allowed the researcher to proceed with the main study. The GeoGebra task showed adequate response distribution, with no further changes needed for the main study.

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SOCIO-ECONOMIC STATUS PREDICTS GENDER DIFFERENCES IN MATHEMATICS ACHIEVEMENT AMONGST GRADE 9 LEARNERS IN OR TAMBO INLAND DISTRICT, SOUTH AFRICA

Ntokozo Kintosh Nogobo- and Jogymol Kalariparampil Alex

Walter Sisulu University

United Arab Emirates University

This study investigates the relationship between gender, socio-economic status (SES), and mathematics achievement among Grade 9 learners in the Eastern Cape, South Africa. Using a quantitative approach, the research aims to explore whether gender differences in mathematics performance vary across different SES levels. The study employed a regression research design, with 351 learners from rural and urban schools participating. The findings revealed no significant gender differences in mathematics achievement, as measured by a Mann-Whitney U test. Additionally, a moderated regression analysis showed that SES did not significantly moderate the relationship between gender and mathematics performance. The study highlights the need for further research into the local context of education and its impact on gender and socio-economic disparities in academic performance.

INTRODUCTION AND BACKGROUND

Mathematics achievement has been a central topic in educational research, particularly concerning gender disparities (Li et al., 2018). While global assessments such as the Trends in International Mathematics and Science Study (TIMSS) 2019 reveal that boys outperform girls in mathematics in most countries, South Africa is one of the few exceptions where girls have shown an advantage (Mullis et al., 2020). However, national assessments at the Grade 12 level contradict this trend, with boys outperforming girls in mathematics (Department of Basic Education, 2024). These inconsistencies raise important questions regarding the underlying factors influencing gender differences in mathematics achievement.

One critical yet underexplored factor is socio-economic status (SES). While numerous studies have examined gender disparities in mathematics, few have investigated SES as a potential mediator (Paterson et al., 2024; Cherian, 1993). SES significantly affects mathematics achievement (Wang et al., 2014) and may shape gender differences in the subject. Research by Paterson et al. (2024) suggests that boys from lower SES backgrounds face greater disadvantages in mathematics compared to their female counterparts, highlighting the potential role of SES in moderating gender disparities.

Despite the recognized impact of SES on academic performance, there is limited research examining SES as a mediator of gender differences in mathematics achievement. Students from higher SES backgrounds often have better access to educational resources, parental support, and enriched learning environments (Singh & Choudhary, 2015), while those from lower SES backgrounds face challenges such as inadequate school infrastructure and financial constraints. These disparities may differentially affect male and female students, leading to variations in gender differences across SES levels.



Given the lack of empirical evidence on the moderating role of SES in gender disparities in mathematics achievement, this study seeks to fill this gap by investigating whether gender differences in mathematics performance vary across different SES levels. Understanding this relationship may provide valuable insights for educators and policymakers to develop targeted interventions aimed at reducing achievement gaps and promoting equitable mathematics education. The following research questions were formulated to guide the study.

Research Questions

1. What are the gender differences in mathematics achievement amongst grade 9 learners in OR Tambo Inland District?
2. To what extent does socio-economic status moderate the relationship between gender and mathematics achievement in OR Tambo Inland District?

THEORETICAL FRAMEWORK: FAMILY INVESTMENT MODEL (FIM)

The Family Investment Model (FIM), first introduced by Mayer (1997), posits that families with greater economic resources are better positioned to invest in their children's development, whereas low-income families prioritize essential needs due to financial constraints. Mayer (1997) argued that children from impoverished families tend to struggle academically, perform poorly on standardized tests, experience behavioural problems, and face increased health risks compared to their affluent counterparts. Over time, researchers such as Conger and Donnellan (2007) extended FIM to include additional socio-economic factors like parental education and occupational status, which also play a crucial role in shaping a child's development. Within the context of this study, FIM provides a useful lens to examine whether socio-economic status (SES) moderates gender differences in mathematics achievement, as it highlights the unequal access to resources that influence academic performance.

The model emphasizes the role of parental investments, which include material, educational, and emotional resources provided to children. Conger and Donnellan (2007) identified key areas of parental investment, such as home learning resources, parental involvement in education, financial support for extracurricular activities, and overall family living conditions, all of which significantly impact a child's cognitive development. Empirical evidence supports this claim, showing that increased parental investment improves educational outcomes (Sohr-Preston et al., 2013). In the South African context, where socio-economic disparities are prevalent, Naite (2021) found that parental involvement in academic activities is often shaped by their socio-economic standing. This study seeks to explore whether similar trends exist in the Eastern Cape province, specifically analysing how SES influences gender disparities in mathematics achievement through variations in parental investment.

Moreover, research by Vasilyeva et al. (2018) and Simons et al. (2016) supports the positive correlation between high SES and strong academic performance, particularly in subjects like mathematics. Higher SES parents can provide children with private tutoring, technology for learning, enriched home environments, and exposure to problem-solving activities, all of which enhance mathematics skills. Conversely, children from low-SES families may lack access to



such resources, leading to lower mathematics achievement, reinforcing the need to analyse whether gender disparities in mathematics are uniform across different SES levels. The Family Investment Model is, therefore, critical to this study as it helps explain the potential mediating role of SES in shaping gender differences in mathematics performance.

Testing the applicability of FIM in the Eastern Cape is crucial, given the region's socio-economic challenges. If SES proves to be a significant mediator of gender disparities in mathematics, it will reinforce the need for targeted educational policies that address inequalities in parental investment. Additionally, while FIM provides a foundation for understanding SES-driven disparities, it is supplemented in this study by the Family Stress Model (FSM), which explains how economic hardship and psychological stress may also influence children's academic outcomes (Arif & Albulene, 2017; Masarik & Conger, 2017). Together, these models offer a comprehensive framework for investigating how socio-economic inequalities contribute to gender differences in mathematics achievement.

LITERATURE REVIEW

The persistent underrepresentation of women and girls in Science, Technology, Engineering, and Mathematics (STEM) fields has been widely documented, with research indicating that only about 30% of females participate in these disciplines (García-Holgado et al., 2019). This gender disparity highlights the need for further investigations into potential contributing factors, such as mathematics achievement, which may either reinforce or mitigate these imbalances (Girelli, 2023; Stoet & Geary, 2018). Given the significance of STEM for economic development and societal participation (Ong & Ling, 2020), understanding the variations in mathematics achievement between boys and girls is essential for shaping interventions aimed at fostering gender equality in education. The current study, therefore, focuses on examining gender differences in mathematics achievement, particularly in the South African context, where evidence suggests that girls perform comparably or even outperform boys in some cases (Mullis et al., 2020).

Gender disparities in mathematics achievement present inconsistent trends across different countries and over time, as reflected in the (TIMSS, 2019). Among 64 participating countries, boys outperformed girls in 31 (48.44%) countries, while no significant difference was observed in 29 (45.31%) countries. However, in only four countries, including South Africa, girls achieved higher scores than boys, though the differences were often not statistically significant (Mullis et al., 2020). The variation in these trends across regions and over time suggests that gender differences in mathematics achievement are complex and influenced by multiple social, economic, and cultural factors (Mullis et al., 2020). However, the precise reasons behind these variations remain uncertain, necessitating further investigation into the underlying factors contributing to these trends.

In South Africa, gender variations in mathematics achievement have shown fluctuating patterns over the past decade. For example, TIMSS 2011 reported that South African girls outperformed boys in Grade 9 Mathematics, scoring 354 and 350 points, respectively, but the difference was not statistically significant (Mullis et al., 2012). In contrast, when measured at age-appropriate



grade levels, boys significantly outperformed girls (Mullis et al., 2012). Similarly, TIMSS 2015 found no significant difference in Grade 9 Mathematics performance between boys and girls (Martin et al., 2015). However, in TIMSS 2019, girls once again achieved slightly higher scores than boys, yet the difference remained statistically insignificant (Mullis et al., 2020). Interestingly, national assessments such as the National Senior Certificate (NSC) School Subject Report (2024) indicate that boys have consistently outperformed girls in Grade 12 Mathematics over the past five years, raising concerns about the persistence of gender gaps at higher education levels.

Beyond gender differences, family socio-economic status (SES) has been identified as a crucial determinant of Mathematics achievement. Studies suggest that higher SES positively correlates with better Mathematics performance due to factors such as access to educational resources, quality schooling, and parental involvement (Ephraim, 2021). However, Naite (2021) argues that SES effects on mathematics achievement largely depend on the degree of parental engagement. For instance, parents with financial means may wish to enroll their children in well-resourced schools, but their income ultimately determines their choices. Likewise, parental education level influences the extent to which they can assist their children with Mathematics homework and learning tasks. Montes and Montes (2021) further emphasize that higher SES parents are often more financially stable and educationally equipped to support their children's academic development, enabling a more structured learning environment.

However, while high SES is generally associated with improved academic outcomes, its impact on mathematics achievement is not always straightforward. Some studies suggest that parents with demanding, high-income jobs may have limited time for direct educational involvement, which could negatively affect their children's performance (Grolnick, 2016). Additionally, Kusaeri et al. (2019) propose that excessive parental involvement may, in some cases, hinder Mathematics performance, as children may develop over-reliance or experience academic pressure. Murphy (2021) further argues that SES does not always predict better Mathematics outcomes, citing studies such as Hernandez (2014), which found no statistical relationship between SES and Mathematics scores. These mixed findings indicate that the relationship between SES and Mathematics achievement remains inconclusive, prompting the need for further exploration. Given that South Africa is among the few countries where girls outperform boys in mathematics achievement (Mullis et al., 2020), the present study aims to investigate the extent to which SES influences Mathematics achievement among Grade 9 learners in Eastern Cape.

RESEARCH METHODOLOGY

This study employed a quantitative research approach to investigate how SES predicts gender differences in mathematics achievement among Grade 9 learners in the Eastern Cape Province. The quantitative approach was deemed appropriate as it enables measurement of SES and Mathematics achievement, facilitating statistical regression analysis. This approach also ensures objectivity, allows for a large randomly selected sample, and utilizes closed-ended questions to minimize bias and enhance generalizability (Rana et al., 2023).



Given the study's aim of assessing the relationship between SES and gender differences in mathematics achievement, a regression research design was adopted. A regression design is a statistical approach used to examine the relationships between one or more independent variables and a dependent variable, typically to predict or explain the outcome of the dependent variable (Brook & Arnold, 2018).

Population and sampling

The study population comprised all Grade 9 learners in the OR Tambo Inland district, estimated at approximately 5 000 learners in the 2021 academic year. A stratified random sampling technique was employed to ensure fair representation across gender and school location. The final sample consisted of 351 learners (171 girls and 180 boys) drawn from six schools (three rural and three urban schools). The stratification process ensured that both gender and school location were proportionally represented in the sample, as recommended by Bhardwaj (2019).

Data collection instruments

Two primary instruments were used for data collection:

- Questionnaire: Used to assess the socio-economic status.
- Standardized Mathematics Test: Used to measure Mathematics achievement.

The validity of these instruments was tested using Pearson correlation analysis, confirming that all items met the critical value for statistical significance. Reliability was tested using Cronbach's Alpha, yielding a value of 0.933, which is well above the acceptable threshold of 0.70 (Singh, 2017). Additionally, the mathematics test was moderated by two Mathematics experts to ensure content validity, reliability, and alignment with the Grade 9 curriculum.

Ethical considerations

Permission to conduct the study was obtained from the Eastern Cape Department of Education and the principals of the selected schools. Informed consent was obtained from all participants, who were made aware that their participation was voluntary and that they could withdraw at any stage. Confidentiality was maintained by using pseudonyms instead of real names. Data security measures, such as password-protected files, were implemented to safeguard participant information. No identifying information was included in the study to ensure anonymity.

Data analysis

The collected data were analysed using IBM SPSS Statistics version 28.0.0.0. Two statistical techniques were applied:

- Regression Analysis: Applied to examine the extent to which SES predicts Mathematics achievement and how this prediction varies by gender (Chatterjee & Hadi, 2015).



FINDINGS

The findings of the study were mainly focusing on answering the two research objectives of the stud:

Objective 1: To examine the gender differences in mathematics achievement amongst Grade 9 learners in the Eastern Cape.

The normality test was conducted to check whether the data is normally distributed or not in order the use the more appropriate statistical tests. The results from normality test are presented as follows:

TABLE 1: Kolmogorov – Smirnov normality test

Tests of Normality							
	Gender	Kolmogorov-Smirnov ^a			Shapiro-Wilk		
		Statistic	df	Sig.	Statistic	df	Sig.
Math_Ach	Female	.108	171	<.001	.969	171	<.001
	Male	.092	180	<.001	.968	180	<.001

a. Lilliefors Significance Correction

The Kolmogorov-Smirnov test was used to assess the normality of the mathematics achievement scores. The results indicated a significant deviation from normality for both female (K-S = .108, $p < .001$) and male learners (K-S = .092, $p < .001$), as the p-values were less than the 0.05 threshold. This suggests that the distribution of scores significantly deviated from a normal distribution. As a result, non-parametric tests such as the Mann-Whitney U test was used to compare the mathematics achievement scores between male and female learners, as these tests do not assume normal distribution.

TABLE 2: Mann – Whitney U test for gender differences in mathematics achievement

Test Statistics ^a	
	Math_Ach
Mann-Whitney U	14339.000
Wilcoxon W	30629.000
Z	-1.106
Asymp. Sig. (2-tailed)	.269

a. Grouping Variable: Gender

A Mann-Whitney U test was conducted to compare mathematics achievement between male and female learners. The results indicated a nonsignificant difference between the groups ($U = 14339$, $Z = -1.106$, $p = .269$), suggesting that gender does not influence mathematics achievement.



Objective 2: To assess the extent to which SES moderates the gender differences in mathematics achievement amongst Grade 9 learners in the Eastern Cape. A moderated regression analysis was conducted to assess the moderating effect of SES on the relationship between gender and mathematics achievement.

TABLE 3: Moderated regression model

Coefficients ^a						
		Unstandardized Coefficients		Standardized Coefficients		
Model		B	Std. Error	Beta	t	Sig.
1	(Constant)	33.246	7.373		4.509	<.001
	Gender_SES	.552	2.376	.047	.232	.817
	SES Level	7.051	3.775	.300	1.868	.063
	Gender	-3.261	4.653	-.093	-.701	.484

a. Dependent Variable: Math_Ach

The results revealed that the interaction term between gender and SES was not statistically significant ($\beta = 0.552$, $p = 0.817$). This suggests that SES does not significantly moderate the relationship between gender and mathematics achievement.

DISCUSSION OF FINDINGS

The findings of this study contribute to the ongoing discourse on gender differences in mathematics achievement and the role of SES as a potential moderator of these disparities. While numerous studies have highlighted gender gaps in academic performance, particularly in STEM fields, the findings from this research provide unique insights into the relationship between gender, SES, and mathematics achievement among Grade 9 learners in the OR Tambo Inland District. Despite global evidence that boys often outperform girls in mathematics, this study reveals a different trend in the local context, where gender did not significantly influence mathematics achievement. Furthermore, SES was not found to significantly moderate the relationship between gender and mathematics performance, challenging the assumption that socio-economic factors might account for gender disparities in this subject area.

Gender differences in mathematics achievement

The first objective of the study sought to examine gender differences in mathematics achievement among Grade 9 learners in the OR Tambo Inland District. The results from the Mann-Whitney U test indicated no significant difference in mathematics achievement between male and female learners. This finding is consistent with certain international studies, such as the TIMSS, where gender disparities in mathematics performance often show no substantial statistical significance. Interestingly, this result contrasts with previous South African studies, where girls were observed to outperform boys in some instances, particularly in TIMSS mathematics achievement (Mullis et al., 2020). In this study, the absence of significant gender



differences suggests that other factors such as individual learner characteristics, classroom dynamics, or school-related influences might have a more prominent effect on mathematics achievement than gender alone.

The role of socio-economic status

The second objective of the study focused on assessing whether SES predicts the relationship between gender and mathematics achievement. A moderated regression analysis revealed that the interaction between gender and SES was not statistically significant, suggesting that SES did not have a substantial moderating effect on the mathematics achievement of male and female learners. This finding contrasts with studies that suggest that higher SES is associated with better academic outcomes, especially in subjects like mathematics, due to the availability of resources such as tutoring, educational technology, and enriched home environments (Vasilyeva et al., 2018; Simons et al., 2016). However, the findings in this study imply that SES may not operate in the same way within the South African context or specifically in the Eastern Cape. It is possible that other contextual factors, such as the quality of local schools or regional socio-economic challenges, may influence students' access to educational resources more significantly than family wealth alone.

Implications for educational policy and practice

The lack of significant findings in terms of both gender and SES highlights important considerations for educational policy and practice. The absence of gender differences in mathematics achievement suggests that interventions aimed at closing gender gaps in mathematics may not be necessary for Grade 9 learners in the Eastern Cape. However, this does not imply that gender equality in STEM should not remain a priority at the national level, particularly at Grade 12 level where gender disparities have been observed (National Senior Certificate Subject Report, 2024). Moreover, the finding that SES does not significantly moderate the relationship between gender and mathematics achievement calls for a deeper exploration of other potential mediators, such as teacher quality, peer support, or curriculum alignment with learners' needs. These findings underscore the importance of understanding the local educational context and the complex interactions between socio-economic factors, gender, and academic performance.

In conclusion, while the study did not find significant gender differences in mathematics achievement nor a moderating effect of SES, the findings serve as a valuable contribution to understanding the nuances of gender and socio-economic factors in educational outcomes. Further research, perhaps longitudinal in nature, is needed to examine the long-term impact of SES and gender on mathematics achievement and to explore additional factors that may influence these relationships in diverse South African contexts.

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USING SONGS, RHYMES AND MOVEMENT STRATEGIES TO ENHANCE LEARNERS' UNDERSTANDING OF TRIGONOMETRY

Selowa Reinhard and Kakoma Luneta

North West University and University of Johannesburg

Abstract

Trigonometry is often viewed as an abstract topic by most learners as they grapple to comprehend its concepts and apply them when given problems. A proper application of instructional practices that relate to learners' social lives appears to be a promising strategy that can enhance learners' understanding of trigonometric concepts. The study intended to use songs, rhymes and movement to enhance learners' understanding of trigonometry. The Embodied Cognition Theory was adopted as it directly supports the idea of using songs, rhymes, as well as physical interactions to construct meaning in learning. However, only five components, namely brain, body, environment, perceptions and actions, were considered to drive the study. A Participatory Action Research design was used to enhance 15 grade 11 and 16 grade 12 learners' understanding of trigonometry. In each class, data were collected through documents and video recordings to check if the five components of Embodied Cognition Theory were considered or not. Essentially, content analysis was used to analyse the two sets of data. Findings revealed that most learners found songs, rhymes and movement interesting as they were given an opportunity to compose trigonometric songs, which later led to movement. As a result, it became easier for them to solve the given trigonometric problems. Despite that, some learners encountered challenges attempting some trigonometric problems fully due to the inability to apply the knowledge of grouping trigonometric-like terms, especially when required to prove. On the other hand, some learners struggled to apply relevant procedures as they were not attending the sessions consistently. Hence, instructional practices that are effective in early childhood also appear to be relevant for teaching mathematics in the FET phase as they relate to their social lives.

Keywords: trigonometry; Songs; rhymes; and movements

INTRODUCTION

Trigonometry is a crucial and practical mathematics topic that enhances learners' problem-solving skills. However, it remains challenging for most learners to recall and apply relevant concepts when given problems. It requires the skill to recall appropriate concepts such as angles, sides (Neel Báez et al., 2023) and equations to solve a given problem. In essence, it prepares learners for the future, especially in the fields of architecture, astronomy, survey, marine biology, and engineering (Dhungana et al., 2023). The Department of Basic Education [DBE] (2015) supports that trigonometry was established in ancient civilisations, such that individuals can apply the knowledge in solving practical or real-life problems. This shows that learners are expected to comprehend various concepts, their importance and how to apply them when given problems. As a result, the implementation of songs, rhymes and movement to



assist learners in recalling the essential trigonometric concepts is mandatory. These include properties of polygons (Herman et al., 2022) applied in different contexts of a problem. Songs and rhymes refer to a method of enhancing learning and assisting learners to remember mathematical concepts (Otchere-Larbi & Amoah, 2020). Subsequently, well-composed songs lead to a movement, which is a positive attitude demonstrating excitement during the process of learning. Hence, it is vital to apply these strategies accurately to maintain a positive attitude among learners.

It is well known that trigonometry involves various forms of representations, such as words, diagrams, and symbols. Büttner and Erath (2023) assert that these representations enable learners to develop a deeper understanding of the topic. The representation of the problems can be provided either as words and diagrams or words only, whereby learners are expected to draw a diagram before solving the problem. Most importantly, both words and diagrams include symbols and variables representing sides as well as angles (Chalmeta, 2021). Hence, the use of songs and rhymes enhances learners' reading skills to comprehend problems and choose the appropriate strategy to solve them. This is ample evidence illustrating that this topic is essential, and it requires skills in accessing relevant concepts to provide correct answers.

Despite that the topic is essential, most learners find it difficult to solve trigonometric problems. They lack skills in reading the given questions with understanding (Liswaniso, 2023; Nurmeidina & Rafidiyah, 2019). During the process of teaching and learning, learners are provided with formula sheets, however, it becomes difficult for them to critically analyse given problems and choose appropriate strategies to solve the given problems (Nanmumpuni & Retnawati, 2021). Additionally, Nurmeidina and Rafidiyah (2019) clearly outlined other difficulties encountered by learners as they are learning trigonometry. Firstly, learners struggle to extract key constructs from the question. Secondly, learners guess answers without following the necessary procedures. Thirdly, they do not like reading long questions. Lastly, learners lack sufficient knowledge and skills in approaching trigonometric questions. Dube and Ngcobo (2024) also discovered that learners tend to provide responses based on intuition rather than applying trigonometric principles and procedures. Moreover, the two authors add on to say, sometimes, lengthy problems tend to discourage learners from fully engaging; this leads to misunderstanding and failure to provide correct responses. This situation highlights a significant concern for the future of learners. Based on the difficulties highlighted above, it can be concluded that appropriate practices to enhance learners' understanding of trigonometry must be implemented. In this study, songs, rhymes and movement methods are viewed as important components of enhancing learners' understanding. Hence, the study, intended to enhance learners' understanding of trigonometry using songs, rhymes and movement strategies. The study asked one research question: How are songs, rhymes and movement used to enhance learners' understanding of trigonometry?

THEORETICAL FRAMEWORK

The study is underpinned by Embodied Cognition Theory explicated by Shapiro (2011). The theory asserts that learning, as well as thoughts, are extremely intertwined to the physique's



interactions with the physical world. Fincher-Kiefer (2019) maintains that the body serves the mind, and the two components guide the mental representations. In this instance, mental representation is the ability to make internal models to comprehend trigonometric concepts. According to Wilson and Foglia (2017), without involving body acting and sensing, thoughts would be empty. Based on these ideas, the recent study contends that learners learn trigonometric concepts appropriately when the body and the physical world are involved. Hence, meaning cannot be distilled away from the body's experiences. There are various components of Embodied Cognition theory; however, the recent study sampled five components, namely: 1) *brain*, processing trigonometric concepts and generating the information while solving problems; 2) *body*, using gestures to recall trigonometric concepts; 3) *environment*, how learners interact with their surroundings to perceive trigonometric concepts; 4) *perceptions*, organising and interpreting information to understand it better; and 5) *actions*, using different materials to explore, reinforce knowledge to solve problems. All these components are embedded within the three themes established by Shapiro, namely conceptualisation, replacement, and constitution (Shapiro, 2011). The theory has been used in several studies (Castro-Alonso et al., 2024; Bhutoria & Hebbani, 2019; Roberts et al., 2024) for unique goals. Castro-Alonso et al. (2024) applied it to demonstrate that learning can be enhanced when body gestures influence each other. Also, Bhutoria and Hebbani (2019) applied the theory to emphasise that intellectual abilities, including gaining knowledge, understanding mathematical concepts, judging, and recalling and solving problems, are not confined to the brain only. On the other hand, Roberts et al. (2024) applied it in assisting instructors to spark learners' interest, create an effective learning environment and foster long-term retention. However, the recent study applied it to enhance learners' understanding of trigonometry through allowing learners to compose trigonometric songs, which later led to movement, an indication of enjoying the process of learning. This shows that it is impossible to allow learners to comprehend various trigonometric concepts without engaging their minds and bodies.

LITERATURE REVIEW

In this section, the reviewed literature is presented to illustrate essential concepts in trigonometry that assist learners in solving problems fluently. Despite that essential concepts are well explicated, previous studies highlighted several difficulties encountered by learners when solving trigonometric problems. However, positive essential skills and knowledge on how to use songs, rhyme and movement methods to enhance learners' understanding of mathematics are clearly explained.

Understanding trigonometric concepts

The Mathematics Curriculum Assessment Policy document [CAPS] (2015) illustrates various sections in trigonometry, namely trigonometric ratios, functions, identities, proofs, and equations. These sections are sequential, meaning how they are taught is important because the knowledge of one section is required for the next section. This is consistent with the finding by Neel Báez et al. (2023), emphasising that the teaching of trigonometry should start with ratios such that it is easier for learners to access the knowledge in the next section. Moreover, the



development of the ratio assists learners in comprehending different trigonometric functions. To ensure that learners comprehend different types of functions, it is important to first check if learners can discover the signs of functions in different quadrants using the trigonometric circle or not (DBE, 2019). Therefore, proper baseline activities for each lesson must be well planned to ensure that prerequisite concepts are processed and can be recalled when generating responses. This is referred to as brain cognition.

Despite the different guidance provided on how to unfold the trigonometric lessons, Orhani (2024) pointed out that learners find it difficult to comprehend the graphical representation as well as the application of trigonometric real-life conditions. This is due to the inability to make perceptions between ratios and angles. As a result, the study suggests the implementation of didactic methods to ensure an in-depth comprehension of trigonometric functions. In contrast, Kim and Lee (2021) argued that a deeper understanding of trigonometric functions can be enhanced through applications of mathematical software such as GeoGebra or dynamic graphics to demonstrate the concepts interactively. This is one of the ideas that is in line with the requirements of the 4IR, which emphasises that it is vital to integrate the teaching of mathematics with technology (Luneta, 2022). However, some teachers find it difficult to consider such innovative instructional practices for various reasons. These include a lack of sufficient equipment and access to the internet [issue of load reductions in rural areas] (Mustafa et al., 2024). Hence, it is the responsibility of educators to make drastic changes in their profession by using techniques that assist learners in comprehending trigonometric concepts rather than complaining about difficulties that cannot be solved in a short period of time.

Carter and Zhang (2022) contend that learners with strong foundational concepts of geometry as well as algebra find it easier to comprehend trigonometry. In contrast, learners with a weak foundational knowledge of the topics mentioned above frequently struggle to solve trigonometric problems. This is a result of the application of traditional teaching methods during the process of teaching and learning (Ferreder et al., 2024). Arguably, some studies postulate that traditional methods enhance learners' understanding, especially when combined with interactive methods (Orhani, 2024). The idea is inconsistent with the recent mathematical teaching framework, which emphasises learning techniques activating learners' attitudes towards mathematics, such as collaborative learning techniques and problem-solving. It is believed that the best strategy to solve this problem is to allow teachers across grades to assist each other in supporting learners to require basic concepts before they proceed to the next grade (Hargreaves, 2019). Similarly, Tachie (2022) revealed that peer collaboration seems to be an effective technique for addressing educators' difficulties and learn from each other. Hence, this enhances their knowledge and skills in teaching mathematics effectively.

Moreover, Ferder et al. (2024) argue that there are numerous documents on trigonometry. However, effective educators' training for guiding the use of available resources is compulsory. This leads to an appropriate implementation of the effective strategies that provide learners an opportunity to retain trigonometric concepts and understand the reasons behind learning these concepts (Amedume et al., 2022). Durmus and Kaeakirik (2006) suggested the use of multiple representations, namely tables, graphs, and algebraic form, to make connections between



trigonometric concepts. However, a good mastery of syntactic and substantive structure to use these representations effectively while teaching is a limit (Tsang & Rowland, 2005). Hence, it is necessary to explain why it is worth knowing mathematical principles and for learners to notice that mathematics is important.

Difficulties in solving trigonometric problems

Trigonometric problems necessitate comprehension of the given figure and the ability to choose the relevant formula to solve the problem. However, learners tend to apply trigonometric equations or identities during the process of computations without understanding how the formula was constructed and struggle to find angles in quadrants (Maknun et al., 2022). For instance, when required to solve general solutions, they become confused and struggle to choose the appropriate identities. This is a result of the failure to interpret the given problem and outline the necessary concepts which assist in choosing relevant constructs. On the other hand, some learners manage to choose correct equations or manage to simplify the first step when required to solve proofs, but find it challenging to complete the whole process. The idea is consistent with that of Dube and Ngcobo (2025), showing that learners often find it easier to apply the initial steps but struggle to subsequently complete the whole process of solving the problem. Hence, learners must be guided on how to approach trigonometric problems to avoid mathematics anxiety. This can be done by allowing learners to acknowledge how the physical and social environment influences their thinking, which is a way of operating within environmental cognition.

Songs, rhymes and movements in trigonometry

How the brain functions when songs are used in learning trigonometry

Taylor (2010) posits that the brain is categorised based on its functions. The author emphasises that there are four components to be considered when using songs or music during the process of learning. First, the *angular gyrus* is responsible for comprehending word meanings as well as combining sensory information. In this case, the angular gyrus is regarded as an ability to perceive trigonometric concepts through organising information received through hearing, seeing and modelling. Second, the *anterior cingulate cortex* regulates empathy as well as emotions. For instance, when trigonometric concepts are well understood, learners display a good mood, showing excitement. Third, the *inferior frontal gyrus* is based on the guidance of movement and assessment of intentions. In this instance, the component relates to dancing as an expression of enjoyment. Four, the *Insula*, is responsible for processing pain and disgust responses. This is observed when learners struggle to solve given trigonometric problems. Hence, songs, rhymes and movements must be implemented in learning mathematics as they activate various areas of the brain.

Empirical studies show that the use of songs and rhymes to teach mathematics is an effective way of engaging learners (Taylor, 2010; Lovemore et al., 2021). Songs in mathematics refer to music composed with mathematical concepts to enhance learning. Meanwhile, Rhyme refers to the acoustic agreement of sounds or words when reading texts (Grofčíková & Máčajová,



2021). The essence of rhymes lies in ensuring that learners hear sounds with words, which allows the process of perception to recognise that words are categorised based on their similarities and differences. As a result, it becomes easier for learners to recognise symbols and choose relevant concepts to solve problems. Goswami (2015) argues that there is a great link between rhyming and reading. Basically, using rhyming as learners are learning trigonometry enables them to have the skill to read questions with understanding. Hence, it can be concluded that it is impossible to discrete reading and rhyming.

Del Barrio and Arús (2024) argued that involving rhymes, songs in mathematics improves learners' attention and makes the learning of mathematics interesting. Taylor & Francis Group (2023) maintains that integrating songs into mathematics makes mathematics enjoyable and reduces mathematics anxiety. However, the attitude of the educator plays an important role in ensuring that mathematics is enjoyable. Hendriks and Cruywagen (2024) contribute that songs foster deeper learning and create an exploratory and engaging learning environment. These studies underscore the importance of songs, rhymes and movement in mathematics. Moreover, the studies emphasise the component of the body, dancing while singing trigonometric concepts and actions, and solving trigonometric problems through recalling how the concepts were perceived during learning. Hence, the methods must be applied appropriately to avoid negative outcomes.

The related literature indicates that rhymes and songs are commonly used in elementary schools for learners to recall math concepts (Walsh & Coleman, 2023). The findings are consistent with those of Moluayonge (2022), arguing that there is a great relationship between the acquisition of knowledge and music in early childhood. These strategies are effective for addressing several educational purposes that align with learners' preferences. Despite that, the strategies are mostly being used in the foundational phase; it is possible to apply its aspects in the FET phase as it assists learners in comprehending trigonometric concepts. For instance, the recent study used rhymes, songs and movements to enhance learners' understanding of trigonometric concepts, as it consists of several formulas and necessitates a skill to solve application questions.

METHOD AND RESEARCH DESIGN

The qualitative study adopted the Participatory Action Research (PAR) design (Kemmis & McTaggart, 2000), to systematically implement songs, rhymes and movement in trigonometric lessons, reflecting on their effectiveness and making improvements based on observations. The study followed the four phases of PAR to implement songs, rhymes and movement techniques for enhancing learners' understanding of trigonometric concepts. These include planning, action, observation and reflecting. Firstly, the problem was identified, which is an inability to choose the correct concepts, such as equations or identities, to solve trigonometric problems. As part of planning, activities were designed which were later given to learners to work in groups, generate songs and present their work. On the implementation step, learners were allowed to sing trigonometric songs, and a reflection was made after the lesson. Data were collected using documents and video recordings to capture learners' emotions during



presentations of the content. Afterwards, learners were given activities to write individually. This was a strategy for checking whether objectives were achieved or not.

Participants and sampling

The study used purposive sampling to sample participants. Purposive sampling refers to a deliberate process of choosing participants with relevant potential to respond to the inquiry about the interest of the study (Creswell & Poth, 2017). The current study purposively sampled 15 grade 11 and 16 grade 12 learners. The learners are expected to comprehend trigonometry; however, they were struggling to make the right choices in terms of concepts and equations when given problems. As a result, songs, rhymes and movement strategies were used to enhance their understanding of trigonometric concepts.

Data collection procedures

Two techniques, documents and video recordings, were used to collect data for approximately three weeks. During the first week (first episode), learners were given learning activities based on the specific sections for each grade. For instance, activities required the application of trigonometric identities and reduction formulae to simplify and prove expressions, interpret trigonometric functions, and solve general solutions. To initiate the process, learners were allocated an hour to independently discover trigonometric knowledge and solve the given problems. In the second and third episodes, each group was given an opportunity to present their responses, providing details on how their responses were obtained. The presentations were used as a tool for checking if all concepts in each section were understood or not.

In the fourth and fifth episodes, learners were given feedback, clarified misconceptions, and highlighted essential ways of tackling questions. For learners to comprehend ratios and various rules, a geometry pad was used to emphasise the relationship between sides and angles. During the second week, learners were allowed to compose songs and rhymes. They were given two days to prepare. Later on, each group was given an opportunity to present their composed songs. After all the presentations in each class, learners were given an opportunity to choose the best songs and rhymes, which would be used every time when revising trigonometric concepts. In the third week, learners were allowed to sing their favourite songs for three days. Thereafter, learners were given an activity to write individually as a way of checking whether the strategies worked or not.

Data analysis

The two sets of data were analysed using content analysis explicated by Erlingsson and Brysiewicz (2017). Firstly, data from documents explored and video recordings were transcribed and shortened but still preserved the core meaning. Secondly, axial coding was used to label data of the same and different characteristics. Thirdly, the codes were categorised based on their similarities and differences. The codes and the meaning of the underlying data are shown below:



Table 1: Showing codes from the two sets of data

Codes	Meaning
Trig competence	Ability to comprehend and apply trigonometric concepts.
Song affinity	Enjoying rhymes and songs leads to movements.
Conceptual gap	Sings well but struggles to apply the knowledge.
Inconsistent attendance	Attending sessions inconsistently.

Four (4) themes emerged from the codes above: 1) recalling trigonometric concepts due to rhymes, songs and movements; 2) finding songs, rhymes and movements interesting and reducing mathematics anxiety; 3) recalling trigonometric concepts but struggling to apply; 4) effects of absenteeism during the implementation process. The next section has more details on each theme.

RESULTS AND DISCUSSION

The purpose of the study was to enhance grade 11 and 12 learners' understanding of trigonometry using rhymes, songs and movement strategies. Related literature and embodied cognition theory components were used to emphasise how rhymes, songs and movements enhanced understanding of trigonometry. Overall, results indicate that the majority of learners enjoy singing and dancing rather than learning mathematical concepts without composing songs; some of the lyrics are shown below. Despite that, some learners enjoyed singing and dancing, but still wrote incomplete responses. More details are explicated below:

Lyrics of some songs

The tables below illustrate some of the lyrics presented by learners during the process of teaching and learning. Hence, their roles are explicated in each theme.

Table 2: Ratios and Pythagorean identities lyrics

Trigonometric ratios	Pythagorean identities
<i>Let's climb the stairs, think of a right-angled triangle, Check reference angle: Sine is SOH Cosine is CAH Tan is TOA Use your angle, then choose a ratio</i>	<i>Sine and cosine are friends: Sine squared, cosine squared make 1, (think about it when required to prove) Let's play.... with each term: You can make sine squared subject of the formula, You can make cosine squared subject of the formula, Check the given problem thoroughly... When you see sine squared, sub 1 minus cosine squared</i>



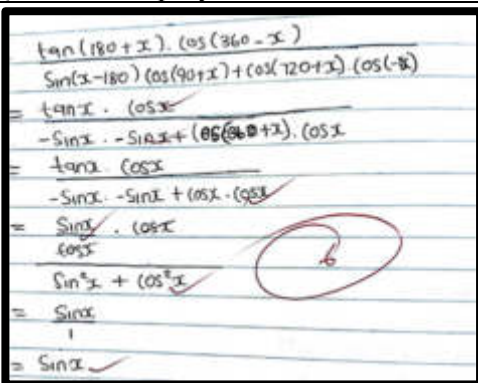
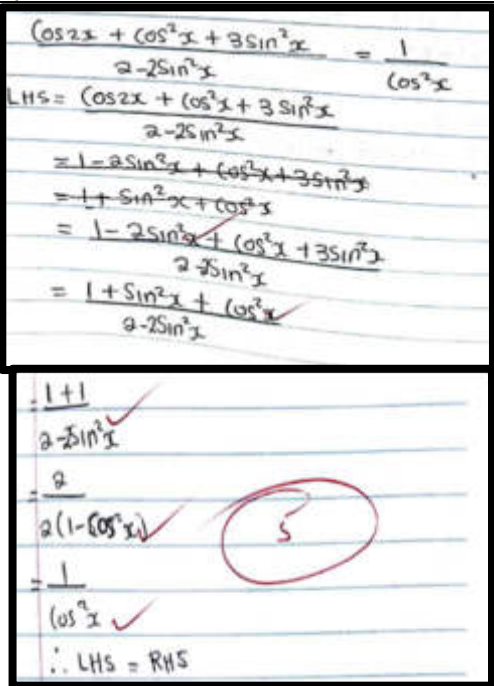
	<i>When you see cosine squared, sub 1 minus sine squared</i> <i>secant squared is, 1 plus tangent squared....</i> $1 + \tan^2\theta = \sec^2\theta$ <i>cosecant squared is 1 plus cot squared....</i> $1 + \cot^2\theta = \csc^2\theta$ <i>You can play with each term..</i> <i>When can I use them? (Check the given problem first)?</i> <i>-when required to simplify,</i> <i>When required to prove</i> <i>-When required to solve trig equations</i> <i>Algebraic knowledge is required.....</i>

Table 3: Double angle identities and Quadrant and graphing keys lyrics

<i>Double angle identities</i>	<i>Quadrants (reduction formulae) and graphing song</i>
$\sin(2\theta) = 2\sin\theta\cos\theta$ simple one ... Check the given problem (which one will be appropriate to use): <i>-when required to simplify,</i> <i>When required to prove,</i> <i>-When required to solve trig equations</i> <i>Algebraic knowledge is required.....</i> $\cos(2\theta) = \cos^2\theta - \sin^2\theta$ or $2\cos^2\theta - 1$ or $1 - 2\sin^2\theta$ <i>You can make..... any term the subject of the formula</i> <i>You can derive them.....</i>	Quadrants: Let's check, All (Sine, Tangent & cosine, Students (Sine) Take (Tangent) Coffee (Cosine) In quadrant I, All of them survive In quadrant II, only Sine survives In quadrant III, only Tangent survives In quadrant IV, only Cosine survives Graphing <i>A few keys to consider:</i> <i>-Amplitude, up, down</i> <i>-period for sine and cosine: p is equal to 360 degrees over b, wave's dance</i> <i>-period for tangent, p is equal to 180 degrees over b, wave's dance</i> <i>-When b goes up, wave gets tight,</i> <i>-When b goes down, wave is slow</i> Very big positive <i>Right is minus -To get an angle less than 360°</i> negative angles <i>Left is plus -To get an angle greater than 0°</i> <i>-To get an angle less than 360°</i>



Recalling trigonometric concepts due to rhymes and songs

Vignette 1.1: Learner A (Grade 12): Question: Simplify	Vignette 1.2: Learner B (Grade 12) Question: Prove
	

Vignette 1: Learner A and B's written responses on the last activity

Carter and Zhang (2022) argue that learners with strong foundational concepts of geometry as well as algebra find it easier to comprehend trigonometry. Similarly, Vignette 1.1 illustrates that learner A demonstrated an ability to simplify the given problem using reduction formulae keys, using a Cartesian plane to interpret. To be precise, the learner managed to recall that: 1) $\tan \tan (180^\circ + x)$ is found in 3rd quadrant; 2) $\cos \cos (360 - x)$ is positive in the 4th quadrant; 3) $\sin \sin (x - 180^\circ)$ is $-\sin x$; has an opposite sign; 4) $\cos \cos (90 + x) = -\sin \sin x$ which can be obtained through deriving or interpreting co-functions; and 5) $\cos \cos (720 + x) = \cos \cos (360 + x)$, which is an application of special angles and understanding how reduction formulae work. All these trigonometric concepts, coded *trig competence*, show that the learner can interpret components of the given expression within the four quadrants and is aware that big positive angles can be reduced through subtracting 360 degrees. This forms part of the brain and perception cognition, and Taylor (2010) classified this as the angular gyrus. DBE (2019) emphasised that it is important for learners to perceive different types of functions in different quadrants and use reduction formulae to simplify. Hence, based on the representation of the responses, it can be concluded that all concepts related to reduction formulae were clearly understood. This is a result of using songs and rhymes, body cognition, to foster deeper learning and exploration (Hendriks & Cruywagen, 2024), which enabled learners to recall relevant trigonometric concepts.

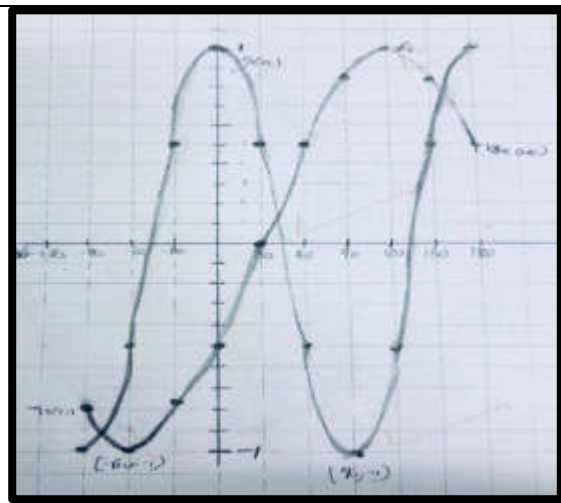
The ability to perceive trigonometric concepts, specifically identities, was illustrated by learner B in vignette 1.2. This was observed as learner B on vignette 1.2 noticed that changing $\cos \cos 2x$ to $1 - 2\sin^2 x$ leads to the addition of like terms, which makes it easier to simplify. Based on the vignette above, one may infer that composed songs also regarded as action,



enabled learners to perceive how and when to apply trigonometric identities when given problems. Moreover, the vignette shows that the topic was well introduced, starting with ratios (Neel Báez et al., 2023), which advanced learners' skills of proving. Durmus and Kaeakirik (2006) also argued that the application of multiple representations, namely tables, graphs, and algebraic form, assists learners in making connections between trigonometric concepts. Similarly, the learner managed to navigate within symbolic representations.

Vignette 2: Learner C: Grade 11

Question: sketching



Vignette 2: Learner C's written response on the last activity

The vignette above illustrates that the learner managed to sketch two trigonometric functions, namely $g(x)$ and $f(x)$, this is coded *trigonometric competence*. Initially, the learner managed to find the concept of finding the correct 2 coordinates for both graphs through organising the information, brain cognition. During the process of learning, learners were advised to read questions with understanding before responding to avoid overlooking essential concepts, which can disadvantage them. As learners were acting, singing and rhyming, their reading skill was enhanced (Grofčíková and Máčajová, 2021). As a result, it was easier for the learner to sketch graphs and include all necessary information, such as intercepts and turning points.

Finding songs and rhymes interesting and reducing mathematics anxiety

Transcripts from video recordings showed that learners find songs and rhymes interesting when learning trigonometry, coded *song affinity*, which is part of the anterior cingulate cortex. During the process of learning, learners were given an opportunity to compose songs using trigonometric concepts (Table 2 and 3). Subsequently, granting them an opportunity allowed them to enjoy the process of learning. This is in line with findings from Del Barrio and Arús (2024) showing that involving rhymes, songs in mathematics improves learners' attention and makes the learning of mathematics interesting. Moreover, Luneta (2022) highlighted the importance of integrating technology in mathematics. This is one of the requirements of the fourth industrial revolution goals, indicating that we are in the 21st century, and available digital tools must be used in ensuring that learners understand the necessary concepts. In this study, a geometry pad software was used to enhance learners' understanding of basic trigonometric



concepts. This forms part of environmental cognition. It is known that learners are exposed to various social platforms illustrating different dancing challenges. During the process of teaching and learning, some learners were singing while imitating recent dance challenges learnt from social media, illustrating some trigonometric concepts using their gestures. Therefore, it is crucial for educators to find ways of ensuring that learners can comprehend trigonometric concepts from different resources.

There are many Trigonometric documents made available for learners to use during the process of learning trigonometry (Ferreder et al., 2024). In this study, learners were given an opportunity to choose guides. However, guidance on how to choose and use them was provided. Learners were guided to consider study guides, considering all CAPS criteria, written in a simplified language, and with engaging problems. This is one of the prerequisites of the mathematics framework that accentuates the issue of engaging learners throughout the lesson (TMU, 2019). Allowing learners to use various materials assists them to learn trigonometric concepts freely, and this reduces mathematics anxiety since learners are not instructed but guided. Essentially, the approach ensures that all components of cognition are considered. Additionally, an appropriate application of rhymes and songs created an exciting learning environment, seen through movement or dancing. Hence, it can be interpreted as a positive attitude towards learning (Roberts et al., 2024), referred to as body cognition.

Recalling trigonometric concepts but struggling to apply

Vignette 3: Learner D: Grade 12

Question: Prove

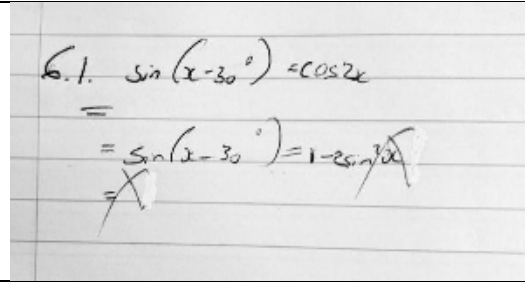
$$\begin{aligned} \cos^2 x + \sin^2 x &= 1 \\ 2 - 2\sin^2 x &= \cos^2 x \\ &= 2\cos^2 x - 1 + \cos^2 x \sin^2 x - 1 + 3\sin^2 x \\ 2 - 2 &= \cos^2 x - 1 \\ &= 2(\cos^2 x - 1) + \sin^2 x - 1 + 3\sin^2 x \\ &= \cos^2 x - 1 \\ &= \cos^2 x - 1 + \sin^2 x + 3\sin^2 x \\ &= \cos^2 x - 1 \\ &= \cos^2 x + \sin^2 x + 3\sin^2 x \\ &= \cos^2 x + 4\sin^2 x \\ &= 1 \end{aligned}$$

Vignette 3: Learner D's written response on the last activity

Dube and Ngcobo (2025) postulated that learners often find it easier to apply the initial steps but struggle to simplify. Similarly, the vignette above shows that learner D managed to apply the knowledge of an identity, however, it became difficult to simplify, coded *conceptual gap*. The learner struggled to perceive the knowledge of grouping like terms from algebra. Such performance indicates limited foundational knowledge of algebra. Hence, it is crucial for learners to revise the basics of algebra before tackling trigonometric problems.



Effects of absenteeism during the implementation process

Vignette 4: Learner E: Grade 11 Question: General solution


Vignette 4: Learner E's written response on the last activity

The vignette above shows that the learner could not determine the general solution; coded as *inconsistent attendance*. The written response shows that it became difficult for the learner to critically analyse the given general solution and choose appropriate strategies to solve it (Nanmumpuni & Retnawati, 2021). The transcripts from the videos show that the learner was quiet when granted an opportunity to work in groups, and this is referred to as the Insula. This is a result of not attending the sessions consistently. Hence, it became difficult to make informed decisions. As a result, the implemented techniques were not effective for learner E, as the learner could not retain trigonometric concepts for solving general solutions (Amedume et al., 2022).

CONCLUSION

The study intended to enhance learners' understanding of trigonometric concepts by following the steps of PAR. Overall findings indicate that learners enjoy songs, rhymes and movements teaching strategies as they accommodate their daily preferences based on the recent trends from social media. This shows that the process of learning involves both the mind and the body. Essentially, whenever songs and rhymes are implemented accurately, a positive attitude, such as movement or dancing, is observed. However, attending the sessions inconsistently leads to confusion or failure to comprehend the linkage between trigonometric concepts. As a suggestion, songs, rhymes and movement should be used in FET to teach thought-provoking topics. Therefore, educators must comprehend how to integrate the three strategies: songs, rhymes and movements in the FET phase to accommodate learners with different learning abilities.

Limitations of the study

There are several strategies for teaching mathematics topics, such as trigonometry. The educator's worldview and viewpoints guide the implementation of instructional practices in the classroom. In this study, rhymes, songs, and movements are seen as critical tactics for enhancing grade 11 and 12 learners' understanding of trigonometry. As a result, the tactics



should be used primarily to support learners who are having difficulty understanding mathematical concepts.

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A SURVEY ON MATHEMATICS OLYMPIAD PARTICIPATION AND CAREER CHOICES IN NELSON MANDELA BAY, SOUTH AFRICA

Vasuthavan Gopaul Govender

(Nelson Mandela University)

Mathematics is a key subject in high schools in South Africa and abroad. Learners who do well in high school mathematics and participate in Mathematics Olympiads tend to negotiate the transition from high school to university programmes, for which a top mark for mathematics is a prerequisite, a bit easier than most. A majority of the participants in this study belong to this category of learners. They were able to provide details on how their school mathematics and exposure to Mathematics Olympiads enabled them to progress through their tertiary studies in some very high-level programmes while those who were still at school at the time of this survey, looked forward to this challenge. This study, thus, provides compelling evidence on why more school learners should participate in Mathematics Olympiads. In this regard, mathematics teachers have a crucial role to play, not only in high school, but from the earlier grades where the problem-solving culture should take root.

INTRODUCTION

According to the Curriculum Assessment Policy Statement, commonly known CAPS, two of the purposes of the National Curriculum Statement Grades R -12 in South Africa are to provide access to higher education and to facilitate the transition of learners from educational institutions to the workplace. One of the key principles of the same curriculum is producing learners who are able to identify and solve problems, and make decisions using critical and creative thinking (DBE, 2011:4)

One of the subjects in the South African curriculum that is likely play a leading role in fulfilling the purposes and principles mentioned in the previous paragraph is Mathematics. In this regard, the CAPS document for FET Mathematics states that one of the aims of Mathematics is to develop learners' problem-solving and cognitive skills. It states that teaching should not be limited to the "how" but should rather feature the "when" and "why" of problem types. It states, further, that the learning of procedures and proofs should be done with the understanding of why this is important so that learners are able to use their knowledge in later life (DBE, 2011). Thus, this emphasis of the South African Curriculum for Mathematics would appear to promote long-term sustainable learning, rather than learning to just pass examinations.

In the South Africa, four cognitive levels of assessment are prescribed. These are *Knowledge* (level 1), *Routine Procedures* (level 2), *Complex Procedures* (level 3) and *Problem Solving* (level 4) (DBE, 2011: 53). These cognitive levels are applied to all school phases. This means that learners should be exposed to problem solving activities from an early age.

While teachers are encouraged to incorporate problem solving within their Mathematics lessons, there are opportunities for South African learners to learn more about problem-solving through participation in Mathematics Olympiads and competitions. The South African



Mathematics Foundation (SAMF), in its website, discusses the importance of Mathematics competitions and the resultant enthusiasm and curiosity generated for Mathematics as a school subject.

Both primary and high school learners participate in competitions organised by the South African Mathematics Foundation. The South African Mathematics Challenge (SAMC) is a competition for grade 4 to grade 7 primary school learners. (SAMF, 2025a).

The benefits for learners taking part in the South African Mathematics Challenge are summarised as follows:

- **The strengthening of learners' problem-solving skills.** Learners are engaged in working through problems that goes beyond conventional problem-solving
- **The building of confidence for Future Competitions:** Learners start competing in the South African Mathematics Challenge from an early age. This helps them build confidence in their problem-solving abilities and they are inspired to take part in future mathematics competitions.
- **Developing Essential Soft Skills:** Children's participation in the South African Mathematics Challenge also nurtures other valuable attributes such as time management and preparation strategies. They are also likely to develop a positive attitude towards Mathematics competitions, from an early age. (SAMF, 2025a)

The South African Mathematics Olympiad (SAMO) is for high school learners in South Africa. It comprises two competitions, a junior one for grade 8 and 9 learners, and a senior one for grade 10 to 12 learners (SAMF, 2025 b). A summary of the benefits for learners who write the SAMO follows:

- **Recognition for their achievements:** Top learners get certificates and awards for outstanding performance.
- **Curiosity-Driven Learning:** The questions in the SAMO are interesting and go beyond classroom boundaries. These questions are also likely to spark curiosity in learners and motivate them to explore topics outside the standard curriculum.
- **Practical Benefits for the Future:** Learners who are successful in the SAMO are likely to take up scarce skills courses at universities and are usually targeted by various companies. The competition emphasises problem-solving skills, and the ability to think critically and creatively, which are key requirements in the 21st-century job market. These important attributes are likely to hold them in good stead for the challenges of the future.

Problem Statement



In South Africa, Mathematics Olympiads are an “add-on” for learners and teachers. Learners tend to work through Olympiad-type questions in their own time and their teachers may or may not give them support. However, these Olympiads tend to enrich the learners’ mathematical knowledge. Schools may use Mathematics Olympiads and competitions to promote excellence in mathematics learning, develop and enhance self confidence in learners and also nurture creativity amongst the learners.

Thus, the enrichment provided by Mathematics Olympiads falls under the Enrichment Triad Model (ETM) of learning (Renzulli, 1977). This model consists of three different kinds of interrelated forms of enrichment activities that are integrated as a complement to the regular curriculum. Type I enrichment consists of general exploratory experiences that are designed to expose learners to topics and areas of study not ordinarily covered in the regular curriculum. Type II enrichment consists of group training in thinking and feeling processes, learning-how-to-learn skills, research and reference skills, and written, oral, and visual communication skills. Type III enrichment consists of first-hand projects or investigations intended to solve real problems (Renzulli, 1999).

All three types of enrichment are appropriate within the context of Mathematics Olympiads and competitions in South Africa. Type I and type III enrichment are relevant in rounds 1 to 3 of the South African Mathematics Olympiad, while type II enrichment would be relevant in the team paper of the South African Mathematics Team competition.

The writer has been involved in local Mathematics competitions since 2003. He also has experience in both the South African Mathematics Challenge and the South African Mathematics Olympiad. Since 2013, the writer has been regional coordinator of the South African Mathematics Team Competition, where regional teams participate in a “Mathematics Olympiad-type” team competition (SAMF, 2025c). The writer has also been local coordinator and facilitator in the Siyanqoba Mathematics Olympiad training programme from 2017 to 2020 (SAMF, 2025d), which ended due to COVID-19. These involvements have enabled the writer to work with some of the best Mathematics Olympiad learners from the local region.

The writer has observed the majority of learners mentioned in the previous paragraph first participated in Mathematics competitions such the South African Mathematics Challenge in primary school. This continued in high school when they participated in the SAMO Junior Division in grade 8 and 9; followed by their participation in the SAMO Senior Division in grades 10 to 12. As a result, they were able to participate in the South African Mathematics Team Competition.

The result of the writer’s work in promoting Mathematics Olympiads in the local region, has meant that the writer has a database of learners who had participated in Mathematics Olympiads over a long period. From examining records for the years, 2017 to 2021, there were 40 such learners who were identified. The writer approached the South African Mathematics Foundation for permission to do a study on these learners. This study would find out the career choices of these learners and to find out to what extent did their mathematics backgrounds, and



their participation in Mathematics Olympiads play a role in these choices. The South African Mathematics Foundation gave the writer permission to conduct the study.

Research question

In the light of the discussions in the previous section, the following research question was posed for this study:

To what extent does participation in Mathematics Olympiads at school prepare learners for tertiary studies in mathematically related fields?

The following subsidiary questions were formulated within the context of the research question:

1. What did they learn from participation in Mathematics Olympiads?
2. What tertiary studies did they undertake and what role did mathematics and the participation in Mathematics Olympiads prepare them for these studies (and career choices)?
3. Were they able to recall some of the Mathematics Olympiad problems?
4. What advice would they give mathematics teachers and learners about participation in Mathematics Olympiads?

LITERATURE REVIEW

This short literature review focusses on the importance of problem-solving in school mathematics over many years as well as the possible benefits for children who participate in Mathematics Olympiads while at school.

In 1980 the National Council of Teachers of Mathematics (NCTM) in the USA recommended that problem solving be the focus of mathematics teaching because it encompassed skills and functions which are an important part of everyday life (NCTM,1980). In 1982, the Cockcroft report in the UK advocated for problem solving as a means of developing mathematical thinking as a tool for daily living, saying that problem-solving ability lies 'at the heart of mathematics' because it is the means by which mathematics can be applied to a variety of unfamiliar situations (Cockcroft, 1982)

According to Caprioara (2014), learners realise the importance of solving mathematics problems when learning mathematics but tend to prefer problems that are solved through algorithmic methods. However, she explains that the true intellectual challenge is given by problems that can be solved through heuristic methods, where the effort is focused on searching for the solution. These types of problems require a greater degree of student emotions and experiences which are likely to prepare them to solve real problems that they will face in life.

Campbell, Cho and Tirri (2018) in their study of mathematics and science Olympiads at an international level found that there are crucial factors when identifying Olympiad talent among school learners. Some of the common factors are:

- Early recognition of STEM talent and conducive home environment



- Specialized challenging programmes for developing strong STEM foundations and opportunities to participate in STEM activities and competitions during the schooling period
- Strong motivation and efforts on the part of these learners for success throughout the developmental stages.

Govender (2019) in his study of learners who participate in Mathematics Olympiads/Competitions outlines the following attributes of such learners:

- They love mathematics as a school subject and tend to do very well in the subject with nearly all scoring more than 90% in their mathematics tests and examinations. However, school mathematics does not appear to be challenging for them. Thus, they seek outside challenges such as Mathematics Olympiads/Competitions to help them develop further in the subject.
- The majority of them have been involved in these competitions for most of their high school careers with some having participated as early as grade 3.
- They are likely to look deeper into the problems based on the mathematical principles involved (Schoenfeld and Herrmann, 1982), and applying the principles of problem-solving steps as proposed by Polya and others had become second nature to them.

Yuret (2024) in her study of the International Mathematics Olympiad (IMO) medallists over the years has noted the following:

- Many medallists choose the United States for their higher education studies, despite having some excellent institutions in their own countries. This is probably due to a number of factors such as international recognition and funding of such high calibre students.
- Medallists pursued their PhD education abroad, and slightly more than a third of them became academics. A majority of these academics (approximately 70%) chose to specialise in Mathematics, probably stemming from their high school aspirations.
- The career paths for medallists have consequences for medallists themselves as well as their countries. A lack of opportunities for the best minds within a country poses a serious hurdle for gifted children within that country. This may also imply that “less able” students may face limited prospects.

There is no doubt that Mathematics is an important school subject and learners who do exceptionally well in mathematics are likely to participate in Mathematics Olympiads, where their problem-solving abilities are likely to be “tested” and further developed. However, it is equally important that there should be a path for such learners to pursue studies, and subsequent careers within their own countries to help in the development of their countries and to motivate other learners.



THEORETICAL FRAMEWORK

Since this study is about Mathematics Olympiad learners' reflections about their experiences in Mathematics Olympiads while at school as well as their career choices, or intended career choices, the following theories apply to this study.

The first is Kolb's theory of experiential learning which focuses on the student's personal development and perspective. Unlike other learning methods, the onus is on the student to guide his/her learning process in experiential learning. Experiential learning allows students to apply knowledge in real life situations and encourages active participation, critical thinking, creativity, problem solving, collaboration, and communication skills (Kolb, 1984).

The second one is Holland's Theory of Vocational Choice which is a person-environment fit approach stating that people seek work environments that suit their interests and that the degree of fit between a person and a specific work environment has substantial effects on favourable outcomes such as job satisfaction and performance (Hartman, Heine and Ertl, 2021).

In this study participants had to reflect on their experiences of participation in Mathematics Olympiads while at school and the possible impact of this participation in their university studies. They then had to indicate whether these experiences had any bearing on their career choices. Thus, an appropriate framework in which to locate this study is:

The relationship between Mathematics Olympiad experiences and future career choices

RESEARCH DESIGN AND METHODOLOGY

This study is about the participants' reflections about their Mathematics Olympiad experiences while at school and the subsequent influence on their university studies and future career choice. Thus, this study is located within the Pragmatic Research Paradigm (Kaushik and Walsh, 2019). This paradigm accepts that there can be single or multiple realities that are open to empirical inquiry (Cresswell and Clark, 2011). It states that there is an object reality that exists apart from human experience. However, this reality is grounded in the environment and can only be encountered through human experience (Morgan, 2014).

Thus, this study sought to determine the current reality of the participants in this study, especially the role of Mathematics Olympiads in this reality. As mentioned earlier, the author identified 40 such participants and sought to determine whether there was any relationship between their participation in Mathematics Olympiads and their career choices. A high percentage of these participants, 34 (85%), responded.

It should be noted that this research examines the above relationships through the participants' own voices, thus, making this research qualitative in nature (Hatch, 2002).



Data

The initial survey took place in early 2024, via a Google form. Of the 40 ex-Mathematics Olympiad learners contacted, 34 responded. All 34 agreed to participate in this study. Of these 6 had already completed their university studies and were in full-time employment, 21 were currently at university and 7 were still at school. There was one learner who was under the age of 18. Her father gave approval for her complete the survey.

Due to space considerations, participants' responses have been categorised into the following broad categories:

What they learnt from participation

Before exploring the participants responses, it is important to examine the years of participation in Mathematics Olympiads/competitions. This is shown in table 1:

Table 1: Years of participation in Mathematics Olympiads/Competitions

Years of participation in Mathematics Olympiads/Competitions	Number
5 years and more	17
3 to 4 years	12
Less than 3 years	5

The majority of the participants have been participating in Mathematics Olympiads/competitions for three years and more, with a significant number having their first experiences in the primary school. Some of the key learning from these experiences are shown here. These are written in their own words:

“Critical and higher order thinking and time management”; “How to break problems down before solving”; “Different ways of thinking while doing problem solving which was very useful when doing tertiary studies and while working”; “I formulated equations based on a logical interpretation of the questions; recognising patterns and trying to figure out what was unique about the question”; “I was able to integrate logic with mathematical knowledge was the best way to approach the Mathematics Olympiads”; Being involved in these Olympiads was beneficial in cultivating a real passion for Mathematics, that extended beyond the scope of what was taught at school as an academic subject”; “It taught me how to approach complex problems and find creative solutions for difficult problems”; “The skills I learnt at the Olympiads/ competitions have helped tremendously when it comes to solving problems even outside of the field of mathematics”; “It helped me learn and improve my problem solving skills and also helped me a lot with my regular mathematics”; “How to think about mathematical problems from different angles as well as using the skills that I learnt to apply to my other subjects at school”; “That there was a lot more I needed to learn in mathematics and it reminded me of how many people were working hard and for me to work hard as well”; “The competitions taught me the value of teamwork that would be applicable in the workplace, especially with the time constraints and the need to delegate tasks to those best equipped”;



“The competitions required a level of thinking that was new and abstract. Difficult questions at school can often be solved using repetitive, iterative methods rather than by making full use of mathematical principals and knowledge, whereas the Olympiad questions demanded a proper understanding in order to get the correct answers. Thus, I learnt how to approach questions theoretically from many different points of view”; “I learned various problem-solving skills. I have learnt to love mathematics. I have been taught that sometimes there are multiple solutions to a problem or that problems can be assessed in different ways to be able to find the best-fitting solution. Mathematics Olympiads have helped me improve my social skills and taught me how to work together in a team to tackle 'difficult' problems”

A summary of these learnings

There is no doubt that participation in the Mathematics Olympiads and competitions was extremely beneficial for all participants in this study. They were able to learn a number of useful problem-solving strategies which helped them in both the Olympiads as well as their school mathematics. This enabled them to interpret “higher order” questions, think “critically and outside the box”, important attributes which has also helped them in problem-solving in their daily life and work situation, outside the mathematical context. Some also enjoyed working with like-minded people in teams, which was developed when they participated in the South African Mathematics Team competition.

Tertiary studies and careers

Table 3 shows the tertiary studies of the participants who were still studying or had completed their studies. It also includes the career choices of those who were still at school when this survey was conducted.

Table 2: Tertiary Studies and subsequent careers

Field of study:	Types	Number	Total
Commerce	B. Com.: Chartered Accounting; Economics; Accounting Science	4	4
Medicine	MBChB	7; 2 completed	7
Engineering	Industrial (3); Electrical (1); Mechanical (2); Aeronautical (1)	3 +1+2+1	7
Computer Sciences	B.Sc. Computer and Information Systems (8); Software developer and quality assurance engineer (1)	9 (1 completed on software development)	9
Architecture	Architecture	1	1
Mechatronics	B.Eng. Mechatronics	1 Completed	1



Actuarial Sciences	B.Sc. (Actuarial Sc)	4(I completed)	4
Arts	B.A (Music)	1	1
	Total	34	34

NB: Table 2 also includes the career choices of the 7 participants who were still at school in 2024. These are included under Actuarial Sciences (1); Computer Sciences (2); Commerce (2) and Engineering (2)

From table 2, we see that all were involved in some form of study or were going to do so once they had completed school. Other than the one participant who did a degree in music, the others had chosen studies for which Mathematics as a school subject would be a key perquisite. All participants agreed that their participation in Mathematics Olympiads had a positive impact on their school mathematics marks. For those who were already busy with their university studies or had completed their studies, this is what they said, in their own words, about the importance of school mathematics and their Mathematics Olympiad experiences on their university studies:

“It helped me break down and understand questions easier”; “It helped to set you up to tackle different form of mathematics by opening up your mind to other solutions and problem solving”; “My degree didn’t involve any mathematics. But I enjoyed keeping up with mathematics and tried to implement it in my everyday life”; “I’m not sure if there was a direct impact the Olympiads had on my university studying, but I’m privileged to have been exposed to different, challenging ways to engage with academic content outside the typically academic school space. It encouraged a different perspective on learning, in that it is lifelong, which is a takeaway I carry into my university approach to studying”; “As an undergraduate medical student, problem solving is the root of my degree. While we don’t necessarily deal with lots of mathematical problems in this degree, medicine is all about problem solving. Doing mathematics Olympiads at school helped improve my confidence in my ability to problem solve. This has made a big difference when I am faced with diagnostic challenges. I am able to think creatively and find unique solutions”; “It helped greatly in making the mathematics I explored in university much easier/bearable”; “It allowed me to problem solve which I use to this very day when confronted with new challenges problems in patient management”; Most of Engineering is actually solving problems and more critical thinking which is what the Olympiads consisted mostly of, so the Olympiads helped a bit in terms of that”; “I don’t really see the Olympiads as way to learn new things but more a way of actually a way of expressing/using problem solving skills”; “Because I feel school mathematics is more of just applying formulas and not making silly mistakes, there is only a few “nice” critical thinking questions whereas Olympiads is just full of these critical thinking questions”; “The Olympiads exposed me to different types of questions that required a more complex understanding of mathematics, which has better prepared me for university level mathematics”; “It helped me with graphs and other things in economics”; “It helped me understand my university mathematics easier as a lot of the topics covered in my university mathematics were topics that I already experienced in mathematics Olympiad tests or classes”; “University level



mathematics and science requires a greater understanding of the subject and a higher level of thinking. These competitions helped me to see that early on and get a head start in university”; “It helped with my mathematical modules as university requires a different way of thinking which I had already gained from my participation in my mathematics Olympiads”; “The type of questions in Olympiads are seen much more frequently at a university level than at a school level and it was therefore advantageous to be exposed to such questions from an earlier age”; “The Mathematics Olympiad has helped me think in a different way about the questions I get. It has also given me the confidence to pursue a Mathematics based degree”

A summary of the importance of Mathematics in tertiary studies

It is important to acknowledge that they participated in Mathematics Olympiads for a number of years while at school due to their potential and performance in school mathematics. Thus, with their exposure to Mathematics Olympiads, their performance in school mathematics went even higher. They were accepted into some very interesting and diverse fields, as a result of these impressive school mathematics results.

This school background of high mathematics achievement and exposure to Mathematics Olympiads/competitions played a pivotal role in them being able to negotiate the various mathematical and non-mathematical aspects of their tertiary studies. They were well-prepared, confident and enjoyed the challenge of engaging in higher level thinking. One of the comments that needs to be highlighted is by one of the newly qualified medical graduates who said:

“As an undergraduate medical student, problem solving is the root of my degree. While we don’t necessarily deal with lots of mathematical problems in this degree, medicine is all about problem solving. Doing mathematics Olympiads at school helped improve my confidence in my ability to problem solve. This has made a big difference when I am faced with diagnostic challenges. I am able to think creatively and find unique solutions”

These and other comments point to the very essence of doing Mathematics at school and participation in Mathematics Olympiads, the ability to think critically and creatively, and considering various options when involved in problem-solving.

Mathematics Problems Recalled

Participants were asked to recall some of the mathematics problems that they had experienced in their Olympiad journey. They were very descriptive about the problems and the thinking involved in working through these problems. Here are some examples of their recollections:

- **“What unit does the number 3^{100} end in?** I first decided to find a common pattern from the following powers of 3: 3^1 ; 3^2 ; 3^3 ; 3^4 . From my observations I could work out what 3^{100} ends in. “
- “I cannot think of any Olympiad problems but rather a matric final question which I feel was on a similar level. The question was the probability question of the 2020 Grade 12 Final Mathematics Paper 1. The question goes along the lines of **“If 2 people play**



darts and the first to hit a bullseye wins and each player has 50% chance to hit a bullseye per throw. What is the chance that player 1 wins if he throws first”?

I believe the answer is 66,6% or $\frac{2}{3}$. I think I used a sum of a geometric series to infinity. I enjoyed the question because it involved some level of critical thinking. Some background: I got an answer of 66.6% and so did my one friend, but another friend and his tutor got an answer of 50%. In the end my first friend got 100% for mathematics in his final results I believe we were correct. My younger sister also told me that a group of classmates spent a whole lesson on that problem and struggled to solve it so that’s why I believe it is a tricky question”.

“I saw this question at university. I believe it was once set in a Student Aptitude Test (SAT) exam in the USA. ***“If disk 1 has a radius of 1 unit disk 2 has a radius of 3 units, and disk 1 rotates one revolution around disk 2 without slipping how many times does disk 1 rotate about its own axis”***

The answer is 4 and not 3 like most believe. There was an interesting YouTube video on this question named “the SAT question everyone got wrong” which explains the solution very well. I used relative velocities and then Integrated it because that concept made sense to me when thinking about the problem”.

- “An interesting question that I learned while preparing for the Olympiads was a question from the Senior Second Round. The question was: ***“ We define $S(n)$ as the sum of the digits of a positive integer, n , for example $S(7) = 7$, $S(19) = 1 + 9 = 10$ and $S(1099) = 1 + 0 + 9 + 9 = 19$. Find the value of $S(1) - S(2) + S(3) - S(4) + \dots + S(1011)$. ”*** I love questions which involve series and patterns. I also enjoyed and learnt from a question that asked about the digit of a decimal, where one would have to realise that it will be a recurring decimal”.
- “ The problem is as follows: ***“In linear land there are 3 currencies. There are gold coins, silver coins and bronze coins. If 2 gold coins and 1 bronze coin is worth 3 silver coins, but 7 silver coins and 2 bronze coins is worth 5 gold coins, then what is 6 gold coins and 5 silver coins worth in bronze coins”***.

The solution to the problem was not so significant but rather my thought processes when solving this problem. I remember I was stuck on it for quite some time, but I eventually got to an answer in the end. This question and solution taught me that no matter how hard something might seem, you have to at least try because you never know what you are capable of until you try and succeed”

- “I enjoyed working on this problem. ***“A cylindrical barrel of radius 1 m lies on its side up against a wall as shown in figure 7. What is the radius of the largest ball that can fit in the gap between the barrel and he wall?”***

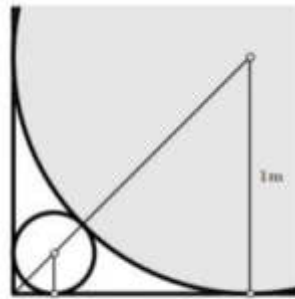
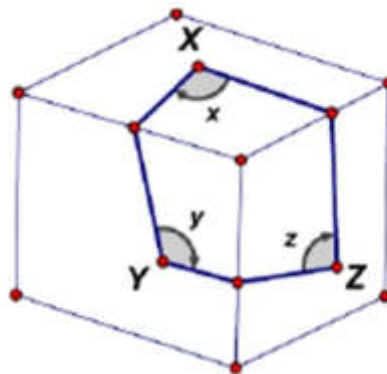


Fig. 7.

I worked on this problem with a partner in the team competition and we were unable to solve it because we over-thought the problem and tried to use trigonometry to get the radius instead of seeing that we could simply use the theorem of Pythagoras. I found it interesting because it was the last question of the paper and we automatically assumed it would be the most difficult. Because of that we were unable to see the simple solution. The problem taught me to always look for the simplest solution to a problem instead of just assuming it will be difficult.”

- “I found the following problem interesting and challenging: *“An ant lives on the surface of a cube that has points X, Y and Z on three of the faces. The ant travels between X, Y and Z along the shortest paths between each pair of points. What is the sum of the indicated angles x, y and z on the ant’s path in degrees?”*



I had never seen it or had to solve a problem like this before. However, I tried different strategies and learned how to solve it. It also taught me how to approach and solve problems similar to it and how to better prepare for 3D questions in future Olympiads and tests”.

Participants were able to list some very interesting and engaging problems and how they were able to work through these problems and the lessons learnt. This showed massive insight on their part as these were problems from three or more years earlier.

Advice given to teachers

The participants in this study had some very good advice for teachers on how they could assist their learners with encouragement and preparation for Mathematics Olympiads. This advice, using the participants’ own words, are outlined here:



“Give learners past year papers or tell them where to find them”; “Be passionate about sharing your love for mathematics with learners”; “Allow learners to be taught some of the things they see in the Olympiads which they don’t recognise as it can help in both mathematics at school and the Olympiad”; “Tell learners that it’s not only about mathematics but teaching you how to think and finding a solution to any problem in a meticulous way”; “Make learners aware of what can become of you if you participate in the Maths Olympiad and participation in the team competition”; “Make sure learners understand that the Olympiad should be seen as an opportunity to gain a deeper understanding of concepts covered as well as not yet covered”; “I would advise them to teach the learners old Mathematics Olympiad questions. To give them questions to think about by themselves or in groups. This will promote group brainstorming. It’ll also encourage learners to think about the problem without giving them the answer right away. Different learners can have different approaches and each student can learn from others’ ideas. This could also allow criticism and thinking out the box”; “I would strongly recommend participation in Mathematics (and various other kinds) of Olympiads, no matter the age or “level”, as they enrich the knowledge we accumulate at school, and help to stimulate out-of-the-box thinking, which is a valuable skill to take out of school and into the working/studying space”; “To work through one or two problems with the students to show them how to ‘think outside of the box’ when it comes to Mathematics Olympiads. I think one of the most off-putting parts of Olympiads was having to figure out the language of Olympiads all on my own”; “Work hard not on recruitment for Olympiads from the beginning but focus more on creating a culture around the idea of what participation in these Olympiads will do for them. In that way recruiting will become come a lot easier.”; “Don’t always make them extra-curricular, include them in the classroom, simply as a fun challenge, and then make further Olympiad activity known and optional”; “Encourage all learners to participate, not only those who are at the top of the class. And congratulate the students who achieve in the Mathematics Olympiads and Mathematics team competitions at the school’s awards ceremonies to encourage more participation”; “It is extremely beneficial to learners; tell them that Mathematics Olympiad problems are ‘closer’ to real world applications than school mathematics; it is a different way to stretch their brains and challenge themselves, ways that calculators and AI cannot easily replicate”; “I would advise teachers to spread awareness about the benefits of Mathematics Olympiads to further encourage pupils to participate in these Olympiads”

A summary of the advice to teachers

It is clear that the participants, from their own reflections, feel that the role of the teacher is key in encouraging learners to participate in Mathematics Olympiads. They offer teachers some tangible steps to make this a reality at schools. This advice is closely aligned with their own learning. It is obvious that these participants revelled in their Olympiad experiences and would like others to have the same experiences. However, much will depend on whether the teachers are, themselves, competent in teaching or facilitating such problems to interested learners. According to Govender (2024), teachers who are trained in problem-solving are likely to incorporate problem-solving in their lessons and prepare their learners for Mathematics



Olympiads and other competitions, thus, developing their learners' creativity and intellect, and raising their achievement levels in their school mathematics.

Learners

It should be noted that the participants in this study had taken part in Mathematics Olympiads over many years. Thus, with their own experiences and subsequent career choices, they were in a position to give some interesting and constructive advice to learners at schools. Once again, the participants' own words are used:

"It's actually a lot of fun to do"; "Try as many questions as possible, maybe go up a grade or level and solve round 1 problems without seeing the options"; "Practice time management and breaking down patterns. Don't be scared to look at all your resources beforehand"; "It is an opportunity to really improve your mathematics skills as well as learning and thinking skills, in general; definitely an opportunity not to be missed and really does not take much time"; "I would encourage them to learn the different approaches to questions, and when to use each approach. For example, how many different ways to arrange the word ALPHABET. What to do about repeating letters, etc."; "Embrace the opportunities to learn new approaches and ways of thinking about Mathematics. It is definitely not like school mathematics, in that it presents fun challenges and teaches you to think differently to solve the problems; even if you don't see the benefits immediately, it's always exciting to embrace the challenge. It's the best part!"; "In life it's always good to challenge yourself. Mathematics Olympiads are a fun and safe space to test your knowledge without worrying about the result. Even if your mathematics marks aren't the best, if you understand basic mathematical principles it can only help you to give Mathematic Olympiads a try. Who knows, you may even find you love them!"; "Apply yourself as much as you can, they are quite educational and will help a lot in your mathematical skills development"; "If you're even remotely academically inclined, they can serve as a fun challenge. Assuming the results are anonymous, there is nothing to lose, so try it out, think outside of the box, using tools that came from within the box"; "If you have an interest in mathematics and problem solving, you should participate. Encourage your friends to join with you and make new friends with likeminded people from other schools who also participate"; "It's super fun! Stay consistent! Can only learn and grow if you stay consistent with the classes and practice!"; "Practice some Olympiad style questions beforehand and attend Olympiad classes if there are any, as this preparation helps a lot to get use to the different styles of questions"; "Learn as much as you can. It is a very interesting experience and it will help you wherever you choose to go in life"; "Practice in some way, discuss these problems with others, appreciate that it's more relevant to the workplace than school mathematics which begins to equip you for it"; "Do so without holding yourself to too high of a level of expectations - these are advanced problems"; "Study hard for it. If you don't study you going to do very badly"; "Enjoy it. It is an awesome experience, and you can definitely learn a lot"; I would advise learners to practice as many questions as possible and to use past papers in preparation for these Olympiads.



A summary of the advice to learners

Participants were very encouraging in their advice to learners. They pointed out the fun aspect of these competitions while acknowledging the “difficult” nature of the problems. However, they believed that learners should embrace this exposure to Mathematics Olympiads as it is likely to result in a positive impact on their school mathematics and raise the likelihood of them doing some high-level university programme, where this experience would come in handy. The participants were probably reflecting on their own positive and fulfilling experiences and wanted others to experience the same.

DISCUSSION AND FINDINGS

This study shows the importance of school mathematics as a subject, as well as the advantages of participation in Mathematics Olympiads, while at school. Thus, findings of this study are written within the context of the research sub-questions, and consequently the research question posed:

What the participants learnt: It is clear from the “voices” of the participants that they learnt a great deal from their participation in Mathematics Olympiads and competitions while at school.

Mathematics and tertiary studies: From a survey of the career choices of the participants, nearly all had chosen tertiary studies for which mathematics was a prerequisite. As mentioned earlier, their exposure to higher level problem types in Mathematics Olympiads and competitions from the early grades probably ensured that their school mathematics results were of the highest order which cleared the way for them to embark on tertiary studies where a top mark in school mathematics is a key prerequisite. This included careers in medicine, commerce, engineering, actuarial sciences

Mathematics problems recalled: While participants extolled the virtues of participation in Mathematics Olympiads, it was remarkable to note that some of them were able to recall actual Olympiad-type problems, how they approached these problems and what they learnt in the process, from a few years ago. These learnings probably prepared them well for other problems that they encountered in both their school mathematics as well as Mathematics Olympiads

Advice given to teachers: Participants provided some very pertinent advice to teachers about Mathematics Olympiads. This includes identifying learners for participation in these Olympiads and giving them some support in preparation for these Olympiads. They also provided some advice on how Olympiad-type problems could be incorporated within normal class lessons as these could also prepare learners for higher order questions in their final mathematics examinations

Advice given to learners: One of the key findings from this research is the Mathematics Olympiad journeys of the participants when they were at school. They acknowledged what these journeys meant to them in their learnings, career choices and associated matters. They



shared these important experiences in the hope that others would follow in their footsteps. However, they did indicate that participation in these Olympiads was not for the faint-hearted and that they needed to put in some hard work. At the same time, they embraced the journey, without giving up. This perseverance and determination have ensured that they were able to pursue some “high-level” careers.

CONCLUSION

This study has shown that top performances in school mathematics, together with participation in Mathematics Olympiads, gives learners a path to accessing high-level tertiary studies and subsequent “in-demand” careers in Actuarial Sciences, Commerce, Computer Sciences, Engineering, Medicine, and Mechatronics. This was explained explicitly by participants with details of what they learnt from Mathematics Olympiad participation over many years and how this influenced their study options, for those who had completed their schooling and others who were about to complete their schooling. This learning played a role, either directly or indirectly, in how those who were busy studying, or had completed their studies, were able to negotiate these usually “tough” university programmes. It is interesting to note that the career options of participants in this study differ slightly from the international study by Yuret (2024) in that none of the participants in this study had become academics.

Thus, it is important for more learners to have exposure to such “high-level” content as provided by Mathematics Olympiads. The participants in this study also recognise the key role of mathematics teachers in this process by offering some constructive advice to them. At the same time, they offer some realistic advice to school learners, in an effort to motivate them. This is crucial in a developing country like South Africa, which needs all the support from its young people in an effort to create more jobs and reduce poverty. The writer is aware that many young South Africans do enter high education institutions and successfully complete these “high-level” university programmes, without having taken part in Mathematics Olympiads or competitions while at school. This is highly commendable. However, the school Mathematics Olympiad experience is likely to make students confident and the journey “less stressful”.

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INVESTIGATING GRADE 10 LEARNERS' MATHEMATICAL MODELLING COMPETENCIES ON MEASUREMENT: A CASE STUDY OF AREA AND COSTING CONCEPTS THROUGH MODELLING ELICIT ACTIVITIES

Williams Y, Moodley M, Hlangwani W, Mhakure D

University of Cape Town

This study examines Grade 10 learners' mathematical modelling competencies in measurement, focusing on area and costing concepts. Using model-eliciting activities, learners engaged in real-world tasks, applying mathematical modelling to contextualize measurement tasks. The study employs Blum and Leiß's (2007) modelling cycle as a theoretical framework and uses Leong and Tan's (2020) rubric to assess learners' modelling competencies. Findings indicate that learners struggle with making real-world assumptions, applying correct computations, and reasoning mathematically. The study contributes to how the integration of mathematical modelling as a teaching strategy in schools can foster modelling competencies among the learners.

Keywords: Mathematical Modelling, Measurement, Grade 10 Learners, Model-Eliciting Activities, Competencies, Assessment

INTRODUCTION

Learners in South Africa often struggle with measurement concepts – this is a process of determining numerical values to attributes of an object using concepts such as area volume and perimeter (Morapeli, 2017). Morapeli (2017) argues that learners mainly struggle with the concepts of area and volume. Similarly, Botha et al. (2023) noted that learners face challenges in determining the area and perimeter of objects. Both studies attribute these learner difficulties to the nature of measurement as an abstract concept.

Mathematical modelling, which connects real-world situations to mathematical representations, provides a meaningful way to make mathematical abstract concepts more accessible (Greefrath et al., 2022). Mathematical modelling is a process in which real-life contextual situations are transformed and represented using mathematical models (Haines & Crouch, 2004). International studies have found that real-life contextual situations deepen learners' understanding of mathematical concepts (Dunn & Marshman, 2020) and help them view mathematics as relevant and practical (Geiger et al., 2022), which equips them with the ability to apply mathematical knowledge to solve real-life contextual situations. However, systematic reviews have revealed that there are very few local studies in South Africa, prescribing how mathematical modelling can be integrated in the mathematics classroom and in the national curriculum (Jakobsen & Mhakure, 2024; Moodley et al., 2024).

To contribute to the discourse this current study used mathematical modelling as a teaching strategy to develop learners' mathematical modelling competencies on measurement tasks. One effective way to foster these competencies is through model-eliciting activities. Model-eliciting activities (MEA) are learning tasks that learners can solve using the stages of a modelling cycle shown in Figure 1 below. During the modelling cycle, learners are afforded the opportunities



to apply mathematical concepts in real-life contextual situations. Leong and Tan (2020) view the modelling process as a way for learners to develop mathematical modelling competencies through making assumptions, computing and interpreting solutions, applying mathematical reasoning, and validating.

This study investigates Grade 10 learners' mathematical modelling competencies in measurement, focusing on area and costing concepts through model-eliciting activities. To address this, the study pursued the following research question: What is the nature of mathematical modelling competencies developed by Grade 10 learners when engaging with model-eliciting activities on measurement and costing?

THEORETICAL FRAMEWORK

This study adopts Blum and Leiß's (2007) mathematical modelling cycle, which serves as the central theoretical lens for understanding learners' engagement with mathematical modelling. The mathematical modelling cycle provides a structured framework for guiding learners through the process of transforming real-world problems into mathematical representations, solving them, and interpreting the results in context (*see Figure 1*). Blum and Leiß (2007) emphasise the iterative nature of mathematical modelling, where learners continuously refine their models based on real-world constraints and mathematical reasoning. This iterative process is particularly relevant in this study, where learners engage in collaborative measurement tasks to build modelling competencies.

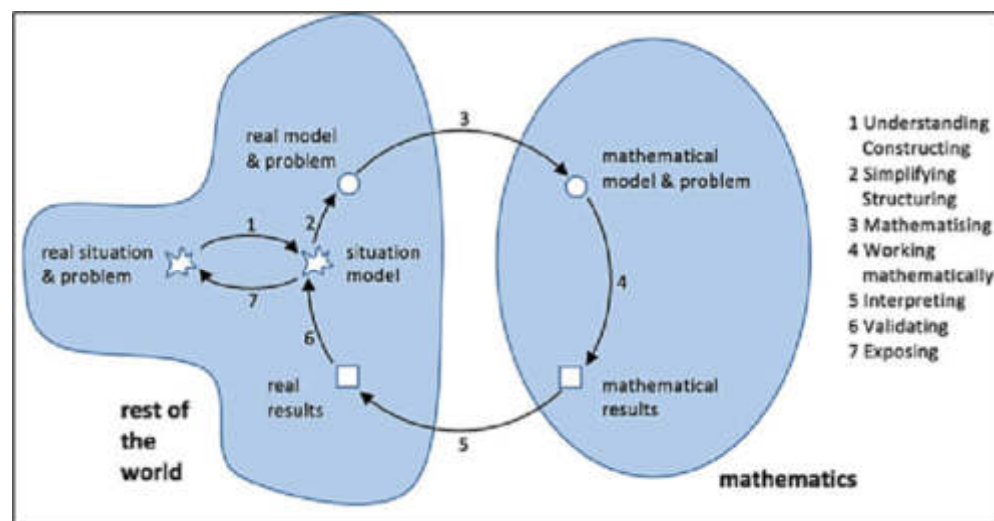


Figure 1: *The Mathematical Modelling Cycle and its competencies by Blum and Leiß (2007)*

The mathematical modelling cycle depicted in Figure 1 shows distinct but interrelated competencies inherent in the modelling process: understanding the problem, simplifying and structuring, mathematising, working mathematically, interpreting, validating, and presenting the solution. These competencies are evaluated through a rubric developed by Leong and Tan (2020) to assess learners' modelling solutions. This rubric classifies the modelling competencies into three categories, namely:



- Making Assumptions: This includes understanding the problem and simplifying and structuring it.
- Computing and Interpreting Solutions: This covers mathematising, working mathematically, and interpreting the results.
- Mathematical Reasoning: This involves validating and presenting the solution.

METHODOLOGY

This study adopts a case study approach to understand Grade 10 learners' mathematical modelling competencies when engaging in model-eliciting activities. This study reports on data from one school where twenty-five learners participated. Ethical approval was granted by the Department of Education, while consent was sought from the participant and their legal parent/guardian.

Procedure and Data Collection

The study was conducted over five consecutive sessions, each lasting 30 minutes. Learners were divided into five groups of five learners. Learners were introduced to the study's objectives in the classroom and completed a baseline survey assessing their mathematical modelling knowledge at the beginning of the study. They were then given the MEA to solve in groups over the five sessions. During the first session, each learner received an individual copy of the model-eliciting activity (see Figure 2) on measurement concepts.

Spinach Planting
Vegetable Farming in South Africa
Modelling Task

Ready To Grow Your Own Food?
EXPLORE OUR RANGE OF VEGETABLE SEEDS FOR YOUR HOME GARDEN
GET STARTED NOW

Amahle is a grade 10 learner at Kleinfield Secondary in Kraaifontein. Amahle's granny recently received a new house from Government in 2023. Motivated by an ad she saw in a local newspaper [article](#), Amahle's granny is interested in running a small vegetable garden. Using half of the available space on her backyard, she decided that she would like to grow spinach to sell to the community for a stable income. Her backyard has dimensions 2m x 1,5m.

Amahle's granny wants to know, how much it will cost her to plant spinach in her backyard. (Hint: in your answer should be represented as a formula or function or model)

Figure 2: Model-Eliciting Activity on Measurement



Learners had five minutes to read it independently before collaborating in groups of five to work on the task. During the five separate consecutive sessions, groups were allowed to access the internet to search for relevant information that is required to develop a mathematical model based on the measurement task. At the end of the five sessions, each group presented their solutions in a chart, and this was assessed by the teacher and researchers who contributed to assessing and evaluating the presented model answers. For the scope of this paper, the work of only one randomly selected group is presented and discussed.

Data Analysis

This study employed a documentary analysis of the model eliciting activity to understand learners' mathematical modelling competencies on measurement concepts. Three researchers assessed the level of learners' modelling competencies using a rubric developed by Leong and Tan (2020).

The rubric categorised learners' mathematical modelling competencies based on the following: making assumptions (*Showing an understanding to the task and listing suitable variables and assumptions*), computing and interpreting solutions (*applying mathematical concepts and solving the problem*), and mathematical reasoning (*explaining the reasoning "why" using symbols, diagrams or vocabulary*). The rubric adapted by Leong and Tan (2020) employed an analytical scale from Level 1, which indicates low competency, to Level 4, which indicates a higher level of competency for each category. The researchers each give the tasks of the a score, discussed and then agreed on a final score represented in the findings below.

FINDINGS AND DISCUSSIONS

This section is divided into three subsections namely; making assumptions, computing and interpreting solutions and mathematical reasoning. The results of learners' modelling competencies are simultaneously presented and discussed against current research findings.

Making Assumptions

This section evaluates the competency of making assumptions by analysing the group's written work (see Figure 3) and scoring it using the criteria in Table 1, based on Leong and Tan's (2020) adapted rubric. The group received a Level 2 competence score for making assumptions irrelevant to real-world problems. This indicates that learners struggled with making relevant real-world assumptions.

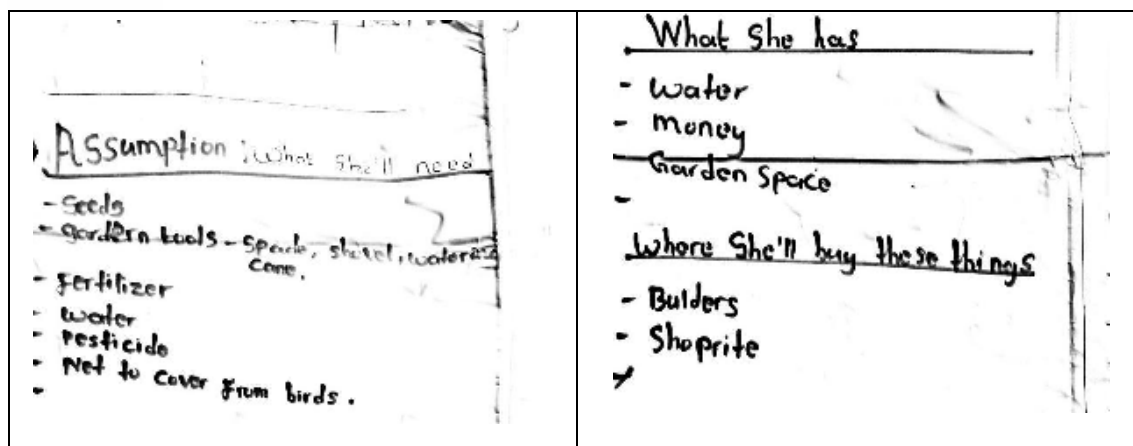


Figure 3: Learners' assumptions presented from the model-eliciting activity.

Table 1: Analytical assessment of group work based on the competency of making assumptions

Competency	Level 1	Level 2	Level 3	Level 4
Making Assumption	-Comprehensive list of variables or assumptions made based on real-world interpretations of task.	-Less than 2 variables listed, or assumptions made based on real-world interpretations of task.	-Less than 2 variables listed, or assumptions made based on real-world interpretations of task.	-No keywords listed.
Level 2	-Assumptions stated are relevant to model	-Assumptions stated are irrelevant to model.	-Assumptions stated are irrelevant to model.	-No variables listed. -No assumptions made. -Incorrect notions of assumptions.

Initially, the learners struggled to compile a list of assumptions based on the task, as some were not familiar with gardening. After their group discussion, learners managed to compile a list of assumptions. Despite the learners having a comprehensive list of variables and assumptions, their assumptions were not all relevant to the task provided, and therefore, they were scored at Level 2 (see shaded region of Table 1). For example, the modelling task mentioned “*Amahle’s granny recently received a new house from government...she decided that she would like to grow spinach to sell to the community for a stable income.*” However, as seen in Figure 3, learners assumed that Granny has money, even though she received a house from the government, which indicates that she is dependent on government funding. Additionally, Granny wants to sell spinach for a stable income. This implies that she needs more money to support herself than she currently has. This shows that learners struggled to connect the real-world context to the assumptions of the modelling task.

Similarly, Leong and Tan (2020) found that secondary school learners in Malaysia did not know how to employ real-world knowledge or they superficially made assumptions. A local study by Govender and Machingura (2020) also found limited competencies in learners making sense of contextual problems. Chan et al. (2012) argue that making assumptions in modelling tasks helps learners in understanding and simplifying the problem as well as situating the modelling task in a more realistic setting. If assumptions are not properly established, the problem's context becomes distorted, making it difficult to solve effectively and further develop



the model (Govender & Machingura, 2020). Therefore, setting assumptions tied to contexts is a crucial aspect of mathematical modelling, which helps learners develop modelling competencies.

Computing and interpreting the solution

This section assesses learners' competency in computing and interpreting the model as seen in Figure 4. The group was scored at a Level 2 competency (see Table 2) as they made minor mistakes in their model constructions.

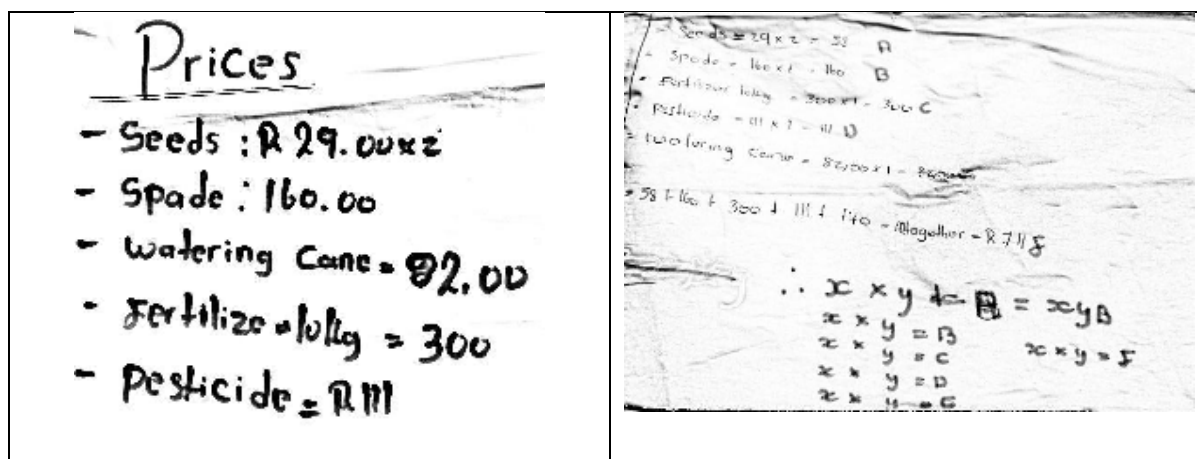


Figure 4: Learners' presentation of computing and interpreting their model based on the model-eliciting activity

Table 2: Analytical assessment of group based on Computing and interpreting solution

Competency	Level 1	Level 2	Level 3	Level 4
Computing and interpreting solution Level 2	- Never show attempt to develop mathematical model. -Errors shown in computation. -There is no evidence of real-world constraint in the presentation of work.	-Show little attempt to develop mathematical model. (fail to connect the variables to form the mathematical model) -Minor errors shown in computation. -Show evidence of 1 real-world consideration in examining variables that will impact interpretation and solution of modelling task.	- Attempt to develop a proper mathematical model. (able to connect the variables to form an initial mathematical model that might be inaccurate) - No errors shown in computation. - Show evidences of 2 real-world consideration in examining variables that will impact	- Develop the most appropriate mathematical model. (able to connect the variables to form the suitable mathematical model) - Computation is clear and accurate. - Show evidences of more than 2 real-world consideration in examining variables that will impact interpretation and solution of modelling task.

Initially, the learners struggled to develop a mathematical model as they were unfamiliar with the word “model”. After their group discussion, learners managed to compute a model based on real-world prices they had found on the internet. However, their mathematical model had some mistakes in their computations. For example, in Figure 4, their model was not represented correctly – as they thought that they should determine the final price by multiplying “x” (price of item) by “y” (number of item) instead of using their part model computations that they had



started with i.e. “ $A+B+C+...+E$ ”. This partial calculation reveals some conceptual understanding where learners had an idea of what was needed; however, due to inaccurate assumptions and errors, their final model was not complete.

Some conceptual understanding exhibited by the learners aligns with the framework provided by Chan et al. (2012), who emphasised that formulating, interpreting, and reflecting on a model is crucial for its development. Additionally, Tañola and Lomibao (2024) highlighted that connecting to prior knowledge and real-life contexts significantly enhances understanding and application of mathematical concepts. Sokolowski (2013) also found that mathematical modelling helps learners understand and apply mathematical concepts in real-life contexts. The learners' errors suggest they had an emerging understanding of mathematical modelling but faced difficulties in refining their computations and assumptions.

Mathematical Reasoning

This section presents learners’ competency of mathematical reasoning by assessing the groups’ written work (see Figure 5). Table 3 shows that the group achieved a competency of Level 3 as they had minor calculation mistakes, and they showed some mathematical skills, but there were some flaws in logic, which are discussed below.

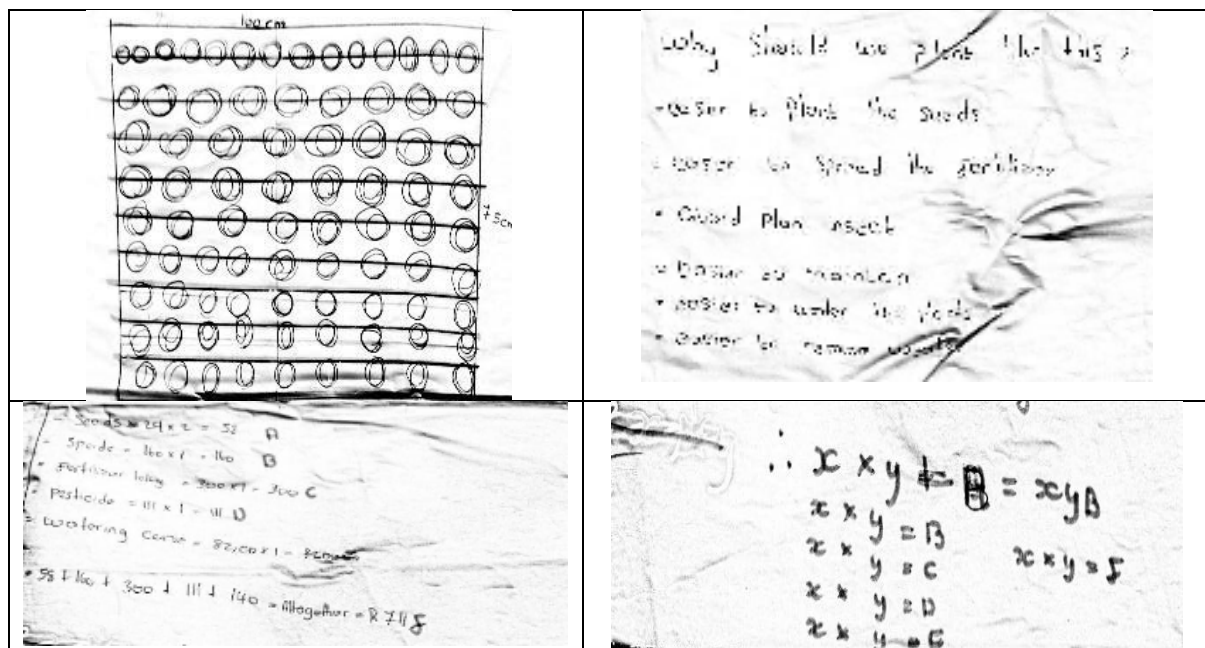


Figure 5: Learners' presentation of mathematical reasoning on their model based on the model-eliciting activity



Table 3: Analytical assessment of the group based on mathematical reasoning

Competency	Level 1	Level 2	Level 3	Level 4
Mathematical reasoning Level 3	-Mathematical reasoning is not logical. - Inappropriate use of mathematics. -No connection for each stage.	- Mathematical reasoning is somewhat logical. - Inappropriate use of mathematics. -Some stages are connected.	- Mathematical reasoning is logical. -Appropriate use of mathematics (minor computational errors) - Some stages are connected.	- Mathematical reasoning is logical. - Correct use of mathematics (no computational errors) - Each stage is connected.

As seen in Figure 5, the learners successfully sketched a diagram of their garden space with correct area measurements and conversions from meters to centimetres, demonstrating an understanding of basic measurement concepts. They also correctly applied the task requirement of using half of the available backyard space ($2\text{m} \times 1.5\text{m}$) for planting spinach. However, their representation of the garden as a square instead of a rectangle suggests some conceptual misunderstanding. Additionally, they struggled with determining appropriate quantities of materials, as seen in Figures 4 and 5, where they assumed that a 10kg bag of fertiliser and two bags of seeds would be sufficient without considering the actual spacing of the spinach as well as the total size of the garden. This oversight highlights their difficulty in analysing spatial constraints and connecting real-world conditions to their model. Although they managed to calculate the overall cost of planting spinach numerically, they did not justify the quantities needed, relying on guesses rather than structured reasoning. These errors indicate a broader challenge in transitioning from mathematical concepts to a fully developed model.

Mathematical modelling literature emphasises the importance of integrating conceptual understanding with real-world applications to develop accurate representations of spatial and quantitative constraints (Blum & Leiß, 2007). The learners' ability to convert measurements and allocate space aligns with foundational modelling competencies but highlights a common struggle in transitioning from abstract calculations to meaningful and context-driven decision-making calculations (Stillman et al., 2013). Their misrepresentation of the garden's shape and miscalculation of material requirements reflect reasoning gaps often observed in early modelling stages, where students rely on intuitive assumptions rather than systematic analysis (Borromeo Ferri, 2018). These errors suggest the need for scaffolded learning approaches that reinforce spatial reasoning and proportional reasoning within applied contexts.

CONCLUSION

In conclusion, this study highlights significant challenges faced by learners in transitioning from mathematical concepts to fully developed models, particularly in making relevant real-world assumptions, computing and interpreting solutions, and demonstrating mathematical reasoning. These findings are crucial for understanding the gaps in current educational practices and underscore the need for targeted strategies to enhance modelling competencies and integrate mathematical modelling in the school curriculum more explicitly. Teachers can apply these insights by integrating real-world contexts into the curriculum, using structured training and iterative approaches. Encouraging teachers to improve their modelling teaching



approaches by using the materials available for mathematical modelling in schools can improve the learners' thinking skills and enhance teacher education. Encouraging reflective practice is also essential for developing robust modelling skills and a deeper conceptual understanding of mathematical concepts. By continuing to refine and implement these strategies, teachers can better equip learners with the skills needed to navigate complex real-world problems through effective mathematical modelling.

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CHALLENGING GRADES-FIRST APPROACH: RETHINKING MATHEMATICS SUCCESS FOR LEARNER EMPOWERMENT IN SOUTH AFRICAN SCHOOLS

Mildret Ncube and Kakoma Luneta

University of Johannesburg

The dominance of the grades-first approach in South African high schools has contributed to a narrow and exclusionary definition of success in mathematics education, often prioritising academic performance at the expense of holistic learner development. The systematic review investigates alternative frameworks that reconceptualise success to include learner empowerment, conceptual understanding, metacognitive development, and real-world application. Drawing on international and South African scholarly literature, educational theories, and policy documents, the review analyses how shifting away from performance-driven models can enhance learner engagement and long-term mathematical proficiency. It also explores the impact of socio-economic disparities and historical inequities on learner outcomes, particularly in under-resourced schools. The review finds that traditional assessment practices contribute to test anxiety, reduced confidence, and disengagement. In contrast, inclusive models grounded in formative assessment, culturally responsive pedagogy, and competency-based education are more effective in promoting equity, critical thinking, and sustained learner engagement. These findings reinforce the need for a more holistic and socially responsive definition of success that centres learner agency, contextual relevance, and inclusion. The paper concludes that rethinking mathematics success through a learner-centred, equity-oriented lens is essential for fostering empowerment and advancing meaningful educational transformation in South African schools.

INTRODUCTION

The grades-first approach refers to an educational strategy in which learners are primarily grouped according to their current grade levels instead of their individual abilities or age before any further differentiation occurs (Francis et al., 2020; Spaul, 2013). The approach has emphasis on numerical achievement and test outcomes as the main indicators of learners' success to the detriment of deeper conceptual understanding, critical thinking, and problem-solving skills. Rote learning is therefore supported on standardising assessments and grading systems instead of holistic learning exposure (Black & Wiliam, 1988; Shepard, 2000). Boaler (2016) argues that grades-first approach steers to surface-level learning, phobia, and poor definition of mathematical competence.

In South African high schools, mathematics education has historically been characterised by a grades-first approach, where the emphasis on academic success, especially in high-stakes examinations, often obliterates the comprehensive development of learners (Reddy, 2006; Spaul, 2013). This emphasis frequently results in over dependency on rote learning and examination-oriented education, which neglects necessary competencies, for instance, logical



reasoning, problem-solving, and the practical implementation of mathematical principles (Bansilal, 2015; Black & Wiliam, 1988). Though this strategy may produce instant improvements in pass rates, it fails to sufficiently prepare learners with the essential skills and assurance needed to do well in a progressively knowledge-driven economy (Boaler, 2016; Venkat & Spaul, 2015).

The South African education system continues to be deeply impacted by historical inequalities stemming from apartheid, which persist to mould discrepancies in access to quality education (Kurtz et al., 2021; Ramoroka et al., 2024; Taylor & Vinjevo, 1999). Regardless of post-apartheid reforms, challenges like unbalanced resource distribution, infrastructure shortages, and variability in teacher quality continue, particularly in underprivileged communities (Fleisch, 2008; Spaul, 2013). These discrepancies contribute to remarkable performance gaps between learners in under-sourced rural and township schools and well-sourced schools. Although recent years have shown improvements in mathematics pass rates, a more concerning and persistent issue is the declining proportion of learners enrolling for mathematics National Senior Certificate examinations. According to the Department of Basic Education (2024)'s diagnostic report, Book 1, only 36,2% of matriculates wrote mathematics in 2024, compared to 40,6% in 2020, 37,0% in 2021, 37,5% in 2022, and 38,3% in 2023. This decline signals a shift in learner subject choices, with a growing number opting for mathematical literacy, often perceived as an easier pathway to achieving a matric certificate.

This trend raises critical questions about the current structure and philosophy of mathematics education in South Africa, particularly in the grades-first approach. This approach typically prioritises academic progression and curriculum coverage based on grade level, often at the expense of foundational understanding, learner readiness, and long-term competence. In doing so, it risks marginalising learners who struggle with the pace or abstract nature of the curriculum especially those from historically disadvantaged and under-resourced communities.

The growing disengagement from mathematics suggests that many learners may not feel adequately supported or confident to succeed within this rigid, performance driven system. It underscores the urgent need to reconsider how success is defined in mathematics education, not solely through grades or examination results, but through deeper learner engagement, conceptual understanding, and real-world application.

The repercussions of a grades-focused model are distinctly severe for learners in marginalised communities, where limited opportunities to educational support worsens the consequences of high-stakes assessments (Mji & Makgato, 2006; Meeran, 2024). In these settings, learners are often evaluated primarily through summative examinations, despite lacking the foundational resources and support structures needed to succeed. This mismatch creates intense pressure to perform without the necessary scaffolding, leading to heightened anxiety, reduced self-efficacy, and eventual disengagement from mathematics (Graven, 2014; Maluleke, 2019). The constant emphasis on examination outcomes over learning processes undermines learners' confidence and reinforces negative mathematical identities. Moreover, the grades-first approach frequently neglects the cultural and contextual relevance of mathematics, disrupting



learners from realising its importance in their daily lives and communities (Mhlolo & Schafer, 2012; Rosa & Orey, 2020). As a result, the system not only limits learners' academic progression but also impedes their personal and cognitive development.

Globally, there is increasing recognition of the necessity to redefine success in mathematics education. Competency-based learning, culturally responsive teaching, and problem-based learning have emerged as feasible choices to the conventional grades-first model (Boaler, 2016; Gutstein, 2006). These approaches accentuate the development of higher-order thinking skills, meaningful problem-solving experiences, and creativity, stimulating deeper engagement with mathematics. Nonetheless, their incorporation into the South African education system remains insubstantial due to systematic challenges, insufficient alignment with local contexts, and rigid assessment structures (Adler & Pillay, 2017; Moleko & Maphalala, 2015). Whilst alternative pedagogical frameworks have been proposed globally, few studies have systematically synthesised their implications for mathematics success and learner empowerment in the South African context. Existing research often focus on performance outcomes or policy critique in isolation. Therefore, this study fills a critical gap by integrating international and logical insights to critically re-evaluate how mathematics success is defined and pursued in schools, particularly under the influence of the grades-first paradigm.

This systematic review critiques the grades-first approach through examining different frameworks for defining and attaining success in mathematics education, with emphasis on empowering learners in South African high schools. By integrating insights from both local and global literature, this study examines how shifting from a grades-centric model to one focused on holistic learner development can assist with persistent educational inconsistencies. The review as well underscores the critical role of educators, policy designers, and communities in promoting an inclusive, and empowering mathematics education system.

Whilst the South African education system is dominated by grades-first approach that prioritises academic performance over deeper competencies, this raises concerns about its impact on learner empowerment. The study seeks to answer the following research questions as a way of challenging this paradigm:

1. What are the main challenges posed by the grades-first approach in mathematics education, particularly regarding learner empowerment in South African schools?
2. What educational frameworks or pedagogical approaches have been proposed to promote problem-solving, critical thinking, and learner engagement?
3. How do cultural and socio-economic factors influence the implementation of learner-empowering approaches in mathematics classrooms?

The interrogation of these questions in this study, aims to redefine success in mathematics education prioritising relevance, equity, and learner agency amidst systematic barriers.



THEORETICAL AND CONCEPTUAL FRAMEWORKS

The study embedded in two theoretical frameworks which are: social constructivism (Vygotsky, 1978) and self-determination theory (Deci & Ryan, 2013), coupled with two conceptual frameworks: culturally responsive pedagogy (Gay, 2010) and competency-based education (Chappell et al., 2020).

Social constructivism provides a foundational lens by viewing mathematical knowledge as socially constructed through interaction, language, and cultural context. This framework is especially relevant to South African classrooms, where learners come from diverse socio-cultural backgrounds. It challenges the grades-first model, which often detaches mathematics learners' lived realities, by advocating for collaborative, culturally situated learning experiences that promote deeper conceptual understanding.

Self-determination theory contributes by emphasising the psychological needs of learners, autonomy, competence, and relatedness as key drivers of motivation and effective learning. Within the context of this review, the theory is used to critique how traditional grading and assessment practices may suppress learner agency and intrinsic motivation. Instead, it supports the shift towards models that promote learner empowerment, confidence, and sustained engagement with mathematics.

Complementing these, culturally responsive pedagogy ensures that instructional methods and content are inclusive, affirming the identities, experiences, and languages of diverse learner populations. This pedagogy counters the depersonalised nature of standardised curricula, making mathematics more accessible and meaningful.

Finally, competence-based education redefines success by focusing not on grades or test scores, but on mastery of skills, understanding, and application. It aligns with the review's critique of the grades-first approach by prioritising personalised learning, formative assessment, and progress at the learner's own pace.

Together, these frameworks inform the study's analysis and guide the identification of educational models that empower learners, promote equity, and support meaningful success in mathematics education.

METHODOLOGY

This study engages a structured literature review to examine key themes in local and international research related to success in mathematics education and learner empowerment in South African schools. A systematic review allows for the identification, appraisal, and integration of findings from existing literature in a transparent and replicable manner (Fan et al., 2022; Pati & Lorusso, 2018; Siddaway et al., 2019). This approach is suitable for informing education policy and practice through evidence-based insights (Davies, 2000; Pellegrini & Vivanet, 2021).



Data selection process

A comprehensive search was conducted using Web of Science, Scopus, and ERIC academic databases. These were selected to ensure broad coverage of both high-impact international literature and regionally relevant studies (Jefferson et al., 2024). The search strategy was guided by these key words: grades-first approach in education, mathematics education in South Africa, mathematics success and identity, constructivist mathematics teaching, culturally responsive mathematics education, learner empowerment in mathematics, and alternative assessment in mathematics education. To refine search results, Boolean operators and/or were utilised to ensure rigorous selection of studies.

Selection process

To start with the selection process, 335 studies were identified. Following the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) approach (Moher et al., 2009; Vrubel, 2015), duplicates and studies that had irrelevant titles were removed, and 93 full text articles were assessed for eligibility. This resulted in 16 studies left based on their alignment with methodology rigour, research questions and relevance to the South African education system.

Inclusion and exclusion

To maintain relevance, transparency, and replicability in study selection, inclusion and exclusion criteria were developed in line with recommendations by Atkinson et al. (2015). The inclusion criteria involved journal peer reviewed articles, conference papers, and policy reports published from 1998 to date; studies addressing mathematics education in South Africa and comparable international contexts, and research exploring frameworks to the grade-first approach, including learner-centred pedagogy, competency-based learning, and formative assessment. It also considered empirical studies, theoretical papers and systematic reviews relevant to the topic. The studies that had the following characteristics were removed: did not focus solely on school level education, lacked empirical or theoretical relevance to the research questions, solely focused on traditional grading systems without discussing alternative approaches, and not available in English.

Data extraction and analysis

A data extraction framework was developed to systematically classify and assess each study based on key features, such as author(s), year, educational level, country, methodology, sample, key findings, and empowerment strategies (Jonnalagadda, 2015). Thematic analysis was then employed to identify cross-cutting patterns and gaps. A narrative synthesis was used to integrate findings across studies (Cruzes et al., 2014), focussing on themes of learner empowerment, equity, and contextual relevance in mathematics education.

Quality Assurance and ethical considerations

Based on the defined inclusion and exclusion criteria, a total of 16 studies with 11 of the studies



having been carried out in South Africa and 5 being international studies were thematically analysed. For quality assurance, each study was appraised using a modified version of the Critical Appraisal Skills Programme (CASP) to ensure their reliability and credibility. Whilst the study did not involve direct participants, ethical guidelines such as accurate representation of sources and fair interpretation were observed (Booth, 2016 Jacobsen et al., 2020; Stommel & Rijk, 2021).

Study limitations

Whilst this systematic review provides valuable insights into the shortcomings of the grades-first approach and alternative frameworks for mathematics education in South Africa, there are several limitations to consider. First, the reliance on published studies means that the findings are subject to the availability and quality of existing research, which may not fully represent all educational contexts within South Africa, particularly rural or under-resourced areas. Additionally, the scope of the review was limited to studies in English potentially excluding relevant research published in other languages (Benzies et al., 2006; Conn et al., 2003).

Moreover, as a literature-based study, it does not include primary data collection, which limits the ability to capture current classroom dynamics or teacher and learner perspectives directly. The heterogeneity of the reviewed studies in terms of methodologies, participant demographics, and educational settings also poses challenges in drawing definitive conclusions applicable across the entire South African education system.

Despite these limitations, the review highlights critical themes and gaps in the current discourse, providing a foundation for future empirical; research and policy development aimed at transforming mathematics education towards a more equitable and empowering model. In addition to that, publication bias may have influenced the range of available perspectives.

RESULTS

The systematic review identified two overarching categories that reflect the scope of the research questions: 1. The challenges posed by the grades-first approach to mathematics education, and 2. The proposed resolutions informed by learner empowerment and inclusive pedagogical models. Four dominant themes emerged within these categories: (i) the impact of traditional assessment on learner identity, (ii) misalignment between summative and continuous assessment, (iii) the role of metacognition on mathematics learning, and (iv) the significance of culturally responsive pedagogy. These themes respond directly to the three research questions and reflect both the limitations of current practices and potential alternative frameworks for mathematics success in South African schools.

Category 1: The challenges posed by the grades-first approach to mathematics education

Impact of traditional assessment on learner identity

Several studies including those by Spaul and Kotze (2015) and Meeran (2024) highlight the damaging effects of performance-driven assessment practices on learners' self-perceptions and



engagement in mathematics. High-stakes testing often increases anxiety, particularly among learners from underprivileged backgrounds, reinforcing a sense of failure and diminishing motivation. These outcomes are consistent with the findings of Black and Wiliam (1998), who argue that summative assessments often fail to capture the breadth of learners' mathematical abilities, contributing to narrow and exclusionary conceptions of success.

For instance, learners who demonstrate deep conceptual understanding during classroom discussions may still underperform in standardised tests due to language barriers or test anxiety. This undermines their mathematical identity and confidence. Sibanda and Rambuda (2021) also observe that learners from under-resourced schools are disproportionately disadvantaged by uniform assessment models that do not consider diverse learning contexts or valid informal mathematical reasoning.

Misalignment between summative and continuous assessment

Findings by Mhlolo and Schafer (2012) reveal a persistent misalignment between summative and continuous assessments in South African mathematics classrooms. Summative assessments often dominate perceptions of success, despite failing to reflect learners' ongoing conceptual development. This misalignment restricts the ability of assessments to inform instructional decisions or support learners' progress meaningfully.

For instance, an end of term test focused on applying analytical geometry formulas in isolation might miss the applied skills demonstrated through continuous assessment tasks such as designing blueprints or solving real-world measurement problems. Van Staden and Motsamai (2017) assert that when continuous assessment is aligned with authentic tasks and learner experiences, it better supports conceptual retention and growth while offering teachers more actionable data.

Category 2: Proposed resolutions for a learner-centred approach

The role of metacognition on mathematics learning

A growing body of research for example Boaler (2011), Yeo et al. (2023), and Moleko and Maphalala (2015), underscores the importance of metacognitive strategies in fostering deeper mathematical understanding. These studies assert that encouraging learners to monitor and reflect on their thinking processes such as the effectiveness of different solution methods or explaining their reasoning promotes long-term competence and learner autonomy.

In the context of algebra, for example, learners are empowered when they are not only asked to solve algebraic equations but also to justify their procedural choices, identify common errors, and compare strategies. This metacognitive engagement shifts the learning focus from correct answers to conceptual growth and self-regulated learning moving from a narrow, grades-first definition of success.



Culturally responsive pedagogy in mathematics education

Studies by Jojo (2024), Maluleke (2019), and Wager (2012) underscore the benefits of embedding cultural and contextual knowledge into mathematics education. For instance, when learners encounter mathematical concepts through familiar real-life scenarios such as using local markets prices or traditional measurement systems they are more likely to engage meaningfully and understand abstract content.

This approach not only demystifies mathematics but also challenges the dominance of decontextualized assessments that fail to reflect learners' lived realities. Mulaudzi (2024) further argues that acknowledging Indigenous knowledge systems promotes learners' confidence and validates their cultural identities, particularly in rural and marginalised schools. These pedagogies advocate for mathematics education that values relevance, inclusivity, and learner voice.

Linking findings to the research questions

The themes align with and directly respond to the study's research questions. The negative effects of grades-first assessment addresses Research Question 1, which interrogates how current success definitions affect performance and empowerment. The emergence of metacognition, continuous assessment, and culturally responsive teaching as transformative tools aligns with Research Question 2, which explores alternative pedagogical and assessment frameworks that move beyond performance metrics. Finally, the emphasis on the role of cultural and socio-economic context as a foundation for learner success speaks to Research Question 3, which examines the contextual influences shaping equitable and meaningful mathematics education in South Africa. These insights underscore the need to redefine mathematics education around learner empowerment, identity development, and contextual relevance, moving decisively away from a narrow grades-first model.

DISCUSSION

The findings of this systematic review point to an urgent need to challenge the entrenched grades-first paradigm that currently dominates South African mathematics education. The research highlights how traditional assessment practices not only limit the scope of mathematical success but also contribute to inequities in learner outcomes and disengagement. In response, the study identifies the potential of alternative pedagogical and assessment approaches particularly those rooted in formative assessment, metacognition, and culturally responsive teaching to support a more equitable and empowering model of mathematics education.

Moving beyond a performance-driven culture

The predominance of summative, high-stakes assessment has shaped a culture where grades are prioritised over understanding. In mathematics education, this often manifests as an overemphasis on procedural fluency at the expense of deep conceptual engagement. The



reviewed studies consistently recommend formative assessment strategies that promote continuous feedback, learner reflection, and opportunities for growth. Practices such as peer review, open-ended questioning, and error analysis create safe spaces for learners to make and learn from mistakes, fostering a growth mind-set and reducing mathematics anxiety. These shifts not only enhance understanding but also restore learner curiosity and agency in the classroom, especially among those previously disempowered by pressure to perform.

Rethinking Success in mathematics

Redefining success requires a paradigm shift from grades-based achievement to a broader focus on mathematical empowerment. A recurring theme in the literature is the power of real-world context-based applications such as budgeting exercises, community-based measurements, or local data analysis to increase learner engagement and conceptual relevance. This approach encourages learners to make meaningful connections between mathematics and their everyday lives. In line with Boaler (2011) and Moleko and Maphalala ((2015), the review affirms that diverse instructional practices like collaborative problem-solving and inquiry-based learning are key to fostering creativity, resilience, and long-term success in mathematics. Within this reimagined framework, success is not simply about scores, but by learners' ability to understand, apply, and adapt mathematical knowledge in varied contexts.

Equity and contextual considerations

The literature underscores the structural disadvantages faced by learners in under-resourced schools, where access to learning materials, support systems, and conducive environments is limited. The grades-first approach disproportionately penalises these learners, perpetuating cycles of underperformance and exclusion. Scholars such as Maluleke (2019), and Vandeyar and Catalano (2020) call for teaching strategies that are both inclusive and context-sensitive. Scaffolded learning tasks, the integration of learners' home languages, and culturally relevant examples provide multiple access points to mathematics content and help close persistent learning gaps. These approaches affirm learners' identities and experiences, offering a more just and responsive mathematics education.

SUMMARY OF FINDINGS

This systematic review's findings highlight the short-comings of the grades-first approach in South African mathematics education. Studies by Black and Wiliam (1998), Boaler (2011) and Spaul and Kotze (2015) highlight how this model promotes rote memorisation, reinforces inequities in learning outcomes, and undermines the development of meaningful mathematical understanding and problem-solving skills. In schools that are under-resourced, where learners already experience substantial educational disadvantages, the grades-first approach increasingly solidifies educational inequalities (Meeran, 2024).

The psychological impact on grades-focused assessments is one of the identified central issues. According to Maluleke (2019), the pressure to achieve high scores has a contribution on test anxiety and reduced self-efficacy, ultimately discouraging persistent engagement with



mathematics. Black and Wiliam (1998) further note that summative assessments in most cases fail to deliver meaningful feedback that foster learner growth, resulting in a cycle of disengagement and underperformance. Conversely, formative strategies such as continuous assessment and reflective peer feedback have been seen to promote deeper understanding and learner motivation (Graham, 2023).

A pivotal finding that emerged from the review is the role of alternative assessment models in promoting meaningful mathematics experiences (Boaler, 2016). Research by Mhlolo and Schafer (2012) and Van Staden and Motsamai (2017) show that problem-based learning, open-ended tasks, and competency-based approaches foster higher-order thinking and encourage mastery over superficial achievement. The practices are especially effective when integrated with culturally responsive pedagogy, which connects learning to learners' lived experiences and linguistic backgrounds.

Culturally grounded teaching strategies emerged as particularly significant in the South African context. Research by Mulaudzi (2024) and Meeran (2024) pinpoint how mathematics instruction that integrates real-world contexts and learners' lived experiences promotes comprehension and motivation. However, challenges continue to exist in implementing these approaches because of rigid curriculum structures and dominance of standardised assessment requirements (Moleko & Maphalala, 2015).

The review also highlighted the need for investment in professional development and teacher training as important factors in shifting towards a more learner-centred approach. Study by Meeran (2024) indicates that many South African educators grapple to balance the demands of high-stakes assessments with innovative teaching practices. Studies by Jojo (2024) and Malema (2024) emphasise the need to empower educators with resources, training, and autonomy to execute alternative instructional strategies as a necessity for meaningful reform. Without systemic change, even the most promising pedagogical models will struggle to take root.

Overall, the findings of this study advocate for a shift away from a rigid, grades-first assessment system towards a holistic learner-centred, and contextually grounded framework for mathematics education. Such a transformation holds the potential to bridge gaps in learning, promote conceptual understanding, and prepare learners for meaningful mathematical engagement both inside and beyond the classroom. Future research should explore long-term impacts of alternative assessment models and investigate how digital technologies can support equitable, locally relevant mathematics instruction in South African schools (Graham, 2023).

IMPLICATIONS FOR POLICY AND PRACTICE

The findings of this systematic review critiques the entrenched grades-first paradigm that continues to dominate South African mathematics education. The evidence underscores that performance-driven assessment models contribute to test anxiety, reduced learner confidence, disengagement, and the reinforcement of existing inequalities, particularly in under-resourced schools. By focussing narrowly on numerical outcomes, these models marginalise learners who



may otherwise thrive under more inclusive and holistic teaching strategies.

In contrast, the review highlights the value of alternative frameworks that prioritise deep conceptual understanding, metacognition, and real-world application. Such approaches not only address the limitations of traditional assessment but also promote equity, learner agency, and mathematical resilience. The review's implications point to the need for systemic changes in both curriculum and instructional practice, including reforming assessment policies to prioritise formative, feedback-rich evaluations, embedding contextually relevant and authentic mathematical tasks into the curriculum, promoting inclusive pedagogies that reflect learners' lived experiences and cultural identities, and supporting teachers to adopt more flexible, learner-centred methodologies.

RECOMMENDATIONS

To realise these shifts in mathematics education, the following recommendations have been proposed:

1. For curriculum transformation, policymakers should prioritise curriculum reforms that move beyond procedural fluency to include reasoning, problem-solving, and communication. Learning objectives must support conceptual depth and authentic application.
2. In terms of professional development, ongoing, contextually responsive teacher training is crucial. Teachers should be empowered with the tools, confidence, and autonomy to implement alternative assessments and culturally relevant pedagogies.
3. As far as resource allocation is concerned, addressing disparities in infrastructure and resource availability, particularly in rural and disadvantaged schools, is essential. Equitable access to teaching materials, digital tools, and learning support systems must be a national priority.
4. In terms of stakeholder engagement, curriculum designers, school leaders, and policymakers must work collaboratively with teachers and communities to ensure that reforms are contextually grounded and sustainable.
5. Lastly, for research and innovation, continued research is needed to evaluate the long-term impact of alternative success frameworks. Particular attention should be paid to how these models function in underrepresented and rural contexts, and how digital tools can support their effective implementation.

CONCLUSION

This study contributes to the growing body of literature advocating for the redefinition of mathematics success in South African mathematics education. It reinforces the importance of moving away from narrow, grades-focused models and embracing inclusive, competency-based approaches that prioritise understanding, equity, and learner empowerment. When success in mathematics is understood as the ability to think critically, reason flexibly, and apply



knowledge meaningfully across contexts, it becomes a transformative force capable of challenging systemic inequalities and unlocking lifelong opportunities for all learners.

By valuing diverse ways of knowing and broadening what counts as success, South African mathematics education can better reflect its democratic ideals and prepare learners not just for examinations, but for active participation in an increasingly complex and dynamic world.

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INTEGRATING TECHNOLOGY TO SIMPLIFY COMPLEX GEOMETRIC CONCEPTS IN MATHEMATICS LECTURING

¹Lokesh Ramnath Maharajh and ²Hlengiwe Mhlungu

¹University of KwaZulu-Natal

²North West University

This paper explores the integration of technology to simplify complex geometric concepts in mathematics, focusing on circle geometry in Technical and Vocational Education and Training (TVET) colleges. Circle geometry is often challenging due to its abstract nature, requiring strong visualization and deductive reasoning. Traditional teaching methods often fall short, but the use of digital tools such as interactive geometry software, visual simulations, and online platforms offers a promising solution. The paper examines the integration of these technologies, drawing on the Technological Pedagogical Content Knowledge (TPACK) framework to highlight the role of content, pedagogical, and technological knowledge in effective teaching. Various teaching strategies, including the use of GeoGebra, interactive whiteboards, and collaborative learning, are discussed as ways to enhance student engagement and understanding. However, challenges such as limited digital infrastructure, lack of professional development, and lecturers' technological competence hinder the full potential of technology integration. Solutions include expanding institutional support, providing targeted professional development, and adopting a blended learning approach. The paper concludes by emphasizing the importance of addressing these barriers to maximize the benefits of technology in mathematics education and recommends future research on the effectiveness of specific digital tools and professional development programs.

Keywords: Technology Integration, TVET Education, Circle geometry, GeoGebra, Technology, TPACK.

INTRODUCTION

This paper reports on PhD research that explored mathematics lecturers' use of technology in teaching circle geometry to level 4 students at a Technical and Vocational Education and Training (TVET) College in the Majuba District, KwaZulu-Natal, South Africa. The study aimed to understand "what" technology lecturers use and the reasons behind their use of technology in teaching circle geometry. In South Africa, mathematics teaching faces numerous challenges, including poor learner performance (Chimuka, 2017) and various contextual factors such as teachers' practices and school resources (Mji & Makgato, 2006). Rural areas, in particular, face challenges like unqualified teachers and inadequate resources (Mulkeen, 2005). Research by Arends, Winnaar, and Mosimege (2017) highlighted the significant impact of teachers' classroom practices on learner performance.

Since 2016, fewer than 50% of matric final exam learners have taken mathematics, with performance consistently lower than other subjects (Letsoalo et al., 2019). In 2018, only 21.7% of mathematics learners achieved 50% or higher, and performance remained between 30% and 35%. Mathematics literacy has also seen a decline in both participation and pass rates from



2017 to 2018. Geometry, a crucial component of mathematics, has not been adequately addressed in teaching, especially after it was downgraded to an optional subject in the National Curriculum Statement (Ma & Johnson, 2008). Although geometry was reinstated as a compulsory part of the CAPS curriculum (Govender, 2016), challenges persist, particularly in TVET Colleges where geometry is only introduced at level 4 (Ngoveni & Mofolo-Mbokane, 2019). This has contributed to poor performance, such as the 15% and 19% pass rates in level 4 mathematics by mechanical engineering students in 2017 (Ngwato, 2020).

Technology use in education, especially in the Fourth Industrial Revolution (4IR), has become essential, yet TVET lecturers often use technology more for socializing than for teaching. Despite this, the use of technology, such as WhatsApp, during the COVID-19 pandemic demonstrated its potential in educational settings. Studies by Godebo (2018) and Govender (2016) show that technology enhances the teaching and learning of geometry. However, challenges such as access to technology and electricity persist, exacerbating educational inequalities. This qualitative study involving five mathematics lecturers aimed to explore how technology is integrated into teaching circle geometry, using questionnaires and semi-structured interviews for data collection. In particular, this paper focuses on TVET lecturers' integration of technology to simplify complex geometric concepts in Mathematics.

Technology plays a vital role in education, but there are difficulties in integrating it with mathematics teaching, particularly in under-resourced settings. Teachers face challenges using both technology and mathematics content effectively (Koekemoer, 2021), and the need for lecturers to be equipped with technology-based teaching skills is urgent (Naidoo, 2020). The COVID-19 pandemic exposed inequalities in access to technology between private and state schools, with many TVET students lacking reliable internet and Wi-Fi for online learning (Mesuwini, 2024). The urgent need for technology in education is evident to bridge the gap in learning opportunities.

METHODOLOGY

This study reports on how lecturers use technology to teach geometry to level 4 TVET students, adopting a qualitative approach to gather textual data. According to Guba and Lincoln (2005), qualitative research emphasizes the socially constructed nature of reality, with researchers actively engaging with the participants. Creswell (2021) further explains that qualitative research involves understanding the meanings individuals or groups ascribe to social problems through emerging questions and inductive data analysis. The research involved one-on-one, semi-structured interviews with mathematics lecturers in their preferred settings to facilitate open and comfortable interactions. De Vaus (2004) and Zhou & Creswell (2012) emphasize that participants' attitudes are evident in qualitative research, and this was observed during the data generation.

Denscombe (2017) supports the use of qualitative methods in behavioural and social fields, making it ideal for this study, which involved semi-structured interviews. The research is aligned with Check and Schutt's (2012) view that qualitative research includes action research, case studies, and ethnography, with the case study method being used here. The study adopts



an interpretive paradigm, which focuses on understanding how participants make sense of their world (Cohen, Manion, & Morrison, 2007). The interpretive paradigm allows for the creation of knowledge through the participants' perspectives, as described by Scotland (2012). Despite some criticisms of interpretivism being subjective (Cohen et al., 2013), the study mitigated these concerns by allowing participants to verify the generated data.

This paper reports on the one-on-one semi-structured interviews conducted to explore the use of technology in teaching circle geometry at a TVET College. The one-on-one semi-structured interviews were conducted to generate more detailed data. Ramrathan (2017) asserts that this method allows the researcher to guide the conversation and probe deeper into participants' views. The interviews followed Creswell and Creswell's (2017) guidelines to ensure effective data generation, emphasizing the importance of participant understanding and researcher listening skills. Menjívar (2000) suggests in-depth interviews are necessary to capture personal experiences. Each interview was scheduled at a mutually convenient time, and participants were informed about the study's objectives and the nature of the questions. Open-ended questions were used to avoid restricting responses, ensuring a comprehensive understanding of lecturers' use of technology in teaching geometry. The interviews were recorded, and notes were taken for transcription, facilitating the analysis of responses. Confidentiality, anonymity, and voluntary participation were also emphasized, with participants informed that they could withdraw at any time (Christiansen et al., 2018).

The Role of Technology in Teaching Circle Geometry

The integration of technology in mathematics education has significantly transformed the teaching and learning of complex geometric concepts, particularly within Technical and Vocational Education and Training (TVET) colleges. Dynamic digital technologies (DDTs) have the potential to enhance learning by addressing issues such as lack of interest and motivation among students (Ntshangase et al., 2024). A study by Ntshangase, Ndlovu, and Oladele (2024) demonstrated that using GeoGebra software in teaching circle geometry to Grade 11 learners resulted in statistically significant improvements in academic performance and increased enjoyment of mathematics.

Circle geometry is often considered one of the most challenging areas for students due to its abstract nature and the necessity for strong visualization and deductive reasoning skills. According to research by Kwadwo and Asomani (2021), circle theorems are among the most difficult topics in geometry, contributing to poor student performance in mathematics.

Traditional teaching methods frequently fall short in making these abstract concepts accessible, leading to persistent struggles in comprehension and application. The use of dynamic geometry software, such as GeoGebra, allows students to visualize and manipulate geometric figures interactively, thereby enhancing their understanding and engagement (Ntshangase et al., 2024). Ntshangase et al. (2024) found that learners taught with GeoGebra performed better academically in circle geometry and reported greater enjoyment in learning mathematics compared to those taught using traditional methods.



Integrating digital tools like interactive geometry software, visual simulations, and online platforms offers a promising approach to bridging this gap. These technologies make abstract mathematical principles more tangible, thereby enhancing student engagement and facilitating deeper conceptual understanding. The study by Motseki and Jojo (2024) supports this, indicating that the use of The Geometer's Sketchpad significantly improved TVET college students' understanding of circle geometry, fostering cooperative and collaborative learning.

Despite its potential benefits, the effective integration of technology in teaching circle geometry is influenced by various factors, including lecturers' technological competence, institutional infrastructure, and the availability of professional development opportunities (Niess, 2008). This paper explores how mathematics lecturers integrate technology to simplify complex geometric concepts, examining both the strategies employed and the challenges encountered in this process. Using the Technological Pedagogical Content Knowledge (TPACK) framework, the discussion highlights the role of content knowledge (CK), pedagogical knowledge (PK), and technological knowledge (TK) in effective teaching (Mishra & Koehler, 2006).

THEORETICAL FRAMEWORK: TPACK AND PEDAGOGICAL INTEGRATION

The TPACK framework provides a lens through which the integration of technology in mathematics education can be understood. This model underscores the interconnectedness of three critical knowledge domains: content knowledge (CK), pedagogical knowledge (PK), and technological knowledge (TK) (Xie & Luo, 2025). Mathematics lecturers must not only possess a deep understanding of geometric concepts but also develop the pedagogical expertise required to deliver these concepts effectively. Moreover, the incorporation of technology necessitates a working knowledge of digital tools that can enhance instructional delivery.

In the context of circle geometry, lecturers must employ pedagogical content knowledge (PCK) to present theorems, properties, and applications in ways that cater to diverse student learning styles. Simultaneously, technological pedagogical knowledge (TPK) enables lecturers to select and implement appropriate digital tools, ensuring that technology supports rather than complicates the learning process. The integration of these knowledge domains is crucial in overcoming common learning barriers associated with circle geometry (Khalil & Osman, 2022).

FINDINGS

Teaching Strategies and Technology Integration

Mathematics lecturers employ various strategies to enhance student comprehension of circle geometry, leveraging technological tools to facilitate the learning process (Mahmoud, 2023). Among the most commonly utilized resources are GeoGebra, Blackboard, interactive whiteboards, PowerPoint presentations, and Learnerscape. These tools enable students to manipulate geometric figures dynamically, offering a visual and interactive learning experience that enhances their conceptual grasp (Farrajallah, 2016).



Visualization and Conceptual Learning play a critical role in simplifying geometric principles. GeoGebra, for instance, allows students to construct and explore geometric relationships dynamically, reinforcing their understanding of theorems and their applications (Pfeiffer, 2017). This digital interactivity bridges the gap between theoretical concepts and real-world applications, making abstract ideas more concrete.

Collaborative Learning and Group Work further support the integration of technology. Many lecturers adopt cooperative learning strategies in which students work in groups to explore geometric problems using digital tools (Woods, 2017). This approach fosters peer interaction and knowledge-sharing, encouraging students to develop a deeper conceptual understanding through discussion and collaborative problem-solving.

Additionally, scaffolding, and real-world applications enhance students' ability to connect theoretical knowledge with practical scenarios (Schmid & Korenova, 2024). Lecturers often incorporate digital models, animations, and physical representations to illustrate circle theorems, ensuring that students grasp the logical progression of geometric reasoning. By linking abstract concepts to tangible experiences, lecturers help students develop a more intuitive understanding of geometry (Hoadley & Jansen, 2013).

Challenges in Integrating Technology

Despite the advantages of technological integration, several challenges hinder its full implementation in TVET colleges. One of the most pressing issues is institutional barriers, including limited access to digital infrastructure, unreliable internet connectivity, and insufficient technological resources (Ralushai, 2021). Many TVET colleges struggle to provide the necessary devices and software required for effective digital instruction, restricting lecturers' ability to integrate technology seamlessly into their lessons.

Another significant challenge is the limited professional development opportunities available for lecturers. Many educators acknowledge the potential of technology but lack the training required to implement it effectively (Mukiri, 2016 &). Continuous professional development programs that focus on enhancing technological proficiency are essential to addressing this gap (Hamzah, Abdullah, & Ma, 2024).

Furthermore, lecturers' attitudes and technological competence play a crucial role in determining the extent to which technology is incorporated into teaching. Some lecturers hesitate to adopt digital tools due to unfamiliarity or a lack of confidence in their effectiveness. Without adequate training and institutional support, the full potential of technology in mathematics instruction remains underutilized (Theron, 2025).

Time constraints and assessment-related challenges further complicate the integration of technology. Many lecturers struggle to balance the demands of covering the syllabus with the additional time required to incorporate technology-based learning strategies. Moreover, assessment practices in mathematics education often remain traditional, limiting the extent to



which digital tools are used to evaluate student progress (Borba, Askar, Engelbrecht, Gadanidis, & Llinares, 2016).

Bridging the Gap: Solutions and Recommendations

To overcome these challenges and maximize the benefits of technology in mathematics instruction, several strategies must be implemented. Expanding institutional support through increased investment in digital infrastructure, software licensing, and reliable internet connectivity is essential. By providing lecturers with the necessary tools and resources, institutions can facilitate more effective technology integration (Yang & Wu, 2024).

Enhancing lecturer training through structured workshops and professional development programs is equally important. Alrwaished et al. (2017) states that training initiatives should focus on equipping educators with the skills and confidence needed to incorporate digital tools into their teaching practices. Additionally, mentorship programs that pair technologically proficient lecturers with those less experienced can foster collaborative learning and skills development.

A blended learning approach, which combines traditional teaching methods with digital tools, offers a balanced and effective solution. By integrating technology gradually and strategically, lecturers can ensure that digital resources enhance rather than replace essential instructional techniques.

CONCLUSION AND FUTURE DIRECTIONS

The integration of technology in teaching circle geometry holds immense potential for enhancing student engagement and comprehension. While mathematics lecturers in TVET colleges demonstrate a willingness to adopt digital tools, significant challenges, including inadequate infrastructure, limited professional development, and institutional barriers, hinder full implementation. Addressing these challenges requires a concerted effort from policymakers, educational institutions, and lecturers to ensure that technology is effectively integrated into mathematics instruction.

Future research should focus on evaluating the effectiveness of specific technological tools in improving student performance in geometry. Additionally, further exploration of professional development strategies and institutional policies will be crucial in ensuring that technology integration is both sustainable and impactful. By adopting a strategic approach to technological integration, educators can bridge the gap between theoretical mathematics and practical application, ultimately improving student learning outcomes in TVET colleges.

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A CASE OF ANALYSING THE GRADE 12 DIAGNOSTIC REPORT AND DEMYSTIFYING RECOMMENDATIONS ON OPTIMISATION

Wisani Hlangwani¹, Miranda Moodley¹, Jacob Jaftha¹, Duncan Mhakure¹

¹University of Cape Town

This study critically examines the Grade 12 Diagnostic Reports on Optimisation in South Africa, highlighting key gaps and their implications for mathematics education. The analysis reveals a misalignment between identified learner difficulties and the recommendations provided, raising concerns about the reports' effectiveness in informing instructional practice. By drawing on empirical research, the study underscores the need for evidence-based strategies that foster deep conceptual understanding rather than relying on generic recommendations. The findings have significant implications for teachers, curriculum advisors, and teacher educators, emphasizing the importance of refining diagnostic reports to provide actionable, research-aligned guidance. This paper advocates for a more structured approach to bridging assessment analysis and classroom practice, ultimately enhancing the quality of mathematics education and improving learner outcomes in South Africa.

Keywords: Diagnostic Report, Optimisation, Grade 12 Exams, conceptual misunderstanding, relational understanding, mathematics education.

INTRODUCTION

This paper presents a documentary analysis of the South African Grade 12 Diagnostic Report focused on sections on Optimisation as published from 2020-2024. Optimisation is an abstract (critical thinking) concept that demands strong foundational skills requiring algebraic and relational understanding (Brijlall & Ndlovu, 2013). The complexity of optimisation contributes to persistent underperformance, as highlighted in successive Diagnostic Reports from 2020 to 2024. Similarly, assessment results' consistently show that South African learners perform below the international average on abstract concepts (Carnoy & Chisholm, 2008; Reddy, 2006). As a result, the South African mathematics curriculum emphasises that learners should engage with practical real-life problems involving abstract concepts through topics such as Optimisation (Department of Basic Education, 2011; Mhakure, 2020).

Although Optimisation is highlighted in the national curriculum, it is often allocated the lowest marks in final high-stakes examinations, overshadowed by other topics that require procedural knowledge. Despite its limited weight in final exams, it remains a consistently weak content area for learners. To improve the poor performance areas covered in Grade 12, the Department of Basic Education releases an annual Diagnostic Report that identifies challenges and suggestions for improvements. However, these reports are often dense and difficult to implement in time for the following year's assessment (Spaull, 2013; van der Berg et al., 2021). This paper presents a document analysis of the Diagnostic Report focusing on the section on Optimisation, published in 2020-2024. This paper aimed to determine if the recommended suggestions for improvements align with the challenges identified by the Diagnostic Report



and current research recommendations. To achieve this aim, the paper addressed the following research questions:

***RQ1:** To what extent does the challenges identified by the Diagnostic Report match the recommended suggestions for improvements proposed to address learners' conceptual misunderstanding of Optimisation?*

***RQ2:** To what extent does the recommended suggestions for improvements in the Diagnostic Report align with current research recommendations in improving learners' understanding of Optimisation?*

METHODOLOGY

This study employs a documentary analysis of the South African Grade 12 Diagnostic Reports on Optimisation from the November Exams published by the Department of Basic Education between 2020 and 2024. A documentary analysis approach was used to systematically review and interpret the Diagnostic Reports to determine if the recommended suggestions for improvements align with the challenges identified and current research recommendations (Bowen, 2009). The raw data from the Diagnostic Report was categorised into a table with the following headings: performance percentage, challenges identified, and recommended suggestions using a case-by-case approach. For each year, the challenges identified by the Diagnostic Report was compared to whether they correspond to the recommended suggestions for improvements – inconsistencies in the Diagnostic Reports were then documented for discussion. The study further compared the recommended suggestions for improvements against current research findings on the teaching and learning of optimisation.

FINDINGS AND DISCUSSION

In this section, the study presents challenges identified by the Grade 12 Diagnostic Report (2020 – 2024) and recommended suggestions for improvements on Optimisation questions. To address the research questions, this section is structured as follows: (1) an overview of the Report findings is presented; (2) followed by four subsections focusing on a case-by-case approach of the Diagnostic Report; and lastly, (3) a synthesis of the key findings of the study is presented. Each case is simultaneously discussed and analysed to assess the alignment between the recommended suggestions for improvements and the challenges identified by the Diagnostic Report. Current evidence-based recommendations from research findings are used to critique the current recommended strategies proposed by the Diagnostic Reports.

Overview of the Diagnostic Reports between 2020 – 2024

South African learners have consistently underperformed in Optimisation questions, with an average performance of approximately 25%. Performance declined sharply in 2021 and 2022, where learners scored below 10%, indicating severe challenges in conceptual understanding and problem-solving.



The Diagnostic Reports attribute this poor performance to learners' conceptual misunderstandings, particularly in algebraic manipulation (e.g. Table 1, item (a) and Table 4, item (b)) and the application of differentiation. (e.g., Table 4, item (a) and Table 1, item (b)). However, the absence of specific challenge identification in 2021 and 2022 raises concerns about the Reports' analytical depth. This overview provides a foundation for the case-by-case analysis that follows.

Case 1: Year 2024

Case 1 examines the findings from the 2024 Diagnostic Report on Optimisation, assessing whether the suggested recommendations effectively address the identified learner challenges. Figure 1 is presented to reference the exam question discussed by the Diagnostic Report. Table 1 below presents the extracted data from the Diagnostic Report, detailing the performance percentage, common challenges, and proposed suggestions from the Grade 12 November exam on Optimisation.

QUESTION 10

A cyclist rode from town P and stopped at town T. The speed (in km/h) at which this cyclist rode, is represented by the equation $s'(t) = -3t^2 + 18t$.

NOTE: Speed is the rate of change in distance with respect to time.

10.1 Calculate the maximum speed that the cyclist reached on this ride. (3)

10.2 Calculate the distance between town P and town T. (5)
[8]

Figure 1: Optimisation question from the Grade 12 November final exam in 2024 (page 8)

Table 1: Challenges and Recommendations for the Grade 12 November exam on Optimisation. Extracted from the Diagnostic Report (2024 p.2-9)

Performance Percentage	Challenges Identified	Recommended Suggestions
23%	(a) In Q10.1, most candidates equated the speed to 0 and solved. This led to $\begin{aligned}-3t^2 + 18t &= 0 \\ -3t(t - 6) &= 0 \\ t &= 0 \text{ or } t = 6\end{aligned}$ (b) Some candidates calculated the second derivative but did not explicitly set it equal to zero in Q10.1. (c) In Q10.2, most candidates did not recognise that the distance was represented by $s(t)$ which	(a) Calculus lends itself to many applications in optimisation and rates of change. Teachers need to expose learners to a wide variety of questions, which include integration of topics including rate of change, analytical geometry, measurement and trigonometry. (b) Rate of change must be taught explicitly as part of the optimisation and application of Calculus.



	is the function from which $s'(t)$ is derived. (d) In answering Q10.2 many candidates tried to use the concept of $speed = \frac{distance}{time}$ or $\Delta s = \frac{\Delta d}{\Delta t}$	(c) In all topics, reading for understanding should be ongoing if learners are to improve their responses to problems. (d) Learners need to be exposed to working from the derivative function to the original function using equating of coefficients.

In Table 1, learners exhibited confusion regarding the differentiation process in optimisation problems. The Diagnostic Report notes that many learners incorrectly equated the speed to zero instead of the second derivative when determining critical points. Similarly, learners struggled to recognize the relationship between the distance function and its derivative, as seen in Q10.2 (see Figure 1), where they failed to establish the function before differentiation.

The recommended suggestions—exposing learners to a variety of questions and integrating multiple mathematical topics—fail to address these specific conceptual misunderstandings. Research indicates that structured practice in reversing differentiation, coupled with scaffolding strategies that link differentiation to real-world applications, improves relational understanding (Hlangwani & Dhlamini, 2024; Mendezabal & Tindowen, 2018). Instead of broad exposure to different problems, explicit instructional interventions should focus on concept formation through active problem-solving techniques, such as the use of interactive technology and visualisation tools (Motseki & Luneta, 2024). Furthermore, as noted in Table 1, the recommendation to integrate multiple mathematical topics should be explicitly connected to structured worked examples demonstrating these relationships.

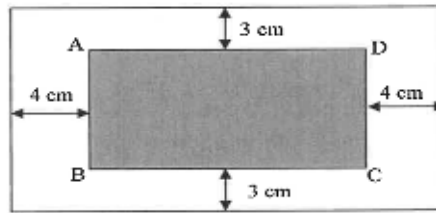
Case 2: Year 2023

Case 2 presents and evaluates the findings from the 2023 Diagnostic Report concerning learner difficulties in algebraic manipulation and over dependence on given formulas. Figure 2 is referenced as the question discussed from the Diagnostic Report. The focus is on determining whether the recommended interventions align with the identified challenges. Table 2 below presents the extracted data from the Diagnostic Report, including performance trends, observed challenges, and suggested improvements.



QUESTION 9

The diagram below represents a printed poster. Rectangle ABCD is the part on which the text is printed. This shaded area ABCD is 432 cm^2 and $AD = x \text{ cm}$. ABCD is 4 cm from the left and right edges of the page and 3 cm from the top and bottom of the page.



9.1 Show that the total area of the page is given by:

$$A(x) = \frac{3456}{x} + 6x + 480 \quad (3)$$

9.2 Determine the value of x such that the total area of the page is a minimum. (3)

[6]

Figure 2: Optimisation question from the Grade 12 November final exam in 2023 (page 8)

Table 2: Challenges and Recommendations for the Grade 12 November exam on Optimisation. Extracted from the Diagnostic Report (2023, p.221)

Performance Percentage	Challenges Identified	Recommended Suggestions
26%	(a) Many of the candidates did not attempt this question because they were unable to formulate the equation required in Q9.1. (b) Many candidates did not use the given equation in Q9.1 to answer Q9.2. (c) Some common errors included: - Candidates simplified the equation required in Q9.2 to $x^{-2} = \frac{1}{576}$ and then incorrectly concluded that $x = \frac{1}{24}$. - Candidates calculated that $x^2 = 576$ and then concluded that $x = -24$ rather than $x = 24$.	(a) Learners appear to be dependent on the formulae being given when solving optimisation problems. It is advisable that learners interrogate the optimum function even when it is given in a question. This should help their conceptual development. (b) Teachers should ensure that there is enough time for learners to understand the application of Calculus fully. (c) Reading for understanding should be ongoing if learners are to improve their responses to word problems.



The 2023 Diagnostic Report (see Table 2) identifies major challenges in algebraic manipulation and learners over dependence on provided formulas. Many learners were unable to derive required equations and often applied incorrect algebraic simplifications, leading to flawed conclusions, such as incorrectly simplifying expressions in Q9.2 (see Figure 2).

The Diagnostic Report suggests that learners should interrogate given functions, yet it does not offer strategies for improving algebraic reasoning. Studies on secondary mathematics instruction suggest that guided inquiry-based learning, where learners explore algebraic relationships through structured tasks, leads to better conceptual retention (Brijlall & Ndlovu, 2013). Additionally, scaffolding algebraic reasoning before calculus application has been shown to reduce reliance on rote memorisation (Motseki & Luneta, 2024). The recommendations should, therefore, focus on stepwise development of algebraic competence and incorporating error analysis activities that encourage metacognitive awareness. Given that Table 2 highlights difficulties in formulating equations, strategies such as progressive complexity in algebraic manipulations should be emphasised in teacher training programs.

Case 3: Years 2021 and 2022

Case 3 analyses the findings from the 2022 and 2021 Diagnostic Reports, which both provided minimal details on the challenges identified. The questions from the exam for both years are referenced below as Figure 3 and 4, respectively. The discussion explores whether the general recommendations made in these years were sufficient to address possible underlying issues. Table 3 below presents the extracted data from the Diagnostic Report for both years, 2022 and 2021, respectively.

QUESTION 9

Given $f(x) = x^2$.

Determine the minimum distance between the point $(10; 2)$ and a point on f .

[8]

Figure 3: *Optimisation question from the Grade 12 November final exam in 2022 (page 9)*

QUESTION 11

After travelling a distance of 20 km from home, a person suddenly remembers that he did not close a tap in his garden. He decides to turn around immediately and return home to close the tap.

The cost of the water, at the rate at which water is flowing out of the tap, is R1,60 per hour.

The cost of petrol is $\left(1,2 + \frac{x}{4000}\right)$ rands per km, where x is the average speed in km/h.

Calculate the average speed at which the person must travel home to keep his cost as low as possible.

[7]

Figure 4: *Optimisation question from the Grade 12 November final exam in 2021 (page 9)*



Table 3: Challenges and Recommendations for the Grade 12 November exam on Optimisation. Extracted from the Diagnostic Report (2022, p. 206 and 2021 p.196).

Performance Percentage	Challenges Identified	Recommended Suggestions
9%	Many of the candidates did not attempt this question because they were unable to determine the distance function from the given information. Some candidates used trial and error to obtain the minimum distance between the points. They were not awarded any marks.	(a) Learners appear to be dependent on the formulae being given when solving optimisation problems. It is advisable that learners interrogate the optimum function even when it is given in a question. This should help their conceptual development. (b) Teachers should ensure that there is enough time for learners to understand the application of Calculus fully. (c) Reading for understanding should be ongoing if learners are to improve their responses to word problems.
8%	Many of the candidates did not attempt this question because they were unable to derive the cost function from the given information.	(a) Learners appear to be dependent on the formulae being given when solving optimisation problems. It is advisable that learners interrogate the optimum function even when it is given in a question. This should help their conceptual development. (b) Teachers should ensure that there is enough time for learners to understand the application of Calculus fully. Reading for understanding should be ongoing if learners are to improve their responses to word problems.

Reports from 2022 and 2021 (see Table 3) provide minimal detail on learner difficulties. The primary challenge noted was the inability to determine distance or cost functions from given information, resulting in low attempt rates on optimisation questions. However, these assumptions become difficult to identify if learners did not attempt the question. The study argues that a high percentage of blank responses in Table 3 suggests a need for further sampling of scripts before making a list of challenges identified.



As noted in Table 3, it is evident that recommendations remain generic, with repeated emphasis on reading comprehension and time allocation for calculus instruction even though there were poor response rates to the questions. Empirical studies highlight that integrating dynamic software tools, such as GeoGebra, to visualise optimisation problems enhances learners' relational understanding and problem-solving ability (Hlangwani & Dhlamini, 2024; Mendezabal & Tindowen, 2018). The reports should advocate for technology integration in mathematics instruction to facilitate exploratory learning. Given that Table 3 shows low engagement with optimisation problems, an emphasis on formative assessment techniques to assess learners' step-by-step reasoning would be beneficial.

Case 4: Year 2020

Case 4 discusses the findings from the 2020 Diagnostic Report, focusing on learners' difficulties in distinguishing between volume and surface area formulas. The analysis evaluates whether the suggested recommendations effectively address these identified challenges. Figure 5 references the exam question discussed from the Diagnostic Report. Table 4 below presents the extracted data from the Diagnostic Report.

QUESTION 9

A closed rectangular box has to be constructed as follows:

- Dimensions: length (l), width (w) and height (h).
- The length (l) of the base has to be 3 times its width (w).
- The volume has to be 5 m^3 .

The material for the top and the bottom parts costs R15 per square metre and the material for the sides costs R6 per square metre.

9.1 Show that the cost to construct the box can be calculated by: $\text{Cost} = 90w^2 + 48wh$ (4)

9.2 Determine the width of the box such that the cost to build the box is a minimum. (6)
[10]

Figure 5: Optimisation question from the Grade 12 November final exam in 2020 (page 9)

Table 4: Challenges and Recommendations for the Grade 12 November exam on Optimisation. Extracted from the Diagnostic Report (2020, p. 193-194)

Performance Percentage	Challenges Identified	Recommended Suggestions
25%	(a) In Q9.1 many candidates used the formula for volume of a rectangular prism instead of the formula for surface area. Some candidates attempted to work backwards	The section on measurement, i.e. surface area and volume, is generally not well taught. Teachers should ensure that there is enough time for learners to



	from the answer but were unsuccessful in their attempts. (b) $C(w) = 90w^2 + 48wh$ as $C'(w) = 180w + 48h$. This was incorrect because: firstly, h is not a constant and secondly, h is a function in w, i.e. h is dependent on w. Candidates made mistakes in making h the subject of the formula in $V = l \times w \times h = 3w^2 = 5$. Instead of $h = \frac{5}{3w^2}$, they obtained $h = \frac{5}{3}w^2$ or $h = 5 - 3w^2$.	understand the application of Calculus fully.

In Table 4, learners demonstrated confusion between surface area and volume formulas, indicating a lack of relational understanding. Many applied incorrect formulas, resulting in computational errors. For instance, in Q9.1 (see Figure 5), learners incorrectly used the volume formula instead of the required surface area formula, leading to incorrect differentiation steps.

The Diagnostic Report suggests improving instruction on measurement concepts, but this recommendation lacks feasibility, as these topics are primarily covered in earlier grades. Research underscores the importance of longitudinal curriculum alignment, where key concepts such as measurement are continuously reinforced in higher grades through interleaved practice (Carnoy & Chisholm, 2008). Instead of merely suggesting additional time allocation, the Reports should propose specific strategies such as real-world optimisation tasks that contextualise volume and surface area applications. The findings in Table 4 indicate a need for explicit contrastive teaching of formula selection in optimisation contexts, ensuring learners differentiate between volume- and surface-area-based constraints.

Synthesis of Findings from the Diagnostic Reports (2020–2024)

The cross-case analysis of the Diagnostic reports between 2020 and 2024 highlights recurring conceptual difficulties in learners understanding of Optimisation—particularly in algebraic formulation, application of differentiation, and interpretation of contexts. However, the recommendations offered by the report are often generic – failing to directly address these identified challenges. Common recommended suggestions, such as “reading for understanding” or “allocating more time,” lack pedagogical precision to improve implementation. Furthermore, the limited detail in the 2021 and 2022 reports further weakens the connection between identified learner challenges and the proposed recommended suggestions, undermining their practical value for teachers.



This analysis points to the need for more targeted, evidence-based approaches in crafting recommendations. Specifically, future reports should promote (1) instructional strategies that link graphical, symbolic, and verbal representations; (2) systematic development of algebraic reasoning through scaffolded and contextualised tasks; and (3) structured teacher professional development focused on exemplars, the use of diagnostic tools, and GeoGebra implementation. Grounding recommendations in research-based pedagogy would enhance the Diagnostic Reports' effectiveness in guiding meaningful instructional responses to persistent learner challenges.

CONCLUSION

This study highlights critical gaps in the Grade 12 Diagnostic Reports on Optimisation and their broader implications for mathematics education in South Africa. The findings reveal a misalignment between the learner challenges identified and the recommendations provided, limiting the reports' practical value for teachers, curriculum advisors, and teacher educators in improving classroom instruction and learner outcomes. To address this issue, the study advocates for a shift from broad, generalized recommendations to evidence-based teaching methodologies that promote deep conceptual learning. By integrating empirical research on effective mathematics instruction, curriculum advisors can refine the structure and content of these reports, ensuring that teachers receive actionable, high-impact strategies tailored to their instructional needs. Strengthening the connection between assessment analysis and classroom practice is essential for enhancing the utility of diagnostic reports and advancing mathematics education in South Africa through a more research-driven, pedagogically sound approach.

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FOSTERING RURAL HIGH SCHOOL MATHEMATICS TEACHERS' COMPETENCIES TO TEACH TRIGONOMETRIC FUNCTIONS USING GEOGEBRA: A THEORETICAL POSITION ABOUT THE LESSON STUDY

Bernard John Ssenyomo¹ & Duncan Mhakure²

University Of Cape Town^{1,2}

This theoretical paper seeks to investigate how the lesson study (LS) can be used as a professional development approach for rural mathematics teachers. The National Senior Certificate (NSC) diagnostic reports for 2024 revealed that trigonometric functions, was one of the poorly performed areas in paper 2. Ineffective teaching methods were cited as one of the key causes of the challenges experienced by learners in this area. This theoretical paper advocates that the use of the LS as a school-based continuous professional development (CPD) can provide a solution to the CPD of rural teachers to develop teachers' competencies to teach trigonometrical functions concepts. School-based CPD is based on the idea that mathematics teachers need real-time, and job embedded instructional practices that are unique to their school and classroom contexts. The school-based CPD that this paper proposes is conducted within the LS cycle using GeoGebra software. The LS approach is an effective CPD learning tool for teachers because it: is school-based, is centred around teachers' needs, provides opportunities for teachers to be researchers, is focused on learners' trigonometric functions learning, and draws on teachers' collective and collaborative experiences from the same school mathematics department. This paper therefore contributes significantly to practical and theoretical debates of the LS as school-based CPD approach for rural teachers. Future research should focus on evidence-based implementation of the school-based CPD of in-service mathematics teachers and its impact on the teaching and learning of mathematics in schools within the South African mathematics education context.

Keywords: Continuous professional development; lesson study; GeoGebra software; research lesson; trigonometric functions

INTRODUCTION

“Rural areas” in the context of this paper are areas nesting learners of low-income parents and/or guardians. Schools located in rural areas are primarily under-resourced. This theoretical paper aims to identify ways in which the LS through the research lesson (RL) can be used for rural high school mathematics teachers' CPD on the topic of trigonometric functions using GeoGebra software. In the context of this paper, a RL is a lesson that is jointly planned, implemented (observed) and evaluated through reflections by the LS group of teachers. This may contribute to deeper theoretical understanding and debates about school-based CPD for rural high school mathematics teachers. Despite evidence that the LS enables teachers to engage in significant CPD, the LS has not been used widely as a CPD approach in South Africa (Posthuma, 2012). In South African education departments, rural teachers attend generic CPD workshops outside of their schools, which are facilitated by external experts that include, but



not limited to subject advisors and provincial coordinators. This paper argues that school-based CPD activities using the RL as a training strategy should be at the centre of rural teachers' trigonometric functions learning and should be purposefully and coherently linked to other external CPD activities (Mhakure, 2019). Firstly, school-based CPD activities offers rural teachers with possibilities to use the minimum resources they already have at their schools to improve trigonometric functions teaching and learning (Posthuma, 2012). Secondly, school-based CPD activities are predicated on the use of teacher-generated, classroom-based, real-time formative and summative assessment data about learner trigonometric functions learning (Woodland & Mazur, 2015). This means that teachers use the most current and meaningful data that exist in their school. Thirdly, literature reveals that when teachers are engaged in CPD activities, which are embedded within their school and classroom contexts, they learn more (Mhakure, 2019). Lastly, the collective participation of teachers in the same school mathematics department in rural schools brings about greater connectedness and alignment of teachers' trigonometric functions learning goals with school policy (Chong & Kong, 2012).

The NSC diagnostic reports (2023-2024) of the Department of Basic Education (DBE) noted that trigonometric functions, were one of the poorly performed areas in paper 2 (DBE, 2024). The reports cited ineffective teaching methods as one of the key causes of the challenges experienced by learners in this topic. Research also shows that in the South African mathematics context, high-school teachers possess inadequate common content knowledge (CCK) and specialized content knowledge (SCK), and that frequent and quality CPD activities are lacking (Fleisch, 2008) thus negatively impacting rural teachers' instructional practices in trigonometric functions in particular. While the notion is true that teachers can learn from social gatherings such as workshops outside their schools (traditional teacher CPD), the reality in the South African mathematics education context is that the quality of trigonometric functions teaching in rural school remains poor. South Africa's mathematics Curriculum and Assessment Policy Statement (CAPS) allocates one-third of the whole curriculum to trigonometry in paper 2 (DBE, 2011). With the emphasis given to trigonometry in the curriculum by the DBE, matching it with what is happening at implementation level that is in the classroom, teachers need professional help to teach trigonometric functions. Keeping in mind the necessity for change in rural teachers' CPD on the trigonometric function topic using GeoGebra software, the position of this paper is that school-based CPD activities can improve the teachers' trigonometric functions instructional practices in rural schools. This paper therefore seeks to theoretically address the research question: How can the LS through the RL be used as a form of school-based CPD of rural mathematics teachers on the topic of trigonometric functions using GeoGebra software?

LITERATURE REVIEW

This paper first discusses technology use in mathematics education. This is followed by a discussion of the use of GeoGebra software as a Dynamic Geometry Software to teach mathematics. The teaching of graphs of trigonometric functions using GeoGebra software is then briefly discussed. Following this is a discussion of the LS as a theoretical framework that



can be used for the school-based CPD of mathematics teachers in rural schools. The section concludes by discussing TDS as theory underpinning the LS.

Technology use in mathematics education

Technology refers to engaging software tools, which can be used to design illustrative trigonometric functions to facilitate learners' conceptual understanding (Chizwina & Mhakure, 2018). The ubiquity of technology highlights technology tools' role in education, particularly in the teaching and learning of trigonometric functions in rural schools (Kearney & Maher, 2013). Whilst there is evidence on the ubiquity of using technology, research shows that the process of embedding it in classrooms is slow and complex (Koh, 2019). Koh (2019) asserts that some practicing teachers are digital immigrants – trained during the time when the technology for instruction was not well integrated into teacher education by either practice or policy. This problem does not only affect experienced teachers alone, but also novice teachers who may be considered as digital natives. Kafyulilo and Fisser (2019) also contend that even in the current initial teacher education, opportunities to training for effective technology integration such as exposure to collaborative design activities are limited. As such mathematics teacher CPD activities – using GeoGebra software and the LS – play a vital role not only in updating their GeoGebra software integration knowledge, but also in enabling digital immigrants to cope up with the current demands of transformation in education (Njiku et al., 2021).

GeoGebra as a Dynamic Geometry Software to teach mathematics

Dynamic geometry is a term coined in response to the new software packages that include, *inter alia*, Geometer's Sketchpad, Cabri Geometry, and GeoGebra, which use motion and change in digital settings (Saralar, 2016). These dynamic geometry softwares [DGS] are powerful technological tools for teaching mathematics topics, particularly trigonometric functions. This position paper uses GeoGebra software in the CPD of rural teachers on the trigonometric function topic. Firstly, GeoGebra is an open source and user-friendly technological tool that requires no licensing and provides the opportunity for both teachers and learners to use it in the classes and at home at no cost (Rahman et al., 2017). Secondly, the software does not require any user to be a master's in computer literacy (Preiner, 2008). Lastly, GeoGebra software is specifically designed for teaching and learning purposes (Hohenwarter et al., 2008).

GeoGebra software serves a myriad of significant roles in mathematics education that include, but not limited to: Firstly, GeoGebra software provides for active trigonometric functions teaching and learning in mathematics classes (Denbel, 2015). Teachers can perform dynamic demonstrations such as dragging trigonometric graphs using GeoGebra software sliders, thus allowing for making conjectures. Secondly, Majerek (2014) asserts that GeoGebra software provides opportunities to learners of distinct mathematical skills and levels to better understand trigonometric functions properties and advances them to do mathematics in new attractive ways. Lastly, Bu et al. (2012) recount that GeoGebra has the potential to create a conducive teaching and learning milieu in which learners can explore and make meanings of



trigonometric functions through multiple representations and dynamic observations that are not possible with pen and paper.

Despite GeoGebra software's pertinence, results from Stols et al.'s (2015) study showed that the use of the software in the South African mathematics classroom is very limited. Stols et al. (2015) noted from participating teachers' own perceptions that their mathematical content knowledge and skills required to apply what they know in the classroom were limited, and as a result, these teachers were hesitant to employ GeoGebra software in their teaching. GeoGebra software has however been found to provide a powerful platform for mathematics teachers to develop content rich pedagogical strategies for proficient trigonometric functions instruction (Bu et al., 2012). This paper therefore gives an alternative to teachers in under-resourced schools to use GeoGebra software, an innovative mathematics software, in their trigonometric function instruction.

Teaching trigonometric functions using GeoGebra software

Trigonometry plays a crucial role in studying mathematics and its applications. For example, trigonometric functions link algebraic, geometric, and graphical reasoning, which can be manipulated by using GeoGebra software that can serve as a vital precursor towards the understanding of pre-calculus and calculus (Bedada & Machab, 2022). Despite the importance of the trigonometry topic, mathematics education literature reveals that trigonometric function representations and an interpretation of graphs of the functions are major challenges to many learners all over the world (Bedada & Machab, 2022; DBE, 2023, 2024; Kepceoglu & Yavuz, 2016). The use of technology tools – GeoGebra software in this case – to teach graphs of trigonometric functions and the related concepts has been found to positively affect learners' achievement and learning than the traditional chalk and talk method (Zengin, Furkan & Kutluca, 2012).

The NSC diagnostic reports (2023-2024) of the DBE in South Africa noted that trigonometry, in particular trigonometric functions, was one of the poorly performed areas in paper 2 (DBE, 2024). Ineffective teaching methods were cited by the report as one of the key causes of the challenges experienced by learners in graphs of trigonometric functions, in rural schools in particular. The average performance of the question on trigonometric functions was 35%, one of the poorly performed questions in paper 2. South Africa's mathematics CAPS document allocates 50 marks to trigonometry in paper 2 (DBE, 2011). With the emphasis given to trigonometry in the curriculum by the DBE, matching it with what is happening at implementation level that is in the classroom, teachers need professional support to teach the trigonometric functions concepts. Keeping in mind the necessity for change in the CPD of rural teachers on the trigonometric function topic using GeoGebra software, the position of this paper is that the LS as a school-based CPD approach can improve rural teachers' instructional practices in this topic.



The lesson study as an approach for professional development

This theoretical paper seeks to address ways in which rural high school mathematics teachers seek to improve the quality of trigonometric functions teaching and learning through engaging in CPD activities. To achieve this CPD, the paper proposes to use the LS as an approach. As a form of CPD for mathematics teachers, Lewis (2009) defines LS as an approach:

... in which teachers work together to: formulate goals for learner learning and long-term development; collaboratively plan a 'research lesson' designed to bring to life these goals; conduct the lesson in a classroom, with a team member teaching and others gathering evidence on learner learning and development; and discuss the evidence gathered during the lesson, using it to improve the lesson, the unit, and instruction more generally (p. 95)

Within the context of this paper, the LS approach as a form of school-based rural teacher CPD is used to improve the quality of trigonometric functions teaching and learning using GeoGebra software. As a form of CPD, firstly, the LS enables teachers to draw on and incorporate all elements of their mathematical knowledge for teaching (MKT) (Ball et al., 2008) during planning, conducting, and reflecting on the trigonometric function RL (Ni Shuilleabhain & Clivaz, 2018) thus developing their MKT. MKT is divided into two main sub-categories: subject matter knowledge (SMK) and pedagogical content knowledge (PCK). In this paper, the researcher does not intend to describe SMK and PCK in detail. SMK is subdivided into three categories. Common content knowledge (CCK) refers to mathematical knowledge not particular to mathematics (trigonometric function) teaching; horizon content knowledge (HCK) deals with the connections of topics in the broader mathematics environment as a discipline; and specialized content knowledge (SCK) is essential mathematical knowledge required to unpack learners' trigonometric functions learning (Mhakure, Jaftha & Manzini, 2019). This paper focuses on SCK for the CPD of rural mathematics teachers because SCK empowers teachers with the ability to: develop alternative solutions; connect concepts of trigonometric functions to prior learning; link trigonometric functions representations to underlying ideas; respond to learners' unexpected why questions; analyse the mathematics behind learners' errors, and solutions to trigonometric function tasks; and make sense of unusual learners' procedures. Furthermore, SCK empowers teachers with the ability to justify, understand, and assess trigonometric functions productions (Ball et al., 2008). Despite the benefits of SCK, it is noteworthy that in the South African mathematics context, high school teachers possess inadequate CCK and SCK, and that frequent and quality CPD activities are lacking (Fleisch, 2008). This paper posits that school-based CPD activities using the LS can support the CPD of rural teachers.

Similarly, PCK is subdivided into three categories. Knowledge of content and students [KCS] refers to teachers' knowledge of their individual learners; knowledge of content and teaching [KCT] entails designing and sequencing trigonometric functions content and instruction; and knowledge of content and curriculum [KCC] refers to an awareness of the trigonometric



functions content in the curriculum and the correct use of GeoGebra software (Mhakure et al., 2019).

Secondly, the LS provides teachers with the opportunity to contextualize representations of their classroom activities, while also making their implicit trigonometric functions knowledge and practices explicit through collaborative conversations (Fujii, 2016). Participating in LS provides teachers with the opportunity to explain, share and develop their trigonometric functions knowledge, where learner learning is at the centre of their activities. Lastly, the LS structure enables teachers to engage in significant CPD activities with a minimum of resources and offers possibilities for rural teachers to use the resources they already have to improve trigonometric functions teaching and learning (Posthuma, 2012). The question is: What is the rationale behind focusing on LS as a CPD approach in this paper? As alluded to earlier, this paper is about the CPD of mathematics teachers in under-resourced schools. This means that teaching and learning contexts in these schools are different from those of other well-resourced schools. Each under resourced school is also unique. Therefore, the use of the LS as a CPD learning approach offers opportunities that do not usually form part of traditional teacher CPD activities, such as the top-down approach where experts take the lead in CPD.

The opportunities include, but are not limited to: Firstly, CPD activities using GeoGebra software and the LS approach, are embedded within the teachers' school and classroom contexts, thereby giving them an opportunity to see trigonometric functions teaching and learning as it happens in real-time in their school contexts (Mhakure, 2019). Secondly, school-based CPD are predicated on the use of teacher-generated, classroom-based, real-time formative and summative assessment data about learner trigonometric functions learning (Woodland & Mazur, 2015). This means that teachers use the most current and meaningful data that exist in their school. Lastly, the collective participation of teachers in the same school mathematics department brings about greater connectedness and alignment of teachers' trigonometric functions learning goals with school policy. It aids to shape a shared culture in which teachers can develop a common understanding of the trigonometric function instruction goals, teaching methods, learning problems, and solutions in the context of their school (Chong & Kong, 2012).

With school-based CPD, the school at an organizational level needs to create a conducive teacher professional learning climate by providing the structures and processes that support instructional change (Woodland & Mazur, 2015). For example, reserving space and time away from curriculum hours to reinforce teacher skills development in the teaching of trigonometric functions. Additionally, school administrators need to avail the necessary resources to support teachers' CPD activities and do away with structural obstacles that hinder processes of change (Erickson et al., 2005).

The theory of didactical situation as a theory underpinning the LS

The LS activities are guided by the constructs of the theory of didactical situations (TDS) (Brousseau, 1997). The TDS in Mathematics, whose foundations were established by Brousseau (1997), models learners' learning, teachers' (trigonometric functions) knowledge,



and its teaching in terms of didactic situations (DS), which is a dynamic triad unfolding in time consisting of an interaction between teachers, students, and a didactic milieu (Clivaz, 2018). The theory offers a quite complex set of constructs that are widely used to describe the teaching and learning of trigonometric functions. These concepts are described in the next paragraphs.

In the TDS, learning is viewed as learners' adaptation of their knowledge to the milieu – solving the trigonometric function problem using the resources it offers (Perrin-Glorian 2008). The function of a milieu depends on the situation and its traits (Hersant & Perrin-Glorian, 2005). A situation may in fact either be didactical or adidactical. According to the TDS, learners' learning of a specific piece of trigonometric function knowledge intended to be taught is modelled with the notion of DS in which interactions occur between three elements: students, teacher, and milieu. Significant learning occurs when the learner interacts with the milieu – trigonometric function problem, as if the teacher is absent for the learner (adidactical situation) (Miyakawa & Winslow, 2009). Adidactical situations are not simply autonomous work by learners on a trigonometric function problem, but rather such situations are devolved by the teacher, in DS called situations of devolution (Bahn & Winsl w, 2019). Devolution entails putting the learners into a relationship with a milieu from which the teacher is able to exclude herself or himself, at least partially (Brousseau and Warfield, 1999). Therefore, adidactical situations are part of the DS. Brousseau (1997) distinguishes four phases of the DS: action, formulation, validation and institutionalization. Learners directly act (situation of action) on the trigonometric function problem; articulate the trigonometric function ideas that have been developing implicitly (situation of formulation); and need to convince themselves or each other of the validity of the ideas they have developed (situation of validation). The final situation, institutionalization, marks the return of the teacher in the DS (Clivaz, 2015a).

An event of teaching such as a RL often comprises several situations (discussed above), including both adidactical and DS (Clivaz, 2018). The RL is a DS, and the milieu for the teachers' learning in LS (Winsl w et al., 2018). Therefore, a GeoGebra trigonometric function RL is at the centre of the LS process. With LS often being concerned with the finer details of the GeoGebra trigonometric function RL, which a group of teachers develop, one may possibly utilize the TDS to dissect the development of this RL, which the teachers plan, realize and observe together (Clivaz, 2018). In other words, the TDS provides tools to compare situations – to analyse how the details of one situation are conditioned by, and condition other situations.

A first discussion of how ideas from the TDS may be brought to bear on LS was given by Miyakawa and Winsl w (2009) – noting that elements of the TDS “could fertilize teachers' practice in the LS” (p. 217). As alluded to earlier, the TDS focuses on the dynamic interplay between a milieu and learners' adaptation to the milieu as it unfolds in time. This allows for a fine-grained analysis of how both learners (in the GeoGebra trigonometric function RL) and the teachers (carrying out this RL, observing learners' learning) develop their knowledge in various phases of LS (Winsl w et al., 2018).



Teacher training on the use of GeoGebra software and the LS

This section presents the two introductory training sessions that need to be conducted per school, each lasting two hours. In the first session, teachers are introduced to GeoGebra software with the aim of empowering them on how to use the software in trigonometric functions teaching and learning. In the second training, the teachers are introduced to the LS because the school-based CPD that the teachers need to undergo is carried out within the framework of the LS cycle. Hence the teachers need to understand the phases of the LS as they are expected to operationalize them using the trigonometric function RL.

Operationalisation of the LS a school based CPD

The LS approach that this paper proposes is implemented on a RL using GeoGebra software on the trigonometric function topic. The LS framework has four phases: viz. identification of (trigonometric functions) objectives, planning, implementing, and reflection. Firstly, in the identification of objectives phase, a group of teachers meet formally in their respective schools to discuss the specific learners' needs, and the trigonometric functions goals (Sierra et al., 2023). In addition, the teachers identify a research question they want to investigate to learn and deepen their knowledge about the learners' learning on the trigonometric function topic (Dudley, 2014).

Secondly, when planning, teachers identify and design a trigonometric function RL. Using the selected goals as a guide, the teachers, with the aid of GeoGebra software develop a trigonometric function RL plan which is informed by the need to create learning opportunities for the learners. During the planning phase, the teachers learn how to use GeoGebra software to: teach trigonometric functions; assess learners' conceptual understanding of trigonometric functions concepts; facilitate the development of learners' conceptual knowledge of trigonometric functions thinking skills; and create new instructional approaches and a classroom environment that promotes and supports the learning of trigonometric functions.

In addition, part of this phase's discussions looks at how to anticipate and respond to learners' conceptual challenges in trigonometric functions using so-called 'case learners'. Case learners in the context of this paper are three learners known to the teachers, around whom the trigonometric function RL is developed using GeoGebra software. In other words, the planning stage looks at specific identified learners in a trigonometric function RL, one in each of the three categories: lower, average, and higher achievers, and a trigonometric function RL with the aid of GeoGebra software is designed around their needs. So, the idea here is that, as teachers plan this RL, they should think about how each of these learners would respond to each of the stages of the RL – essentially, the teachers must predict the trigonometric function learning behaviours of each of the three learners (Dudley, 2014) and by extension the rest of the learners in the group. The strict planning discipline required of a trigonometric function RL has the potential to reveal that the predicted trigonometric function learning behaviours of learners can differ from the behaviours observed by teachers during the RL. Therefore, in the absence of the trigonometric function RL, some of these variations between predicted and observed may not be noticed (Mhakure, 2019).



Thirdly, the planning stage is followed by an implementation stage, in which the trigonometric function RL plan collaboratively developed in Phase 2 is voluntarily taught by one of the teachers, while the other teachers observe. Observing learners' learning is at the heart of the LS; it provides a "detailed picture of the effect of the RL on learners' trigonometric functions thinking and cognition" (Dudley, 2015, p. 10). The teachers create a learning environment where learners interact and share their thoughts with other learners when solving trigonometric function tasks with the aid of GeoGebra software, in addition to initiating whole class discussions of learners' work and aiding them to build nexuses between trigonometric function concepts. The RL activities are both audio – and video-recorded. Samples of learners' trigonometric functions work are also collected as evidence for their ways of thinking. The observing teachers keep a keen interest on the three 'case learners' interactions, and their performance during the RL is noted (Mhakure, 2019).

Fourthly, the implementation stage is followed by the *reflection phase*. During this phase, the mathematics teachers hold a post-RL meeting, where data collected on the use of GeoGebra software on the trigonometric function topic and notes made during the previous three phases – setting goals, planning and implementation – are analysed. Additionally, the phase includes discussions relevant to the trigonometric function RL itself, such as: classroom discourses, learner and teacher learning moments and/or strategies, and meeting of trigonometric function goals. Discussions around the three case learners take place, looking specifically at whether the predictions about their trigonometric function interactions were correct.

DISCUSSION AND CONCLUSION

This theoretical paper supports and acknowledges the LS role in the CPD learning of rural high school mathematics teachers in South Africa on the topic of trigonometric functions using GeoGebra software. The paper sought to illustrate, through using examples on the trigonometric functions, how the LS can be used as a form of school-based CPD to foster in-service rural mathematics teachers' MKT when GeoGebra software is adopted as a technological tool in the learning and teaching of the topic (Mhakure, 2019).

School-based CPD are predicated on the use of teacher-generated, classroom-based, real-time formative and summative assessment data about learner learning (Woodland & Mazur, 2015). This means that teachers use the most current and meaningful data that exist in their school and classroom context. Literature has also revealed that teachers learn more when they are engaged in CPD activities, which are embedded within their school and classroom contexts (Erickson et al., 2005; Mhakure, 2019). Additionally, the collective participation of teachers in the same school mathematics department in rural schools brings about greater connectedness and alignment of teachers' trigonometric functions learning goals with school policy (Chong & Kong, 2012; Woodland & Mazur, 2015). This paper makes significant contributions to the body of knowledge about the LS as a school-based CPD approach for in-service mathematics teachers irrespective of where their schools are located. Future research should focus on evidence-based implementation of the school-based CPD of in-service mathematics and its



impact to the teaching and learning of mathematics in schools within the context of mathematics education in South Africa.

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